Electromagnetically Induced Transparency in a Bounded Plasma and its Relation to Beatwave Physics*

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Abstract

The theory of electromagnetically induced transparency (EIT) in a plasma [1] is examined in the context of a bounded system via particle simulations. It is found that when boundaries are introduced into the problem, the requirements of causality preclude the transmission of radiation through an overdense plasma as conceived in the original theory. However, a two-frequency laser, or "beatwave", will cascade into a Stokes satellite with a frequency below the cut-off frequency. This can lead to an *apparent* EIT signature if a specific set of parameters are chosen.

1 INTRODUCTION

In a cold unmagnetized plasma the cut-off frequency for electromagnetic radiation is equal to the plasma frequency, ω_p. It was suggested by S.E. Harris [1] that an electromagnetic wave with a frequency $\omega_- < \omega_p$ will nevertheless propagate in the presence of an intense pump wave with a frequency $\omega_0 \approx \omega_- + \omega_p$. The proposal was based on the fact that a pump wave at ω_0 and a "Stokes" wave at ω_- combine to drive a plasma wave at the beat frequency $\Delta\omega = \omega_0 - \omega_-$. When the plasma wave is driven below resonance ($\Delta\omega < \omega_p$), it is phased such that the beat current associated with the pump wave and the plasma wave tends to cancel the current associated with the Stokes wave. This reduces the amplitude of the current at the Stokes frequency which allows the Stokes wave to propagate despite the relation $\omega_- < \omega_p$. The term "electromagnetically induced transparency" (EIT) was used to describe this process because of its similarity to the analogous process of EIT in a neutral gas which has been studied theoretically and experimentally for some time [2].

Recently, the theory of EIT in a plasma was extended to account for relativistic effects and the presence of the "anti-Stokes" wave with frequency $\omega_+ \approx \omega_0 + \omega_p$ [3]. This was done by considering the relativistic Raman dispersion relation of Ref. [4] in the limit where the density exceeds quarter-critical. It was found once again that an anomalous passband appears when the pump wave is sufficiently intense. It was also found that the system exhibits a Raman-type instability even when the plasma is overdense with respect to the Stokes wave. We note, however, that the dispersion relation used to arrive at these conclusions was derived under the assumption $\omega_0 \gg \omega_p$. We will present a more accurate dispersion relation in a longer paper [5, 6].

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In this paper, we examine the effect of boundaries on the theory of EIT in a plasma. We find that in the case of a bounded system, no dispersion relation is an adequate measure of the transparency or opacity of a plasma. This is because causality plays a fundamental role in the EIT process. In particular, the plasma wave is caused by the Stokes wave, yet the Stokes wave cannot propagate until the plasma wave is present. The result is that the Stokes wave is not transmitted even though the dispersion relation predicts transparency. On the other hand, when the anti-Stokes wave drives the density perturbation, we find that a Stokes wave can be *generated* with a frequency less than ω_p. We equate this process with the cascading of a beatwave (two-frequency laser) into Stokes and anti-Stokes satellites. Particle-in-cell (PIC) simulations are used to verify the findings.

2 UNBOUNDED PLASMA

From the point of view of classical electrodynamics, the transparency or opacity of a medium is related to the magnitude and phasing of the currents driven within it by electromagnetic waves. In one dimension, electromagnetic radiation is described by the wave equation

$$(\partial_{xx} - \partial_{tt})A = -j \tag{1}$$

where A is some transverse component of the vector potential and j is the corresponding component of current density. Here, and in all that follows, velocity is normalized to the speed of light, the electron has unit mass and charge, time is measured in units of ω_p^{-1}, and the vector potential is normalized to mc^2/e. By rewriting the wave equation in frequency space and requiring real wavenumbers, one obtains the inequality

$$\omega^2 > -\frac{\hat{j}}{\hat{A}} \tag{2}$$

where the circumflex denotes a Fourier-transformed quantity.

Now consider a cold unmagnetized plasma consisting of electrons with density $1 + n_1(x,t)$ and an immobile uniform background of neutralizing ions. Conservation of transverse canonical momentum gives the current as

$$j = -\frac{(1+n_1)A}{\gamma} \tag{3}$$

where γ is the relativistic Lorentz factor associated with a fluid element. Rewriting this in frequency space and inserting the result into equation (2) one obtains

$$\omega^2 > 1 + \frac{\hat{n_1} * \hat{A}}{2\pi \hat{A}} + R(\hat{A}) \quad (4)$$

where the asterisk denotes convolution and R is some operator that accounts for relativistic effects. We henceforth drop the relativistic term since for the parameters considered it is not important. An algebraic relation can be obtained by considering two discrete electromagnetic modes

$$A = \frac{1}{2} A_- e^{i(\omega_- t - k_- x)} + \frac{1}{2} A_0 e^{i(\omega_0 t - k_0 x)} + cc \quad (5)$$

