COHERENT CHERENKOV RADIATION AS A TEMPORAL DIAGNOSTIC FOR MICROBUNCHED BEAMS

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Abstract

Cherenkov radiation of a relativistic e-beam traversing a thin section of aerogel is analized, putting the stress on the coherent contribution due to the intra-beam, transverse and longitudinal structure. The use of this tool as a temporal diagnostic for micro-bunched beams makes possible to improve the amount of collected power at the microbunching frequency several orders of magnitude more respect to the uncoherent Cherenkov contribution. The non-idealities of a real beam are taken in account, and some techniques aimed on enhancing the coherent part of radiation are proposed and analized analitically.

COHERENT LONGITUDINAL DIAGNOSTIC

The use of e-beam emitted radiation (e.g. SR, TR, CR) as a tool for complete reconstruction of the bunch structure, must take in consideration the coherent contribution of such a radiation, no matter which is the physical source generating the phenomenon. More in detail, assuming a process in which every particle of the beam radiates in the same way [1], just an intra-beam particle displacement (i.e. delay) will affect the total coherent contribution.

In such a way it is possible to write a general form of such a radiation, indipendently of the specific physical process that generates it. Let's assume a single particle angular spectrum $S(\boldsymbol{k})$, where $|\boldsymbol{k}|$ is the vacuum wave vector, then the whole bunch far field spectral response [Desy] will be:

$$T(\boldsymbol{k}) = S(\boldsymbol{k})(N + N(N-1)F(\boldsymbol{k}))$$

with N is the number of particles in the bunch, and $F(\boldsymbol{k})$ the so-called form factor , i.e. 3-D Fourier transform of the bunch particle distribution f. The second term in the spectrum expression, the one to deal with, is the radiation coherent contribution with its characteristic N^2 scaling.

The coherent contribution has extensively been used for longitudinal diagnostic (i.e. bunch lenght measurement), and, in some case, for the whole beam reconstruction purpose, for example employing transition radiation [2][3]. Hence, the total beam spectral response will experience both the contribution of the single particle emission and the collective bunch effect.

Cherenkov Radiation Coherent Diagnostic

The advantage in the use of Cherenkov radiation would be given by the wide and flat spectral response (cut-off wavelenghts on the order of the electron classical radius)

$$\frac{\partial N_{ph}}{\partial k \partial \vartheta} = L_d \alpha \left(1 - \frac{1}{\beta^2 n^2}\right) \delta(\vartheta - \vartheta_c)$$

in terms of number of photons per unit frequency, where α is the fine structure constant, n the medium index of refraction, β the particle velocity in speed of light units θ_c the Cherenkov angle, and L_d the lenght of the radiator. Defining the refracting index of the Cherenkov radiator as n=1+Δ, where delta ranges inside the interval 0.006-1.13 [4] the Cherenkov angle can be easily expressed as

$$\vartheta_c = \sqrt{2\Delta}$$

Hence it is natural to use this property to explore high frequency components (very fine bunch details), such as the microbunching deriving from the FEL and IFEL processes (Fig. 1).

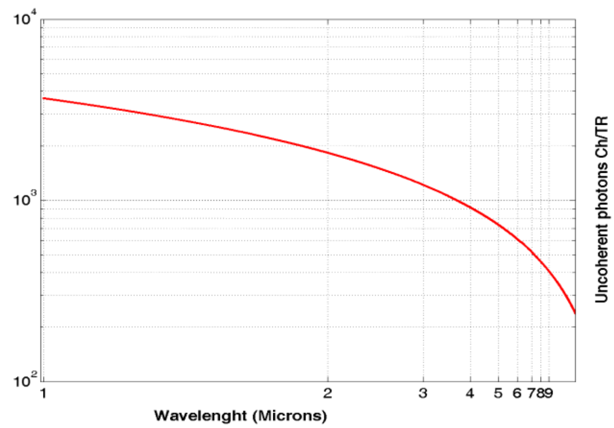

Figure 1: Uncoherent Cherenkov over uncoherent transition radiation photons vs. wavelenght

