BEAM BREAKUP SIMULATIONS FOR THE CORNELL X-RAY ERL*

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Abstract

Multi-pass, multi-bunch beam-breakup (BBU) can limit the current in linac-based recirculating accelerators. We have made the computation of the transverse and longitudinal BBU-threshold current available in BMAD, Cornell's main accelerator simulation library. The coupling of horizontal and vertical motion as well as time of flight effects are automatically contained. Subsequently we present a detailed simulation study of transverse and longitudinal BBU in the proposed 5GeV Energy Recovery Linac light source at Cornell University, including the use of frequency randomization, polarized cavities and optical manipulations to improve the threshold current.

INTRODUCTION

In an Energy Recovery Linac, the electron beam goes through the RF cavities more than once. Each electron bunch first passes the linac's cavities for its acceleration, and after a return loop it enters the linac a second time for the bunch's deceleration. Here we will analyze the single-turn ERL that is being planned at Cornell University where a 100mA beam is to be accelerated up to an energy of 5GeV, at which it is used to generate highly-brilliant x-ray beams [1, 2].

The return of the electron beam for a second pass through the linac can excite both the transverse and longitudinal beam-breakup instability. When an electron bunch gets a transverse kick by dipole HOMs in a cavity, it will return to the same cavity during its second pass through the linac with a transverse displacement, which may pump more energy into the HOMs. If HOMs get enhanced by the bunch on its second pass, they may kick the following bunch even harder. When the current becomes so large that more energy is transfered into a HOM by bunches than is taken out by the HOM couplers, the HOM power will start to grow exponentially.

Longitudinal HOMs can change the bunch energy, which can cause a change in the time of returning to the cavity. Below the threshold current, the longitudinal HOM power is driven only by the current generated by the unperturbed sequence of bunches. Above the threshold, the beam current is modulated by the longitudinal HOMs with frequency matching the HOM frequency. Thus the longitudinal HOMs can be enhanced by this modulated beam current, which in turn leads to an even larger current modulation. This self-enhancement process will eventually cause loss of the beam.

In addition, HOMs of higher order multipoles can also cause beam breakup instability in an ERL. A quadrupole HOM, for example, can be excited if the bunch has a nonzero quadrupole moment. Such a quadrupole wakefield can induce quadrupole moments in the following bunch, which could in turn add more energy to the HOM on its second pass through the cavity. Thus a feedback loop similar to that of the transverse BBU can be formed. A detailed analysis of this process can be found in [3] and we have extended its approach to rotated multipoles of any order.

Several computer programs are available, many of which only consider dipole HOMs. We have extended the library BMAD to simulate BBU, based on the code bi (as discussed in [8]). All simulation results presented here were obtained with this library.

TRANSVERSE BBU INSTABILITY

If we assume that the transverse HOMs behave independently and do not interfere with each other, we can get an approximate formula of the threshold current in the presence of a single higher order mode [4, 5, 6],

$$I_{\mathrm{th}} = -\frac{2c^2}{e\left(\frac{R}{Q}\right)_\lambda Q_\lambda \omega_\lambda} \frac{1}{T_{12}^* \sin\omega_\lambda t_r} \tag{1}$$

$$T_{12}^* = T_{12}\cos^2\theta_\lambda + \frac{T_{14}+T_{32}}{2}\sin 2\theta_\lambda + T_{34}\sin^2\theta_\lambda \tag{2}$$

where $(R/Q)_\lambda$ is the shunt impedance, Q_λ is the quality factor, θ_λ is the polarization angle from the x direction, ω_λ is the HOM frequency, t_r is the bunch return time, and the matrix $\mathbf{T}$ describes how a transverse momentum is transported to a transverse displacement after one turn. According to this formula, a small Q_λ or $(R/Q)_\lambda$ can increase the threshold current. In addition, we can also adjust the lattice to have a small T_{12}^*. But this approach often does not work when there are many cavities because it is not always possible to minimize T_{12}^* for every cavity simultaneously. One example where this is possible is the case of polarized cavities with a fully coupled optics. There we arrange all modes either in x ($\theta_\lambda = 0$) or in y direction ($\theta_\lambda = \frac{\pi}{2}$), which includes the case of unpolarized HOMs for which θ_λ can be chosen arbitrarily. And we couple the x component of a transverse momentum into a y offsets after one turn ($T_{12} = 0$) and vice versa ($T_{34} = 0$). This leads to $T_{12}^* = 0$ for all cavities and Eq. (1) diverges. Formulas applicable in this case are analyzed in [6].

