SPACE CHARGE EFFECTS IN DILUTED BEAM*

A. A. Mikhailichenko†
Wilson Laboratory, Cornell University, Ithaca, NY 14850

Abstract
For the particle acceleration with a laser-based source, the beams with particle's population required around one million only. The length of the bunch of the order of a micrometer yields such spacing between the particles, that the pancakes of the electromagnetic fields of individual particles are not overlapped at the cross section of the bunch. The space charge effects in this diluted beam considered. A comparison made with usual formula for betatron tune shift in self-field.

1 INTRODUCTION

The beams with minimal emittances required for successful operation of a laser driven accelerator. This is due to tiny dimensions of accelerating structure. The length of the bunch must be a fraction of the laser wavelength also. So one may expect that the space charge effects can be strong here. From the other hand, the number of the particles required in laser acceleration is about 10^6-10^7 only. This defined by a loading the acceleration structure. So one may hope that space charge effects are not strong enough to destroy the beam emittance.

2 DILUTED BEAM

Average distance between particles in a moving bunch can be defined as l_b / N, where l_b–is the bunch length in Laboratory frame, *N*–is the number of the particles in the bunch.

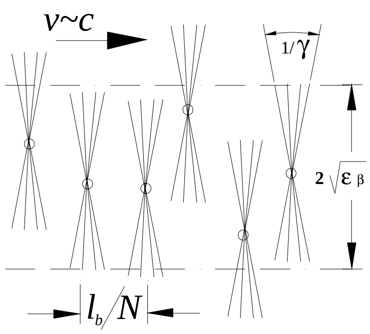

Figure 1: Schematic view of the field in diluted bunch.

We called beam *diluted*, if the particles separated in longitudinal direction so, that the pancakes of its individual fields having an opening angle $1/\gamma$, are not overlapped at the cross section $\sqrt{\varepsilon\beta}$ of the bunch,

$$\frac{l_b}{N} \geq \frac{\sqrt{\varepsilon\beta}}{\gamma}. \qquad (1)$$

Fig. 1 illustrates the condition (1). One can see that this condition is Lorenz-invariant, as the (1) could be rewritten as $\gamma l_b / N \geq \sqrt{\varepsilon\beta}$, where γl_b –is now a bunch length in its rest frame, and transverse size $\sqrt{\varepsilon\beta}$ is an invariant under Lorentz transformations. The last means that the closest particle in rest frame located at the distance of the order of the transverse bunch size.

In [1] there was presented a scheme of a cooler, what is mainly a sequence of wigglers and accelerating structures. The straight sections squeezed together for a compact size, so the back and forward trajectories are congruent. The longer the straight section is–the smaller influence of bends could be made. The bends itself could be made to give a small input into cooling dynamics. Here the particle needs to re-radiate its full energy a few times if the wigglers are dipole type. Let the wigglers have period $2\pi\lambda\!\!\!^{-}$ and the wiggler factor $K_x = eH_\perp \lambda\!\!\!^{-} / mc^2$, $H_\perp$–is a magnetic field value in the wiggler. Then the cooler promises equilibrium emittances as low as

$$(\gamma\varepsilon_x) \cong (\tfrac{1}{2}) \cdot \lambda\!\!\!^{-}_C \bar{\beta}_x (1 + K_x^2/2) K_x / \lambda\!\!\!^{-}$$

$$(\gamma\varepsilon_y) \cong (\tfrac{1}{2}) \cdot \lambda\!\!\!^{-}_C \bar{\beta}_y K_x / \lambda\!\!\!^{-},$$

where $\bar{\beta}_{x,y}$ – are averaged envelope functions in the wiggler, $\lambda\!\!\!^{-}_C = r_0/\alpha$ – is the Compton's wavelength, $r_0 = e^2/mc^2$, $\alpha = e^2/\hbar c$. The cooling time is $\tau_{cool} \cong (\tfrac{3}{2}) \cdot (\lambda\!\!\!^{-2} / c r_0 K_x^2 \gamma)$, $\gamma = E/mc^2$, what is not a function of the wiggler period. Substituting here for estimation $\gamma \cong 2 \cdot 10^3$ (1 GeV), $\bar{\beta}_{x,y} \approx 1$m, $\lambda\!\!\!^{-} \cong 5$cm, $K \cong 5$, one can obtain for *quantum emittances* and for damping time the following estimations

$$(\gamma\varepsilon_x) \cong 2.5 \cdot 10^{-8} \text{ cm rad},$$

$$(\gamma\varepsilon_y) \cong 2 \cdot 10^{-9} \text{ cm rad},$$

$$\tau_{cool} \cong 8.6 \cdot 10^{-3}\, s, \text{ or } 8.6\ ms.$$

This cooling time obviously does not depend on the length of the straight section, as the influence of the

* Work supported by National Science Foundation
† Phone: (607) 255-3785, Fax: (607) 255-8062, e-mail "mikhail@lns62.lns.cornell.edu

bends was neglected: shorter the length, faster the revolution. With such small emittances the beam size is about $\sqrt{\varepsilon\beta} \cong 1.6\cdot 10^{-6}$ cm, $\gamma \cong 10^4$ and this may give a reason for estimation of the transverse effects as a betatron tune shift and longitudinal effects. The length of the bunch required for laser acceleration is a fraction of the laser wavelength. Even suggesting the bunch compression one can have the bunch length in a cooler about 100μm.

