

# LASER ACCELERATION WITH OPEN WAVEGUIDES

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## Abstract

A unified framework based on solid-state open waveguides is developed to overcome all three major limitations on acceleration distance and hence on the feasibility of two classes of laser acceleration. The three limitations are due to laser diffraction, acceleration phase slippage, and damage of waveguide structure by high power laser. The two classes of laser acceleration are direct-field acceleration and ponderomotive-driven acceleration. Thus the solutions provided here encompass all mainstream approaches for laser acceleration, either in vacuum, gases or plasmas.

## 1 MODE PROPERTIES

The open waveguides of interest to laser acceleration have the following characteristics in common. First of all, they are over-sized in all dimensions compared to the laser wavelength. As such, field amplitude of a waveguide mode can be much smaller on waveguide surface than in the core. Secondly, they are electromagnetically or even structurally open. As a result, only low order modes are effectively guided. In other words, these waveguides are over-sized but not over-moded for being electromagnetically open. Laser acceleration with two particular types of open waveguides was studied separately in our previous works, one for capillary waveguide (CW) [1] and another for open-iris loaded waveguide (OILW) [2]. In this paper we present a unified framework for laser acceleration in vacuum, gases, and plasmas with both open waveguides. In particular, we propose to use a hybrid of the two to overcome the limitation due to acceleration phase slippage. The notations used here follow that of [1, 2] unless otherwise stated.

**Capillary Waveguide** The capillary waveguide considered here is made of a hollow core with an index of refraction  $\nu_1$  and radius  $R$ , embedded in a medium of dielectric or metal with a complex index of refraction  $\nu_2$ . The eigenmodes of the waveguide can be solved following the same procedure by Marcatili et al.[3] under the conditions  $\lambda_1/R \ll 1$  and  $|\sqrt{\hat{\nu}^2 - 1}| \gg \lambda_1/R$ , where  $\lambda_1 = \lambda/\nu_1$ ,  $\hat{\nu} = \nu_2/\nu_1$ , and  $\lambda$  is the wavelength in vacuum. For eigenmodes of the form

$$\begin{cases} \mathcal{E}(r, \phi, z, t) \\ \mathcal{H}(r, \phi, z, t) \end{cases} = \begin{cases} \mathbf{E}_{lm}(r, \phi) \\ \mathbf{H}_{lm}(r, \phi) \end{cases} e^{i(\beta_{lm}z - \omega t) - \alpha_{lm}z}, \quad (1)$$

the eigenvalues are given by

$$\beta_{lm} = k_1(1 - 1/2\gamma_g^2), \quad \alpha_{lm} = \text{Re}(\Lambda)/\gamma_g^2 R, \quad \psi \quad (2)$$

where  $k_1 = \nu_1 k$ ,  $k = 2\pi/\lambda$ ,  $\gamma_g = 2\pi R/U_{lm} \lambda_1 \gg 1$ , and  $U_{lm}$  is the  $m$ th root of the equation  $J_{l-1}(U_{lm}) = 0$ . There

are three types of modes, corresponding to

$$\Lambda = \begin{cases} \frac{1}{\sqrt{\hat{\nu}^2 - 1}} & : TE_{0m} \quad (l = 0) \\ \frac{\hat{\nu}^2}{\sqrt{\hat{\nu}^2 - 1}} & : TM_{0m} \quad (l = 0) \\ \frac{\hat{\nu}^2 + 1}{2\sqrt{\hat{\nu}^2 - 1}} & : EH_{lm} \quad (l \neq 0) \end{cases} \quad (3)$$

For laser acceleration, we are interested primarily in two low-order modes:  $TM_{01}$  mode for direct-field acceleration in vacuum and gases with its on-axis  $E_z$  component, and  $EH_{11}$  mode for ponderomotive-driven acceleration in plasmas. Accordingly, we consider three cases:  $\delta\nu_1 = 0$  when the core is in vacuum,  $\delta\nu_1 > 0$  and  $\delta\nu_1 < 0$  when the core is filled with uniform gases and plasmas, respectively, where  $\delta\nu_1 = \nu_1 - 1$  and  $|\delta\nu_1| \ll 1$ .