Unpolarized HOMs

In the following simulations we use the Q values and shunt impedances that are computed for unpolarized 7-cell

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cavities of the TESLA type [7]. The four most significant unpolarized HOMs are listed in Tab. 1. It is not expected that these values will change significantly when a polarized design is implemented.

Table 1: The four dominant transverse HOMs for the 7-cell ERL cavity.

	f_λ[GHz]	Q_λ	$(R/Q)_\lambda[\Omega]$
1	1.87394	20912.4	109.60
2	1.88173	13186.1	27.85
3	1.86137	4967.8	71.59
4	2.57966	1434.2	108.13

Table 2 shows what threshold current can be expected for the x-ray ERL with unpolarized HOMs that do not have any frequency spread for the more than 300 cavities. One strategy to increase the threshold current for a long linac is to avoid contributions to BBU from different cavities adding up coherently. This can be done by introducing a random distribution of HOM frequencies by fabricating each cavity slightly differently, as for example analyzed in [8]. In our simulations we randomize the HOM frequencies according to a Gaussian distribution with an rms width $\sigma_{\rm f}$. As a result, the threshold current increases significantly.

The problem with this approach is the significant statistical fluctuations due to the limited number of HOMs. But this fluctuation can be diminished by calculating the threshold current for the same frequency spread many times, in our simulation 500 times, and finding the average threshold current, as well as its distribution and rms spread. Fig. 1 shows that the average threshold currents and the width of the threshold current distributions form two smooth curves. Thus we conclude from these curves that a 10MHz frequency spread is reasonable because the average threshold current starts to saturate at this frequency spread.

Table 2: Threshold currents for the four most significant unpolarized HOMs of the Cornell ERL.

$\sigma_{\rm f}$[MHz]	$I_{\rm th}$[mA]		$\sigma_{\rm I}$[mA]	
	mode 1	mode 1-4	mode 1	mode 1-4
0	25.8	25.8	0	0
10.0	427.7	405.5	71.1	68.2

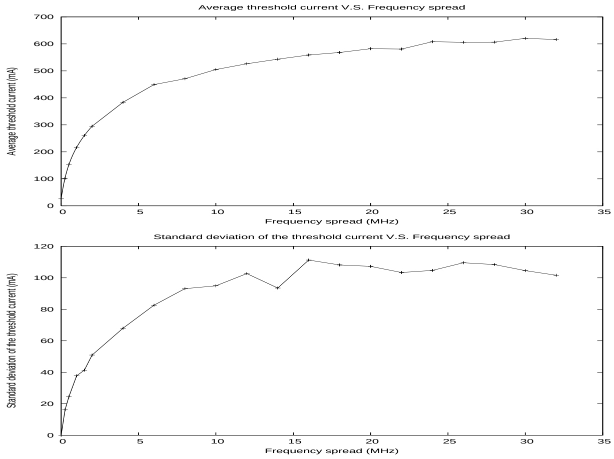

Figure 1: The threshold current I and its standard deviation $\sigma_{\rm I}$ v.s. frequency spread

Polarized HOMs

A dipole HOM with a polarization angle θ_λ can only kick the beam in that direction. By manipulating the shape of the RF cavity we obtain different frequencies for the x and y direction. As pointed out above, one way to raise the threshold current is to introduce x/y coupling into the lattice. If the HOMs in x and y direction have different frequencies, the beam motion will have different characteristic frequencies in these two directions. Therefore the kick from the HOMs in x direction will be less destructive if we can manipulate the lattice so that this momentum change will cause a displacement in y direction instead of in x direction, when the bunch returns to the same cavity. Since the frequency of the x and y modes are different, the beam oscillation in x produced during the first turn does not have the correct frequency to excite the y mode resonantly during the second pass.

If we use the four main unpolarized modes and separate x and y polarized modes according to $f_y = f_x - \Delta f$, we obtain the data listed in Tab. 3. The cavities have HOM frequencies that have a Gaussian distribution around these values with rms width $\sigma_{\rm f}$. We use 500 different random distributions of the frequencies and list the average threshold current $I_{\rm th}$ as well as the rms $\sigma_{\rm I}$ of the 500 resulting thresholds.