and one discrete plasma wave mode

$$n_1 = \frac{1}{2} n e^{i(\Delta\omega t - \Delta k x)} + cc \quad (6)$$

If the frequencies and wavenumbers are related by $\Delta\omega = \omega_0 - \omega_-$ and $\Delta k = k_0 - k_-$, then by inserting $\hat{A}$ and $\hat{n_1}$ into equation (4) one obtains

$$\omega_-^2 > 1 + \frac{n^* A_0}{2A_-} \quad (7)$$

The nonlinear term represents current at the frequency ω_- driven by the beating of the pump wave A_0 with the plasma wave n. If the phasing between A_0, A_-, and n is chosen correctly, this term could be negative. In this case the inequality is satisfied for a range of frequencies with $\omega_- < 1$ and anomalous transparency occurs for those frequencies. If the nonlinear term is positive, the inequality is *not* satisfied for a range of frequencies with $\omega_- > 1$ and anomalous opacity occurs for those frequencies.

In order to specify the density n in terms of the electromagnetic field amplitudes, one must self-consistently determine the frequencies and amplitudes of all the waves in the system. The classic Raman dispersion relation for a cold plasma [7] provides an immediate solution to this problem. It is well known that in the context of the Raman dispersion relation, one has

$$n \propto A_0(A_- + A_+) \quad (8)$$

where A_+ is the amplitude of the anti-Stokes wave with frequency $\omega_+ = \omega_0 + \Delta\omega$. It is clear that for EIT applications, one must have $A_- \gg A_+$ since only then will the transparency condition of equation (7) be independent of the probe amplitude A_-.

Also well known is the fact that the Raman dispersion relation generally has four branches when solved for complex wavenumbers and real frequencies. As will be discussed in Ref. [5], and as has been partially addressed before [1, 3], two of the branches indicate EIT passbands in the regime $A_+ \gg A_-$, while the other two branches indicate EIT passbands in the regime $A_- \gg A_+$. We will also show in Ref. [5] that this remains true when the relativistic dispersion relation is used. In the next section, however, we show that these passbands effectively disappear when boundaries are introduced into the problem.

3 BOUNDED PLASMA

The description of EIT in terms of the Raman dispersion relation is limited in that it describes only the normal modes of an infinite plasma. This information is only useful if it can be related to the problem of the transmission of an electromagnetic pulse through a finite plasma. In the nonlinear regime, it is not necessarily the case that a bounded medium will transmit radiation for which the dispersion relation predicts real wavenumbers. Consider first the the case where $A_- \gg A_+$. In this case, the plasma wave is driven mostly by the Stokes wave, yet the Stokes wave cannot propagate until the plasma wave is present. One might conclude therefore that the Stokes wave never penetrates the plasma. This conclusion is supported by one-dimensional PIC simulations, as discussed below.

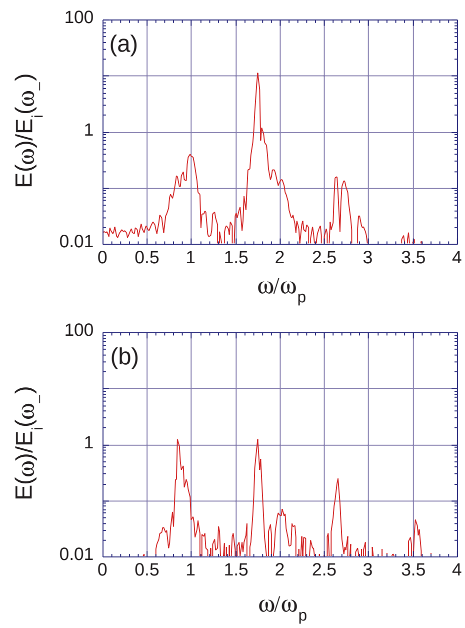

Figure 1: Simulated attempt at EIT with $A_- \gg A_+$ (a) transmitted spectrum of the electric field (b) reflected spectrum of the electric field. Frequencies 1.75 and 0.85 were injected. The data is normalized to the spectral amplitude of the incident wave at $\omega = 0.85$.

In the case where $A_+ \gg A_-$, the plasma wave is driven mostly by the anti-Stokes wave which can enter the plasma unaided. Since the plasma wave is independent of A_-, so is the nonlinear current source at the Stokes frequency, and so too are any waves radiated by this source. In other words, the pump and the anti-Stokes act independently to *generate* a Stokes wave. This suggests that the pump and the anti-Stokes might be regarded as a single two-frequency pump, or "beatwave", which drives a polarization wave at the Stokes frequency. When the polarization wave oscillates at a frequency less than ω_p, it radiates an evanescent wave from every point in the plasma. As will be discussed in detail in another paper [5], the sum of these evanescent waves is a propagating wave. Although within the plasma this wave propagates only in the direction of the pump, it nevertheless exits the plasma equally in both directions. This is because of the fact that signals due to sources deep within the plasma cannot interfere with signals due to sources near the vacuum. In other words, the normal phase matching conditions do not apply. This will be shown analytically in Ref. [5].