The electric fields within the core  $r \leq R$  are given by

$$TM_{01} : \begin{cases} E_z = E_a J_0(k_{r1}r) \\ E_r = -i(\Gamma/k_{r1})E_a J_1(k_{r1}r), \end{cases} \quad (4)$$

$$EH_{11} : \begin{cases} E_y = E_0 J_0(k_{r1}r) \\ E_z = -i(k_{r1}/\Gamma)E_0 J_1(k_{r1}r) \sin \phi, \end{cases} \quad (5)$$

where  $E_a$  is the peak acceleration field for  $TM_{01}$  mode,  $E_0$  is the peak transverse field for  $EH_{11}$  mode,  $\Gamma = \beta_{lm} + i\alpha_{lm}$  and  $k_{r1} = (U_{lm} - i\Lambda/\gamma_g)/R$ . To leading order,  $\Gamma/k_{r1} = \gamma_g$ . Given electric field, magnetic field of a mode can be determined by  $\mathbf{H}_t = \hat{\mathbf{z}} \times (\Gamma \mathbf{E}_t + i\nabla_t E_z)/kZ_0$ ,  $H_z = (i/\Gamma)\nabla_t \cdot \mathbf{H}_t$ , where subscript  $t$  denotes transverse component,  $\hat{\mathbf{z}}$  is a unit vector in  $z$ -direction, and  $Z_0$  is the vacuum impedance. To evaluate surface field  $E_s$  at  $r = R$ , we expand Bessel functions in the transverse fields of Eqs.(4,5) using the expression for  $k_{r1}$  and keeping the larger one of the two components

$$\begin{aligned} E_s/E_a &= \max\{1, |\Lambda|\} |J_0(U_{01})| & : TM_{01}, \\ E_s/E_0 &= \max\{1, |\Lambda|\} |J_1(U_{11})|/\gamma_g & : EH_{11}. \end{aligned} \quad (6)$$

Here we come upon one of the most important advantages of the capillary waveguide: for  $TM_{01}$  mode, surface field can be smaller than peak acceleration field, superior to other acceleration structures including even microwave linac; and for both modes, surface fields are much smaller than peak transverse fields. Power in each mode can be expressed as  $P(z) = P_0 e^{-z/L_{attn}}$ , where  $L_{attn} = 1/2\alpha_{lm}$  is power attenuation length and to leading order

$$P_0 = \begin{cases} \pi R^2 \gamma_g^2 E_a^2 J_0(U_{01})^2 / 2Z_0 & : TM_{01} \\ \pi R^2 E_0^2 J_1(U_{11})^2 / 2Z_0 & : EH_{11} \end{cases} \quad (7)$$

The on-axis intensity for a free-space  $TEM_{00}$  mode falls as  $I(z)/I(0) = 1/[1 + (z/Z_R)^2]$  away from the waist due to diffraction, and the on-axis longitudinal field for a  $TEM_{01}$  mode also falls as  $E_z(z)/E_z(0) =$

$1/[1 + (z/Z_R)^2]$ , where  $Z_R = \pi w_0^2/\lambda_1$  is the Rayleigh length. Assuming a  $TEM_{00}(TEM_{01})$  mode is coupled to a  $EH_{11}(TM_{01})$  mode at the optimal condition  $w_0/R = 0.64(0.56)$ , the effectiveness of guiding can be measured by taking the ratio of the relevant e-folding lengths of the waveguide mode to the free-space mode, yielding  $L_{EH11}/L_{TEM00} = L_{TM01}/L_{TEM01} = 2R/\text{Re}(\Lambda)\lambda_1$ . Despite the fact that the waveguide modes are leaky, optical guiding can be made quite effective to overcome diffraction for low order modes with sufficiently large  $R/\lambda_1$ . In addition, for waveguide material with anomalous dispersion at certain wavelength, it is possible to have  $\text{Re}(\Lambda) \ll 1$ . For example, we have  $\text{Re}(\Lambda) = 0.1$  for sapphire at  $\lambda = 10.6\mu\text{m}$  with  $\nu_2 = 0.67 + i0.04$ .