Since Eq. (1) for this case has $T^*_{12} = 0$, an extremely large threshold current would be expected when different HOMs can be considered independently. It has thus been argued (for example in [9]) that a separation of HOM frequencies by more than their resonance width, i.e. about a MHz, would make modes independent and extremely large threshold currents can be expected. Our simulations prove otherwise. Even for extremely large mode separation of 60MHz, the threshold current only increases by about a factor of four, which means the two modes couple to each other and cannot be treated separately even with very significantly different frequencies. A theoretical explanation of this effect can be found in [6].

LONGITUDINAL BBU INSTABILITY

Similar to Eq. 1 for the transverse BBU, an approximate formula of the threshold current in the presence of a single

Table 3: Threshold currents for the four most significant HOMs of the Cornell ERL.

Δf[MHz]	Coupling	$\sigma_{\rm f}$[MHz]	$I_{\rm th}$[mA]	$\sigma_{\rm I}$[mA]
10	NO	0	25.8	N/A
10	YES	0	93.4	N/A
60	NO	0	25.8	N/A
60	YES	0	117.6	N/A
60	NO	10	409	69
60	YES	10	2227	380

longitudinal HOM is derived in [10] as

$$I_{\rm th} = \frac{2\beta c E_0}{r_{56}\omega_\lambda (R/Q)_\lambda Q_\lambda} \tag{3}$$

where the time of flight term r_{56} replaces T_{12}.

Table 4: The four dominant longitudinal HOMs in the 7-cell ERL cavity

	$f_\lambda(\mathbf{GHz})$	Q_λ	$(R/Q)_\lambda[\Omega]$
1	3.85763	13728	31
2	2.45658	1778.8	134.5
3	5.93396	27887	5.99
4	3.85758	40172	2.94

As a demonstration, the lattice we use in our simulation for the Cornell ERL has a rather large $r_{56} = -1.055 \times 10^3$cm. We run the program with different beam currents and track the bunch motion until the HOM voltage in the cavity becomes stable. The four most important longitudinal HOMs are listed in Tab. 4 and the results of up to four modes in each cavity are shown in Fig. 2. The threshold current with only the most dominant mode $f_1 = 3.85763$GHz is about 92mA and adding the other three less dominant modes could reduce the threshold current to about 80mA, largely due to the fact that f_4 is very close to f_1.

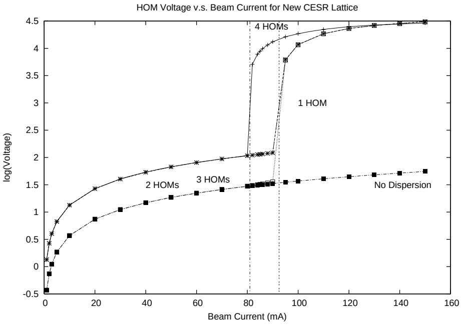

Figure 2: HOM Voltage v.s. Beam Current

To suppress the longitudinal BBU, r_{56} must be small. Figure 3 shows the influence of the r_{56} on the longitudinal BBU threshold current. It is very clear that the threshold current is proportional to the inverse of the r_{56}, as indicated by Eq. (3).

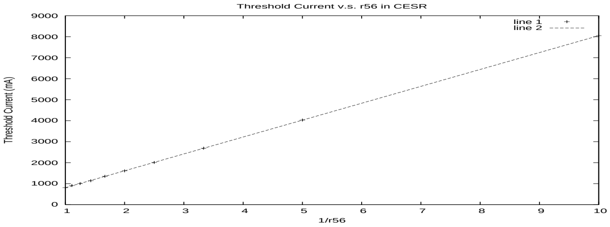

Figure 3: Threshold Current v.s. $1/r_{56}$

Similar to the transverse BBU case, the longitudinal BBU threshold current is affected by the frequency spread of HOMs in different RF cavities. Figure 4 shows that the threshold current increases almost linearly with the frequency spread.

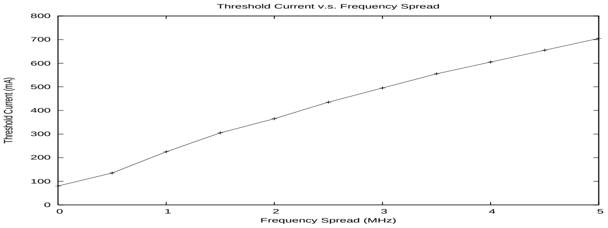

Figure 4: The threshold current v.s. Frequency spread

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