Form Factor Influence

Restricting the present framework to the case of a microbunched beam at the fundamental wavelenght λ_r, and assuming that the electrons distribution can be splitted in the product of longitudinal and transverse part, the complete form factor expression can be written as: $F(k)=F_t(k_t)F_l(k_l)$, with k_t, k_l respectively $k\sin\theta$ $k\cos\theta$. As a first example, assuming both a gaussian radial (transverse) and longitudinal distribution (whose standard deviation are, respectively $\sigma_x=\sigma_y=\sigma_t$ and σ_l), the form factors become:

$$F_t(k_t) = \exp[-(\sigma_t k_t)^2]\,,$$

$$F_t(k_l) \approx \sum_{-\infty}^{+\infty} |A_n|^2 \exp[-\sigma_l^2 (k_l - n_h k_r)]^2$$

Here, the longitudinal part is expressed as a superposition of the different microbunching harmonics with weight A_n on the n-th component. The trivial integration over emission angles gives for the coherent contribution:

$$\frac{\partial N_{ph}}{\partial k} \approx L_d N_b^2 \alpha (1-\frac{1}{\beta^2 n^2}) \exp[-(\sigma_t k \sin \vartheta_c)^2]$$
$$\sum_{-\infty}^{+\infty} |A_n|^2 \exp[-\sigma_l^2 (k \cos \vartheta_c - n_h k_r)^2]$$

It is worth noting that the main limitation in generating high microbunching frequencies is given by the transverse term strong suppression. Moreover, it can be defined a "coherence angle": for a given wavelength this is the emission angle in which the transverse part "cuts" half of the photons emitted. Beyond this limit there's very strong suppression of coherence. Looking at the limitation for the longitudinal (microbunching) wavelenght, in the gaussian case above discussed, it is, for small values of the Cherenkov angle

$$\tan(\vartheta_{cohr}) \approx \vartheta_{cohr} \le \frac{\sqrt{2}}{\sigma_t k_r}$$

This result shows how the transverse part influence grows for bigger angles. On the other side, when the emission is strongly peaked on a small angle, the transverse particles don't influence each other. One way to "restore" coherence at a given wavelenght and a given Cherenkov angle is to transversely squeeze the beam, and make it small, compared to such a wavelenght so that the transverse particle displacementes contributions can add inside a coherent lenght.

PRACTICAL APPLICATIONS

It can be useful to compare the difference in the employment of TR and CR for some experimental situations.

Considering a typical setting for the UCLA Neptune accelerator: γ=28, N_b=6E9 (i.e. Q=1nC), Δ=0.008, L_d=2.5 mm, σ_t=50 μm (that is a well focused beam), σ_l=500 μm and looking at the contribution of the single n-th microbunching harmonic, Cherenkov coherent photons are

$$N_{ph}^{CH} \approx \sqrt{\pi} L_d N_b^2 \frac{\alpha}{\sigma_l} |A_n|^2 (1-\frac{1}{\beta^2 n^2}) \exp[-(n_h k_r \sigma_t \vartheta_c)^2]$$

transition radiation ones, in the same case are:

$$N_{ph}^{TR} \approx \frac{\alpha}{2\sigma_l \sqrt{\pi} n_h k_r} N_b^2 |A_n|^2 [\frac{\gamma}{n_h k_r \sigma_t}]^4$$

It must be noted, anyway, that for such a situation, θ_c=7.2 deg, while the peak of TR is for θ_{TR} =2.04 deg, that explains a stronger suppression. In Fig. 2 the two curves are represented in a wavelenght range of 0-16 μm for a microbunching factor $|A_n|^2$ =1. It can be seen that TR dominates for the wavelenght range (0.1-15 μm), even though the decay of the Cherenkov due to the transverse exponential factor is quite abrupt. This suggests that the employment of different transverse shapes could overcome this drawback, extending the cut-off frequency.