**Open-Iris Loaded Waveguide** The open iris-loaded waveguide considered here is made of a series of thin screens separated by distance  $L$  and each having a circular aperture of radius  $R$ . The eigenmodes of such a waveguide are identical to that of a Fabry-Perot resonator. Two distinctively different methods have been used to calculate the transverse field  $E_t$  of the modes: the numerical method of Fox and Li [4] and the analytical method of Vainshtein [5]. It is known that the two methods agree well in eigenvalue [6], but differ in detail in mode profile [5, 7]. The fine ripples in Fox-Li's profile are absent from Vainshtein's solution. Based on numerical results, Pantell proposed that advantage may be taken of these ripples of high spacial frequency for direct-field acceleration in vacuum [8], since the longitudinal field  $E_z$  is proportional to the transverse gradient of  $E_t$ . However, Pantell failed in providing solution to the phase slippage problem. His claim that net energy gain can be achieved by terminating the structure without terminating the interaction is a direct violation of the well-known theorem for laser acceleration in vacuum [9].

On the other hand, we calculated the acceleration mode [2] taking the analytical approach. We argue that the fine ripples, although neglected from Vainshtein approximation, are of less importance for laser acceleration. First, it has been shown [7] that the ripple magnitude is a decreasing function and the frequency of occurrence is an increasing function of Fresnel number  $N = R^2/\lambda_1 L$ . At the large value of  $N$  required for low loss mode propagation, the high frequency ripples can become very sensitive to slight variation and fluctuation in system parameters, misalignment, and spread in wavelength. Even the mathematical assumption of infinitely sharp aperture may need to be modified. All these factors tend to smooth out the high frequency ripples and what an electron see on average is the smooth profile predicted by the analytical solution. Last, the validity condition for the numerical method is more restrictive than for the analytical one. In addition to the common conditions  $R/\lambda_1 \gg 1$  and  $L/\lambda_1 \gg 1$ , the Fox-Li method further requires that  $L/R \gg 1$  and  $(L/R)^2 \gg N$  [4]. In the parameter regime of interest for laser acceleration, these extra conditions are often violated.

Analytical solution for OILW can be obtained by sim-

ply taking  $\Lambda = \eta_e \sqrt{\pi L/2\lambda_1} (1 - i)$  in all previous results given for CW, where  $\eta_e = 0.824$ . Note here we have changed the mode designation, to be consistent with that for CW. In comparison, CW has lower loss and surface field since generally  $|\Lambda_{oilw}| > |\Lambda_{cw}|$ , whereas OILW is desirable for allowing side access with its open structure. Thus a hybrid waveguide can be conceived in which sections of OILW are inserted in an otherwise uniform CW, wherever necessary. Power coupling coefficient between modes in CW and OILW is given by  $\alpha_c = 1 - C_m |\Lambda_{oilw} - \Lambda_{cw}|^2/\gamma_g^2$ , where  $C_m = 0.33(0.39)$  for  $TM_{01}(EH_{11})$  mode. The second term on the right, being  $\mathcal{O}(1/N)$ , can be made quite small, thus allowing significant reduction in mode coupling loss due to waveguide interruption.

## 2 DIRECT-FIELD ACCELERATION

Acceleration phase slippage length in vacuum is defined by

$$L_a = \frac{\lambda}{1/\gamma_g^2 + 1/\gamma^2}, \quad (8)$$

over which a relativistic electron with energy  $W_0 = \gamma mc^2$ , while being accelerated, slips  $\pi$  phase with respect to the fast acceleration wave in  $TM_{01}$  mode. Energy gain of the electron on the axis is  $\Delta W_a = eE_a L_a T_a$ , where  $T_a = 2/\pi$  is a transit factor. In parallel, a deceleration phase slippage length can be defined over which the electron slips another  $\pi$  phase while losing energy amounted to  $\Delta W_d = eE_a L_d T_d$ . Average acceleration gradient during a period of  $2\pi$  phase slippage is then

$$G = \frac{\Delta W_a - \Delta W_d}{L_a + L_d} = \frac{eE_a T_a [1 - L_d T_d / L_a T_a]}{1 + L_d / L_a}. \quad (9)$$