For $N \cong 10^7$, the bunch will be diluted if it has the length $l_b \geq N\sqrt{\varepsilon\beta}/\gamma$, what in our case is

$$l_b \geq 10^7 \cdot 1.6\cdot 10^{-6}\cdot 10^{-4} = 1.6\cdot 10^{-3} cm \cong 16\mu m .$$

One can expect that any space charge phenomena can look in diluted beam different from a dense beam. First of all the space charge effects what tolerate to the transverse beam dynamics. The last one includes the betatron tune shift and Intra-Beam Scattering.

One can see also that for diluted beam the effects of radiation at IP of collider may look different also. In this publication we consider the transverse space charge effects.

3 SPACE CHARGE PHENOMENA

The field of the moving charge could be expressed as the following [2]

$$\vec{E} = \frac{e\vec{R}_t}{R_t^3}\frac{1}{\gamma^2(1-\beta^2 Sin^2\vartheta)^{3/2}}, \quad \vec{H} = \vec{\beta}\times\vec{E}, \quad (2)$$

where R_t –is the distance between the observation point and the present instant position of the charge (moment *t*), ϑ –is an azimuthal angle to the observation point, β –is a speed of the charge normalized to the speed of light. From (2) one can obtain that

$$\vec{E}_\perp = \vec{e}_\perp \gamma \frac{e}{R_t^2}, \qquad \vec{E}_\parallel = \vec{e}_\parallel \frac{e}{\gamma^2 R_t^2}, \qquad (3)$$

where first expression corresponds to $\vartheta \cong \pi/2$ and the second one to $\vartheta \cong 0$, $\vec{e}_\perp, \vec{e}_\parallel$ –are unit vectors in rectangular to the trajectory and parallel directions respectively.

Let us calculate the flux of rectangular component $\vec{E}_\perp$ of electrical field over a strip having the width $2R_t/\gamma$ (see Fig.2). As the area of the strip is $2\pi R_t\cdot(2R_t/\gamma)$, from (3) one can obtain estimation as

$$Flux \cong \oint \vec{E}_\perp dS \cong \vec{E}_\perp \cdot 2\pi R_t \frac{2R_t}{\gamma} \cong 4\pi e . \qquad (4)$$

This is exactly as it must be from the equation $div\vec{E} = 4\pi\rho$, where ρ –is a macroscopic charge density. So for external observer in the laboratory frame at the distances $r \geq l_b\gamma/N$ can use a macroscopic definition of electric field as $E_\perp 2\pi r\cdot l_b = 4\pi eN$. The field at smaller transverse deflections represents much more complicated picture.

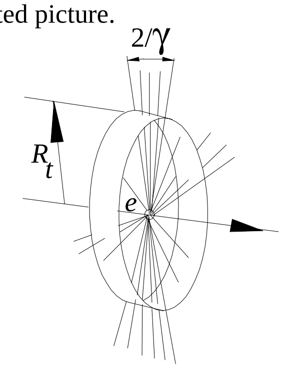

Figure 2. The strip ring around the relativistic charge.

Using (2), (3) one can see that the net force between two charges which longitudinal position is the same (so the fields are overlapping), could be expressed as

$$F_\perp \cong e\vec{E}_\perp - e\vec{\beta}\times\vec{H} = \gamma\frac{e^2}{R_t^2} - \gamma\cdot\beta^2\frac{e^2}{R_t^2} = \gamma\frac{e^2}{R_t^2}(1-\beta^2) \cong \frac{e^2}{\gamma R_t^2}, \qquad (5)$$

what is obvious, as the magnetic field is growing proportionally to the electrical field value, so the ratio has the same factor $1/\gamma^2$.

One can see that the interaction between the particles is going more likely as impact, not as a smooth interaction.

To satisfy the condition that the energy of Coulomb interaction between the electrons is small compared with Fermi energy it needs to be $\rho \geq \left(e^2 m/\hbar^2\right)^3$ [3].

Now we consider the namely space charge effect, i.e. self-focusing. Let us, first, consider the *non-coherent tune shift*. This tune shift exists even when the beam is going exactly in the axis of the round chamber.

So let we calculate the tune shift with (2) and suggesting the beam diluted and compare this with calculations for the overlapping field lines.