The preceding assertions were tested using one-dimensional PIC simulations. In the two simulations presented here, the Stokes and anti-Stokes amplitudes are ramped linearly for $10\omega_p^{-1}$, and remain constant thereafter. The pump wave is gaussian with a standard deviation of $100\omega_p^{-1}$. The three waves are copropagated into a uniform slab of plasma $30c/\omega_p$ long. The electrons are hot ($v_{th} = 0.1c$) while the ions are held fixed.

Figure 1 shows the results of a simulation in which a pump wave with $A_0 = 0.3$ and $\omega_0 = 1.75$ was copropagated with a Stokes wave with $A_- = 0.03$ and $\omega_- = 0.85$. According to the dispersion relation, the plasma is transparent to the Stokes wave for these parameters [5, 6]. However, all the energy in the incident Stokes wave is accounted for by the peak in the reflected spectrum at $\omega = 0.85$. The peak in the transmitted spectrum at $1\omega =$ is due to instability. The associated wave emerges from the plasma late and continues long after the pump disappears. Several other simulations were attempted with $A_- \gg A_+$, including cases with longer pulse lengths, gradual plasma-vacuum boundaries, and higher pump intensities. In every case similar results were obtained.

Figure 2 shows the results of a simulation in which a pump wave with $A_0 = 0.3$ and $\omega_0 = 1.75$ was copropagated with an anti-Stokes wave with $A_+ = -0.1$ and $\omega_+ = 2.85$. The data plotted in figures 2(a) and 2(b) was evaluated $10c/\omega_p$ beyond the plasma. The data of figure 2(c) was evaluated $10c/\omega_p$ before the plasma. We see that as stated above, the beatwave generates a Stokes satellite which exits the plasma equally in both directions. We note that the amplitude of the Stokes satellite is consistent with the ratio A_+/A_- given by the Raman dispersion relation.

Note also that an apparent EIT signature can be obtained if in addition to injecting the pump wave and the anti-Stokes wave, one also injects a "probe" wave at the Stokes frequency [5, 6]. When the amplitude and phase of the probe wave are chosen such that the reflected probe wave destructively interferes with the backward propagating Stokes satellite, the probe wave appears to be transmitted with a transmission coefficient proportional to the pump intensity. This is not the same as true EIT, however, since it only works for one particular probe amplitude.

4 CONCLUSIONS

The consideration of boundaries is crucial to a full understanding of EIT in a plasma. The dispersion relation only determines the ability of a particular mode to propagate, not the ability of a pulse to penetrate a bounded medium.

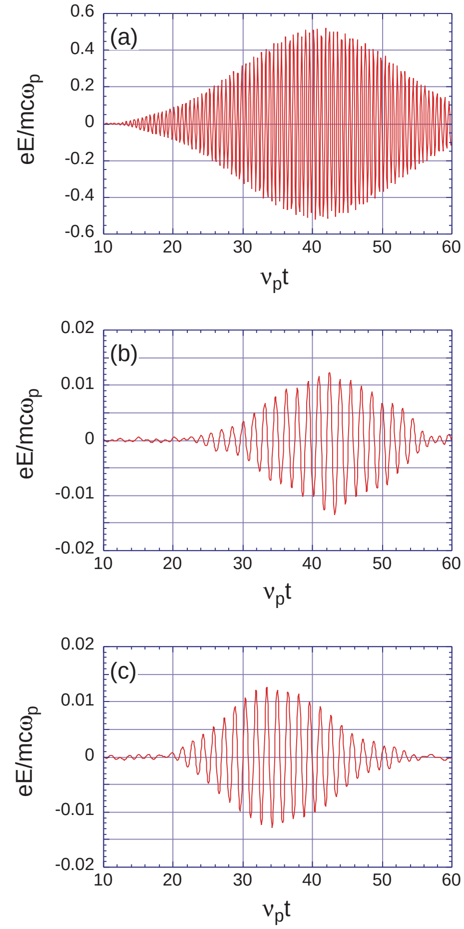

Figure 2: Generation of Stokes radiation in a stopband (a) waveform of the transmitted pump wave (b) waveform of the forward Stokes (c) waveform of the backward Stokes. The injected anti-Stokes wave is not shown.

In the case of a plasma, radiation can be generated in the stopband, but apparently cannot be transmitted through it. A consequence of these findings is that a two-frequency laser will cascade into Stokes satellites not only above the plasma frequency, but also below it.

5 REFERENCES

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