To have net acceleration, the ratio  $L_d/L_a$  should be made small. This can be done with two methods. The idea is to enhance phase slippage during the half period of deceleration, thus taking a shorter distance  $L_d$ . The first method, presented previously [1], works on reducing the longitudinal velocity of an electron by introducing a static transverse magnetic field during deceleration. Instead of tempering electron orbit which could cause significant radiative loss at high energy, the second method works on enhancing phase slippage by increasing phase velocity of the wave during deceleration. This can be done by introducing a plasma layer of thickness

$$L_d = \frac{\lambda}{1/\gamma_g^2 + 1/\gamma^2 + 1/\gamma_p^2}, \quad (10)$$

where  $\gamma_p = \omega/\omega_p \gg 1$ ,  $\omega_p = c\sqrt{4\pi r_e n_0}$  is the electron plasma frequency and  $n_0$  is the plasma density. In this case,  $T_d = T_a$ . The dominant energy loss for an ultrarelativistic electron traversing a plasma is due to bremsstrahlung [10]. The rate of energy loss is given by  $dW/dz = -W/L_R$ , where  $L_R$  is the radiation length defined by  $1/L_R = 4\alpha r_e^2 n_i Z(Z+1) \ln(233/Z^{1/3})$ ,  $n_i$  is density of ions with atomic number  $Z$ , and  $\alpha$  is the fine structure constant. For

Hydrogen plasma with density  $n_i = n_0 = 10^{17}/\text{cm}^3$ ,  $L_R$  is as long as  $4 \times 10^6 \text{m}$ . Reflectance of laser power off a sharp interface between vacuum and an underdense plasma at normal incidence is also negligible, according to the Fresnel formula  $R_p = (1 - \nu_1)^2 / (1 + \nu_1)^2 = 1/16\gamma_p^4$ .

The assumption of a sharp vacuum-plasma interface is not necessary. More rigorous treatment can be obtained with WKB method [11]. For underdense plasma, the only modification required is to replace the factor  $\exp(i\beta_{01}z)$  by  $\exp[i\int\beta_{01}(z)dz]$ . Assuming a density profile  $n_e = n_0 f_p(z)$ , where  $f_p(z) = 1/[1 + e^{-(z+L_d/2)/\delta}] - 1/[1 + e^{-(z-L_d/2)/\delta}]$ , the phase advance for the mode can be calculated by making use of the integral  $\int_{-\infty}^{\infty} f_p(z)dz = 2\delta \ln[(1 + e^{L_d/2\delta})/(1 + e^{-L_d/2\delta})]$ . It is seen here that Eq.(10) is accurate enough as long as  $L_d/2\delta \gg 1$ . In addition, the validity of WKB method requires  $|d\nu_1/dz| \ll 2\pi\nu_1^2/\lambda$  [11], which gives  $\delta \gg \lambda/16\pi\gamma_p^2$  for  $df_p/dz|_{max} = 1/4\delta$ , also easily satisfied.

### 3 PONDEROMOTIVE ACCELERATION

Two methods are presented here to overcome the limit on acceleration distance set by the phase slippage length

$$L_a = \frac{\lambda_p}{1/\gamma_g^2 + 1/\gamma_p^2 - 1/\gamma^2} \quad (11)$$

for laser wakefield acceleration in an open waveguide [1]. The first method requires inserting plasma layers of higher density, each of length  $L_d$ , as drift sections in between acceleration sections, each of length  $L_a$ . Two conditions need to be satisfied. First, the length of a drift section is given by

$$L_d = \frac{\lambda_p}{1/\gamma_g^2 + 1/\gamma_{pd}^2 - 1/\gamma^2}, \quad (12)$$

where  $\gamma_{pd} = \lambda_{pd}/\lambda$  and  $\lambda_{pd}$  is the plasma period corresponding to the plasma density in the drift section. This condition guarantees continuous energy gain in each acceleration section, since  $L_d$  is the distance for the particle to slip  $\pi$  phase with respect to the acceleration wave of period  $\lambda_p$ . Second, the plasma density in the drift section is set according to  $\lambda_p/\lambda_{pd} = 2m$ , where  $m$  is an integer. This condition ensures that there is no net energy exchange between the particle and the laser wakefield excited in the drift section, since  $L_d$  is also the distance over which a particle slips  $2m\pi$  phase with respect to the wakefield with period  $\lambda_{pd}$  in the drift section. Thus the average gradient that can be maintained over multiple slippage lengths is  $G = \Delta W_a/(L_a + L_d)$ , where  $\Delta W_a = eE_a L_a T_a$ . In a limiting case with  $(\gamma/\gamma_p)^2 \gg 1$  and  $(\gamma_g/\gamma_p)^2 \gg 1$ , we have  $L_a/L_d = (\lambda_p/\lambda_{pd})^2$ .