First let us make a formal estimation with the formula which does not take into account the specific properties of diluted beam. Estimating the electrical field with (4) when the charge density $\rho \cong eN/(a_\perp^2 l_b)$ where $a_\perp^2 \approx (\gamma\varepsilon)\cdot\beta/\gamma$ –is the transverse beam size one can obtain

$$E_\perp \cdot 2\pi R\cdot l_b \cong \rho\cdot\pi R^2 l_b . \qquad (6)$$

So the transverse force arising from space charge will be according to (5)

$$F_\perp \cong \frac{e^2 N}{2l_b(\gamma\varepsilon)\beta_x\gamma}\times R . \qquad (7)$$

Equation of motion of tested particle will be

$$\ddot{x} + Q^2\omega_0^2 x = \frac{F_\perp}{m\gamma} \cong \frac{e^2}{m}\frac{N}{2l_b(\gamma\varepsilon)\beta_x\gamma^2}x\,. \qquad (8)$$

So the corresponding tune shift will be

$$\Delta(Q\omega_0)^2_{usual} \cong \frac{e^2}{m}\frac{N}{2l_b(\gamma\varepsilon)\beta_x\gamma^2}\,. \qquad (9)$$

Now let us make estimations directly with (2). For small oscillation around equilibrium one needs to suggest $\vartheta << 1$ and for electrical field one can take

$$\vec{E} = \frac{e\vec{R}_t}{R_t^3}\frac{1}{\gamma^2(1-\beta^2\vartheta^2)^{3/2}} \rightarrow \frac{e}{R^2\gamma^2}\cdot\frac{x}{R}\,. \qquad (10)$$

For distance between neighboring particles one can suggest estimation in denominator as the following

$$R \cong [(l_b/N)^2 + x^2]^{1/2} \approx l_b/N\,.$$

Now as the beam supposed diluted the only neighboring particle suggested to give an input as the field has a cubic dependence on distance. So for the force one can obtain an estimation

$$F_\perp \cong \frac{e^2}{(l_b/N)^3\gamma^4}\cdot x\,.$$

For the tune shift one can obtain

$$\Delta(Q\omega_0)^2_{diluted} \cong \frac{e^2}{m}\frac{1}{(l_b/N)^3\gamma^5}\,. \qquad (11)$$

For comparison (9) and (11) one can make some transformations

$$\frac{\Delta(Q\omega_0)^2_{diluted}}{\Delta(Q\omega_0)^2_{usual}} \cong \frac{l_b(\gamma\varepsilon)\beta_x}{N(l_b/N)^3\gamma^3} \cong \frac{N^2}{l_b^2\gamma^2}\frac{(\gamma\varepsilon)\beta_x}{\gamma} \approx 1,$$

where we used condition (1). Of cause exact value of *one* is out of accuracy of our calculations. This result however is a reflection of the fact that from the point of view of the observer in the moving frame there is no *qualitative* difference if the beam is diluted or not. In this system magnetic *focusing magnetic* field of the cooler transferred into electrical field as $E'_x = \gamma B_y$. So the electrical focusing field is horizontal now. Magnetic field $B'_y = \gamma B_y$. So the focusing system in the moving frame looks as incoming electromagnetic wave with a period, which now is γ times higher: $\omega' = \gamma 2\pi\cdot c/\lambda_F$, where λ_F –is a longitudinal period of the focusing system.

So one can use usual formulas for betatron tune shift. Taking into account the ratio between electrical and magnetic field (2) one can obtain the change in local focusing parameter

$$k(z) \cong -\frac{1}{pc}\frac{\partial\Delta F}{\partial r} \cong -\frac{2r_0N}{a^2l_b\gamma^3} \cong -\frac{2r_0N}{\beta_x(s)(\varepsilon\gamma)l_b\gamma^2}\,.$$

This is a defocusing effect. The corresponding *non-coherent* tune shift is

$$\Delta Q_{NC} \cong -\frac{1}{4\pi}\int_0^C \beta_x(s)k(s)ds \cong -\frac{1}{2\pi}\frac{r_0NC}{l_b\varepsilon\gamma\cdot\gamma^2}$$

where C –is the circumference of the cooler, or the length of the transport line. For the cooler $C \cong 2\pi Q\overline{\beta}$.

So, the resulting formula for relative betatron tune shift will be

$$\frac{\Delta Q_{NC}}{Q} \cong -\frac{r_0N\overline{\beta}}{l_b(\varepsilon\gamma)\gamma^2}\,. \qquad (12)$$

Substitute here for estimation $\overline{\beta}_{x,y} \approx 100$ cm, $\gamma \cong 10^4$, $l_b \cong 100\mu$ m, one can obtain

$$\Delta Q_{NC}/Q \cong -\frac{2.8\cdot 10^{-13}10^7 100}{10^{-2}2.5\cdot 10^{-8}10^8} = -0.011\,.$$

For vertical relative tune shift one obtains $\Delta Q_{NC}/Q \cong -0.038$.

So one can see, that even for such a tiny emittances (and dimensions) the space charge effects does not look as a drastic ones.

4 REFERENCES

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