# REDUCTION OF NON-LINEARITIES IN THE DA $\Phi$ NE MAIN RINGS WIGGLERS* 

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#### Abstract

\section*{Abstract}

The wigglers of the DA $\Phi$ NE main rings have been the main source of non-linearities for the beam dynamics in the collider. This paper describes a method to reduce the integrated odd multipoles (the even ones tend to vanish for the periodicity of the magnet) by alternatively displacing the magnetic axis of the poles to compensate the integrated odd multipoles in each half-period of the wiggler. In order to check the effectiveness of this approach, tracking studies have been performed. Tracking results have been used to tune the MAD model of the wiggler.


## INTRODUCTION

Since the beginning of $D A \Phi N E$ operation the eight normal conducting wigglers, used to reduce the damping time of the rings, have been an important source of non linearity for the lattice of the collider, because of the field rolloff combined with the large excursion of the beam trajectory from the axis (about 1.3 cm ). This was experimentally demonstrated in fall 2000, when tune shift measurements, performed by creating closed orbit bumps, evidenced a large integrated octupole in these magnets [1].

This non-linearity has been reduced by a factor 2.5 by improving the transverse field uniformity by means of pole shims. To further reduce the non-linearities another more drastic approach has been studied [2].

After briefly recalling the method, a more convenient optimization, which is easier to be implemented and allows to produce higher fields, is presented.

## METHOD

Let $\bar{z}$ be a position along the longitudinal axis of the magnet, $y$ the vertical, $x$ and $\tilde{x}$ the horizontal transverse coordinates with respect to $\bar{z}$ and to the beam trajectory respectively. The magnetic field seen by a particle passing in the magnet can be expanded around the beam trajectory $\left(x_{T R}, 0, \bar{z}\right)$ in the mid-plane as:

$$
\begin{equation*}
B_{y}(\tilde{x}, \bar{z}) \equiv \sum_{n=0}^{\infty}{b_{n}}^{T} \tilde{x}^{n} \tag{1}
\end{equation*}
$$

where the coefficients $b_{n}{ }^{T}$, defined as:

$$
\begin{equation*}
\left.b_{n}^{T} \equiv \frac{1}{n!} \frac{\partial^{n} B_{y}(\tilde{x}, \bar{z})}{\partial \tilde{x}^{n}}\right|_{\tilde{x}=0} \tag{2}
\end{equation*}
$$

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correspond to the multipoles with respect to the beam trajectory $\left(b_{0}{ }^{T}\right.$ to the dipole, $b_{1}{ }^{T}$ to the quadrupole, $\left.\cdots\right)$.

It can be shown that each multipole with respect to the beam trajectory can be written as a function of those calculated with respect to the axis of the wiggler, $b_{k}{ }^{A}$, as:

$$
\begin{equation*}
b_{n}^{T}=\sum_{k=n}^{\infty} \frac{k!}{n!(k-n)!} b_{k}^{A} x_{T R}^{k-n} \tag{3}
\end{equation*}
$$

The effects of the non-linearities on the beam dynamics depend on the integral of these coefficients, $I_{n}$, defined as:

$$
\begin{equation*}
I_{n} \equiv \int_{\text {Magnet }} b_{n}^{T} d s \quad n=2,3, \ldots \tag{4}
\end{equation*}
$$

where $s$ is the coordinate along the reference trajectory.
Because of the alternating sign of the current from a pole to the next one and the left-right symmetry of the magnet, the integral of the even terms $b_{2 j}^{T}$ tends to cancel in each period. On the contrary the integral of the odd terms $b_{2 j+1}{ }^{T}$ does not, because in Eq. (3) both the even terms with respect to the axis and the powers of $x_{T R}$ change sign from a pole to the next one, so that their contributions add along the magnet.

In particular from these considerations and substituting Eq. (3) in Eq. (4) the only integrals to be reduced have the form:

$$
\begin{align*}
I_{2 j+1}=\int & \left(c_{2 j+2} b_{2 j+2}{ }^{A} x_{T R}+\right. \\
& \left.+c_{2 j+4} b_{2 j+4}{ }^{A} x_{T R}^{3}+\cdots\right) d s \tag{5}
\end{align*}
$$

The method consists in alternatively displacing the magnetic axis of each pole so that the odd powers of $x_{T R}$ change sign inside each half-period of the magnet. In this way the contributions to $I_{2 j+1}$ from the regions inside the poles tend to be compensated by those coming from the regions between the poles.

## OPTIMIZATION

Several methods to displace the magnetic axis of the poles can be envisaged:

- Apply pole shims;
- Cut the poles;
- Shift the poles.

The first option has been rejected because the solution would strongly depend on the current and on the specific properties of the iron used in the simulation. The second possibility has been already explored and described in detail in [2]. In this case the pole width is reduced, so that pole
shims need to be added to reduce the strong dependence of the integrated multipoles with respect to the beam-wiggler misalignment. The new proposal of shifting the poles consists instead in displacing the axis of each pole without reducing their width, so that the shims are no longer necessary and, as a consequence, the wiggler gap can be smaller.