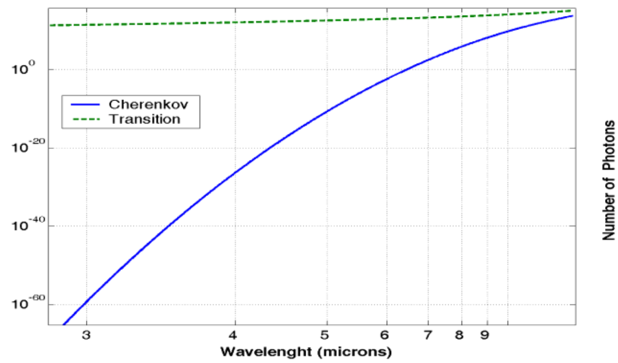

Figure 2: Gaussian beam coherent Cherenkov and TR contribution

Looking at a hard edge uniform distribution beam on the transverse dimension, with the same gaussian distribution on the longitudinal one from the previous example, the form factor changes. Being the electron radial distribution

$$f(\rho) = \frac{1}{\pi \sigma_t^2} rect_{\sigma_t}(\rho)$$

the form factor becomes

$$F(\boldsymbol{k}) = 4N_b^2 [\frac{J_1(\sigma_t k_t)}{\sigma_t k_t}]^2 \sum_{-\infty}^{+\infty} |A_n|^2 \exp[-\sigma_l^2 (k_l - nk_r)^2]$$

with J_1 first order, first type Bessel function.
Once again, taking in consideration the number of photons inside a (narrow) longitudinal microbunching peak:

$$N_{ph} \approx 4\sqrt{\pi} L_d N_b^2 \frac{\alpha}{\sigma_l} |A_n|^2 (1-\frac{1}{\beta^2 n^2}) [\frac{J_1(\sigma_t n_h k_r \vartheta_c)}{\sigma_t n_h k_r \vartheta_c}]^2$$

The improvement given by the form factor is showed in Fig. 3, where the two distributions are compared.

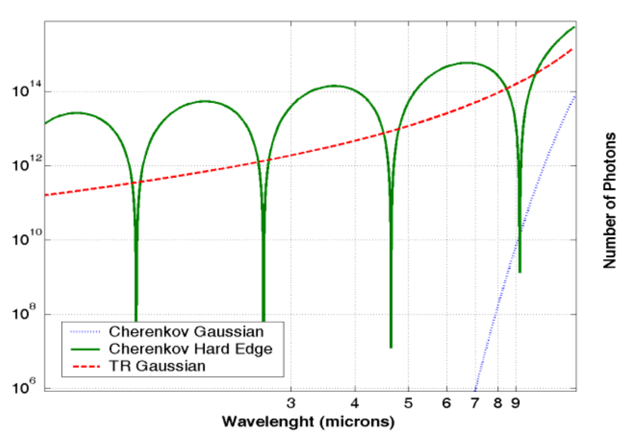

Figure 3: Hard edge/Gaussian beam Cherenkov coherent contributions. Gaussian TR is also showed

BEAM CUT EFFECT

Improvement of the bunch response at short wavelenghts could be achieved through transverse cutting of the beam. Modelling the trasverse cut with a delta functions comb gives an idea of the spectral response extension given by acting a cut. Anyway, since the delta cut is not realistic, the spectral extension would be infinite, that is obviously a not physical result. Let's assume a periodic trasverse modulation of the beam (e.g. Grid of wires). The only assumption is for the cutting period (λ_0, λ_1) to be shorter than the beam dimension. In such a way the distribution along the cartesian coordinates will be

$$f_t(x) = f(x)\sum_{-\infty}^{+\infty} A_L \exp(jLk_0 x)$$

$$g_t(y) = g(y)\sum_{-\infty}^{+\infty} C_M \exp(jMk_1 y)$$

using the Fourier expansion for the periodic modulation. Still the longitudinal contribution will have the same form *h(z)* very similar to the transverse one, but given by the microbunching components. Taking in consideration the gaussian case (but it would be valid anyway) and using the previous assumption on the cut period:

$$\lambda_0 \le 2\sigma_x$$

it leads to

$$k_0 = \frac{2\pi}{\lambda_0} \ge \frac{\pi}{\sigma_x} \ge \frac{2}{\sigma_x}$$

This gives the possibility to write the expressions of the trasverse spatial spectrum dropping out the cross product terms:

$$\left|F(k_x)\right|^2 = \left|\sum_{-\infty}^{+\infty} A_L \tilde{f}(k_x - Lk_0)\right|^2 = \sum_{L=-\infty}^{+\infty} \left|A_L\right|^2 \left|\tilde{f}(k_x - Lk_0)\right|^2$$

Assuming once again a full 3D gaussian distribution, as beam dimensions will be larger, in order to allow a physical easy way to cut. It will be σ_x, σ_y = 250µm for a squared train of cut of period λ_0, λ_1 equal to 500µm period and cut width Δ equal to 50µm so to get

$$A_L = \frac{\Delta}{\lambda_0} \sin c\left(\frac{\Delta}{\lambda_0} L\right)$$

with longitudinal dimension of the beam still 500µm. Assuming the microbunching spectral line very sharp respect to trasverse spectral distribution variation, the number of photons over this line is

$$\frac{\partial N_{ph}}{\partial \phi} = \frac{\sqrt{\pi}}{2\pi\sigma_l} L_d N_b{}^2 \alpha \left[1 - \frac{1}{\beta^2 n^2}\right] \exp[-(k \sin \vartheta_c \sin \phi \sigma^y)^2]$$
$$\sum_{-\infty}^{+\infty} \left|A_L\right|^2 \exp[-(k_r \sin \vartheta_c \cos \phi - Lk_0)^2 \sigma_x^2]$$
$$\sum_{-\infty}^{+\infty} \left|B_M\right|^2 \exp[-(k_r \sin \vartheta_c \sin \phi - Lk_1)^2 \sigma_y^2]$$

The cut of the beam results in a big improvement respect to the plain gaussian contribution; moreover it must be noted that, for modelling a realistic cut, the beam size has been assumed one order of magnitude bigger than the plain gaussian case. Even though, the spectral response of the cut beam is extended beyond the one of the tight focused beam.

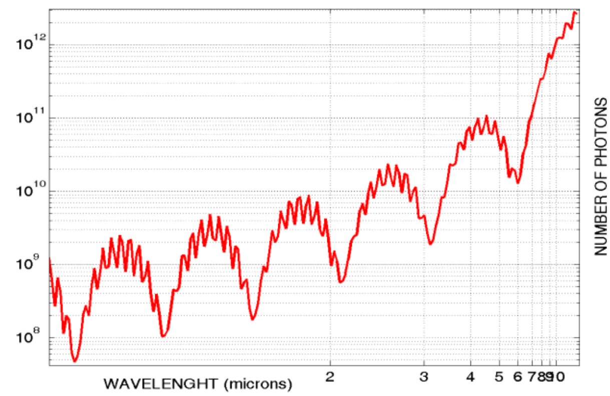

Figure 4: Coherent Cherenkov photons on a Periodic cut gaussian beam

CONCLUSIONS

The use of coherent Cherenkov radiation could be extremely useful as a longitudinal diagnostic tool, expecially moving to short wavelenghts thanks to the Cherenkov flat spectral response. Moreover, the limiting factor is given by the trasverse form factor of the beam. Different beam shapes cases have been taken in consideration, showing the possibility of drastic improvements, for example acting a transverse cut on the beam. Moreover still some effect such as divergence of the beam, radiator dispersion, electrons and light scattering inside radiator will be taken in consideration, since they are likely to make the form factor suppression less steep.

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