The second method utilizes longitudinal modulation in laser intensity due to beating of two waveguide modes. The idea is to choose the beating period same as the distance for a  $2\pi$  phase slippage, such that the wakefield is stronger when the particle is in accelerating phase, and weaker in decelerating phase, resulting in net energy gain

over multiple slippage lengths. When two modes are included, Eq.(24) of [1] is modified to

$$a^2(\rho, \zeta, z) = \frac{a_0^2}{2} |f_b(\rho, z)|^2 e^{-\zeta^2/2\sigma_z^2}, \quad (13)$$

where the profile, normalized to  $f_b(0, 0) = 1$ , is given by

$$\begin{aligned} f_b(\rho, z) &= \frac{1}{1+\eta} [E_{11}(\rho, z) + \eta E_{12}(\rho, z)], \\ E_{11}(\rho, z) &= J_0(U_{11}\rho) e^{i\beta_{11}z - \alpha_{11}z}, \\ E_{12}(\rho, z) &= J_0(U_{12}\rho) e^{i\beta_{12}z - \alpha_{12}z}. \end{aligned} \quad (14)$$

Assuming  $(\gamma_{g11}/\gamma_p)^2 \gg 1$ ,  $(\gamma_{g12}/\gamma_p)^2 \gg 1$  and  $(\gamma/\gamma_p)^2 \gg 1$ , where  $\gamma_{g11}$  and  $\gamma_{g12}$  are  $\gamma_g$  factors for  $EH_{11}$  and  $EH_{12}$  modes, respectively, the group velocity and slippage length then become same for both modes, i.e.,  $v_g = c(1 - 1/2\gamma_p^2)$  and  $L_a = \gamma_p^2 \lambda_p$ . There are three characteristic length scales:  $l_1 \sim \{1/\alpha_{11}, 1/\alpha_{12}\}$  is due to mode attenuation;  $l_2 \sim 1/(\beta_{11} - \beta_{12})$  is due to beating of the two modes; and  $l_3 \sim 1/k_p$  is the plasma period. As they satisfy  $l_1 \gg l_2 \gg l_3$ , we deduce from Eq.(23) of [1] that

$$E_{wz} = E_a |f_b(\rho, z)|^2 \cos(k_p z - \omega_p t), \quad (15)$$

and, in particular, for acceleration field on the axis

$$|f_b(0, z)|^2 = \frac{1 + \eta^2 + 2\eta \cos[(\beta_{11} - \beta_{12})z]}{(1 + \eta)^2}. \quad (16)$$

By requiring  $\beta_{11} - \beta_{12} = \pi/L_a$ , we have the matching condition  $\gamma_{g11}^2/\gamma_p^3 = U_{12}^2/U_{11}^2 - 1$ . Energy gain over  $2L_a$  distance is then  $\Delta W_{2\pi} = eE_a 2L_a T_{2\pi}$ , where  $T_{2\pi} = \eta/(1 + \eta)^2$ . As expected,  $T_{2\pi}$  vanishes when there is only one mode with  $\eta = 0$ , and it reaches a maximum when the two modes have equal amplitude on the axis with  $\eta = 1$ . The relative mode amplitude can be adjusted easily by changing  $TEM_{00}$  mode waist according to

$$\eta = \frac{J_1^2(U_{11}) \int_0^1 J_0(U_{12}\rho) \exp[-\frac{\rho^2}{(w_0/R)^2}] \rho d\rho}{J_1^2(U_{12}) \int_0^1 J_0(U_{11}\rho) \exp[-\frac{\rho^2}{(w_0/R)^2}] \rho d\rho}. \quad (17)$$

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