Until 2006 the wigglers have been powered at 693 A, and a full map at this current has been measured. Recently the wigglers work at 550 A , and the corresponding map is not yet available. The field maps of both the shifted and unshifted poles configurations have been simulated to compare the results of the optimization.

## The shifted poles configuration

The main operational parameters of the wigglers are shown in table 1.

Table 1: Main wigglers parameters.

|  | Full pole | Half pole |
| :--- | :---: | :---: |
| Number of poles | 5 | 2 |
| Nominal current (A) | 550 | 390 |
| Peak field (T) | 1.70 | 1.43 |
| Gap (cm) | 3.7 |  |
| Pole width (cm) | 14 |  |
| Beams particles | electrons and positrons |  |
| Beam energy (MeV) | 510 |  |

To move the magnetic axis each pole has to be horizontally shifted, as shown in Fig. 1 in the $x-y$ plane and in Fig. 2 in the $z-x$ one.


Figure 1: Drawing of a pole in the $x-y$ plane (a) before and (b) after the modification. The case refers to a pole where the particle is on the right of the vertical symmetry axis (geometric axis). Dimensions are in mm .

To find the displacement of the magnetic axis which makes the integrated octupole vanish, several configurations of the wiggler [3] with different displacements have 01 Circular Colliders


Figure 2: Section of the modified wiggler in the $z-x$ plane (solid lines). The configuration of the poles before the modification is also shown (dashed lines).
been simulated to determine $I_{3}$ as a function of the $x$-shift, as shown in Fig. 3.


Figure 3: Integrated octupole as a function of the magnetic axis displacement. The straight line which fits the points and the value corresponding to the optimal position of the magnetic axis are also indicated.

The values of the multipoles integrated over the entire wiggler after the $\pm 7.3 \mathrm{~mm}$ shift (shifted poles) compared to those of the starting configuration (aligned poles) are reported in table 2.

Table 2: Integrated multipoles in the aligned poles and in the shifted poles configuration. The units of $I_{j}$ are $\mathrm{Tm}^{1-j}$.

|  | $I_{0}$ | $I_{1}$ | $I_{2}$ | $I_{3}$ | $I_{4}$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Aligned poles | 0.00 | 2.58 | -1.10 | 279.61 | 314.4 |
| Shifted poles | 0.00 | 2.10 | -1.38 | 0.07 | -62.9 |

The behavior of $b_{3}{ }^{T}$ as a function of $z$ in the whole wiggler is shown in Fig. 4.

To verify the impact of the modification on the beam trajectory inside the wiggler, test particles with different initial conditions have been tracked through the simulated field maps. The position and the angle at the end of the wiggler are shown in Fig. 5 and in Fig. 6 as a function of the initial position for both the original and the modified configurations.

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Figure 4: $b_{3}{ }^{T}$ as a function of $z$ in the configuration of poles shifted by $\pm 7.3 \mathrm{~mm}$.


Figure 5: $x$ exit as a function of the entrance $x$ with respect to the reference trajectory.


Figure 6: Exit angle as a function of the entrance $x$ with respect to the reference trajectory.

## The MAD model

A MAD model of the magnet has been written to be inserted in the model of the whole ring for beam dynamics calculations.

Each half-period of the wiggler has been split in bending magnets (one for the terminal poles and two for the full poles), whose bending angle and length have been determined to reproduce the reference trajectory determined by Tosca, thin lenses (two for the terminal poles and three for the full poles), to take in account quadrupole and higher order terms of the field expansion, and drift spaces.
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The tracking curves obtained from the MAD model have been compared to those obtained by tracking with Tosca. The results are shown in Fig. 7 and Fig. 8 respectively.


Figure 7: $x$ exit as a function of the $x$ entrance with respect to the reference trajectory. The difference is also shown.


Figure 8: Exit angle as a function of the $x$ entrance with respect to the reference trajectory. The difference is also shown.

The agreement of these curves to those obtained from the Tosca tracking using the coefficients from the expansion of the fit is satisfactory.

## CONCLUSIONS

A method to reduce the non-linearities in the wigglers of $D A \Phi N E$ has been presented. The simulation indicates that the shifted poles method could strongly reduce the nonlinearities in these wigglers. The modification should be implemented in DA $\Phi$ NE to confirm the results of the simulations, and, if successful, the same method could be used in future design of more ideal wigglers.

## REFERENCES

[1] A. Battisti et al., The modified wiggler of the DAФNE main rings, DA $\Phi$ NE Technical Note MM-34 (2004).
[2] S. Bettoni, Reduction of integrated odd multipoles in periodic magnets, Phys. Rev. ST-AB, 10, 42401 (2007).
[3] The models have been implemented in Opera 3D, Vector Fields Analysis, version 11.009.


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