INTERACTION OF TM_{01} AND HEM_{11} IN A TWT

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Abstract

We investigate the interaction and the coupling of TM_{01} and HEM_{11L}, with an electron beam in a high-efficiency traveling-wave output structure operating at 9GHz. The coupling between the symmetric and asymmetric mode may be characterized by a single parameter that represents the correlation of the transverse and longitudinal phase-spaces. In order to examine the coupling we consider a pre-bunched beam injected in a uniform structure. For a specific set of parameters simulations indicate that 0.5MW of HEM_{11L} power at the input is sufficient to deflect to the wall a beam of 300A/0.85MV guided by a 0.5T magnetic field.

1 INTRODUCTION

In high-power and high-efficiency traveling-wave amplifiers the electron beam is assumed to interact with the lowest symmetric TM mode. Efficiencies as high as 70% and even higher, may be achieved in coupled cavity TW structures when high order modes do not play a significant role. However, asymmetry may occur either due to the input or output arm or azimuthal electrons' distribution. As a result, asymmetric modes may develop. Such modes are called hybrid electric and magnetic (HEM) modes. The main problem with HEM modes, is their ability to deflect the beam to the wall. Since pulse shortening was observed experimentally, as reported by Wang *et. al.* [1], we investigate in this study, some of the "cold" characteristics of asymmetric modes, and their interaction with the electron beam and the symmetric mode; specifically the beam blow up due to the hybrid mode.

2 DISPERSION RELATION

In the internal region ($r < R_{int}$) of a disk-loaded structure all the components of the electromagnetic field may be derived from the longitudinal components:

$$\begin{pmatrix} E_z \\ H_z \end{pmatrix} = \sum_{n,\nu=-\infty}^{\infty} \begin{pmatrix} \mathsf{E}_{n,\nu} \\ \mathsf{H}_{n,\nu} \end{pmatrix} e^{j\omega t - jk_n z - j\nu\phi} I_\nu(\Gamma_n r) \ , \qquad (1)$$

$k_n = k + 2\pi n / L, \quad |k| < \pi / L, \quad \Gamma_n \equiv \sqrt{k_n^2 - (\omega / c)^2}$ and $I_\nu(\xi)$ is the modified Bessel function of the first type and order ν. The dispersion relation of a periodic structure may be written in a matrix form as follows:

$$\begin{pmatrix} D^{TM}(\nu) & C_{12}(\nu) \\ C_{21}(\nu) & D^{TE}(\nu) \end{pmatrix} \cdot \begin{pmatrix} \mathsf{E} \\ \mathsf{H} \end{pmatrix} = 0 \qquad (2)$$

where, in principle, the matrices D^{TM}, D^{TE}, C_{12}, C_{21}, are infinite. This notation is convenient since in the case of symmetric modes $(\nu = 0)$ the coupling matrices (C_{12},C_{21}) are identically zero and this equation has two *uncoupled* solutions Det (D^{TM}) = 0 and Det (D^{TE}) = 0 that represent all the symmetric transverse magnetic (TM) and transverse electric (TE) modes, respectively. For any other value of ν the coupling matrices are not zero and as a result, the non-trivial solution of (2) implies that each eigen-mode is a superposition of the two modes (TE & TM). From the perspective of the interaction with the electrons, the main problem with such a mode is that it has a non-zero transverse magnetic field on axis and consequently, electrons may be deflected [2-5].

Similar to the symmetric modes for each radial number ν there are two modes, only that here we can no longer distinguish between TE and TM but rather they are referred to as "lower" and "higher" modes. Figure 1 illustrates the dispersion relation of all the modes up to 20GHz in a structure with internal radius of 8mm. The structure was designed to operate at 9GHz with phase advance per cell of $\pi/2$ and phase velocity of 0.933c; the disk thickness is 1.5mm. In such a relatively small internal radius the TM_{01} and HEM_{11L} modes are well separated and do not intersect. For higher radii the modes get closer to each other.

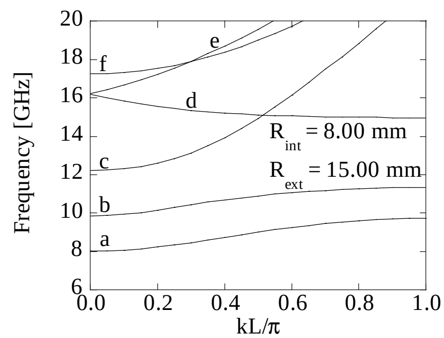

Figure 1: All modes up to 20GHz; a-TM_{01}, b-HEM_{11L}, c-HEM_{11H}, d-HEM_{21L}, e-HEM_{21H}, f-TM_{02}.

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The relative weight of each basic mode (TM and TE) composing the hybrid mode changes at different frequencies. In order to illustrate the "character" of the HEM mode, it is convenient to define the quantity $h_0 = \eta_0 \,|\, \mathsf{H}_{n=0} / \mathsf{E}_{n=0} \,|$. When h_0 is smaller than unity, the system behaves as a "TM" mode whereas values larger than unity its behavior resembles the "TE" mode. Figure 2 illustrates the value h_0 for the same structure presented above. The "TE" behavior is primarily in the lower part of the pass band of HEM_{11L} and "TM" behavior in its upper region.

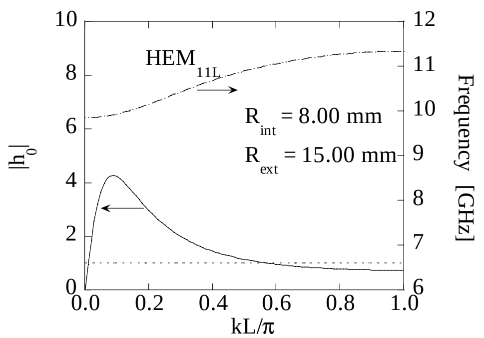

Figure 2: The value h_0 for HEM_{11L} as presented in Figure 1.

Another aspect that is critical in the design of a slow-wave structure is the group velocity of the HEM_{11L} mode. If the latter is negative an inherent positive feedback develops in the system and the system will oscillate. This problem is in particular vital in tapered structures where even if initially the system was designed for a positive group velocity, as the phase velocity of TM_{01} mode is reduced, the group velocity of the HEM_{11L} may become negative. Increasing the internal radius eliminates the positive feedback but it also reduces the interaction impedance; the latter is defined as:

$$Z_{\mathrm{int}}^{(TM_{01})} = \frac{S_W}{2P^{(TM_{01})}} \,|\, \mathsf{E}_{n=0,\nu=0} \,|^2,$$
$$Z_{\mathrm{int}}^{(HEM_{11L})} = \frac{S_W}{2P^{(HEM_{11L})}} \,|\, \mathsf{E}_{n=0,\nu=1} \,|^2 \tag{3}$$

where S_w is the area the wave propagates and P is the total power which flows in the system in the mode.

3 DYNAMICS OF THE SYSTEM

In order to describe the coupling between TM_{01} and HEM_{11} we developed a quasi-analytic macro-particle model that describes the interaction of a beam of electrons with both TM_{01} and the low branch of the HEM_{11}. In the framework of this approach the dynamics of the particles is fully 3D but the variations effect in the amplitude of the electromagnetic field are assumed to occur only in the longitudinal direction (1D). Additional assumptions of the model include; positive group velocity of both modes, the basic form of both modes is preserved, the energy conversion is controlled by the longitudinal motion and no electrons are reflected. Space limitations will constrain the explicit formulation to the 1D case however subsequently simulation results from a 3D simulations will be presented. In the case of 1D motion the mode coupling is only due to azimuthal asymmetry therefore the governing equations read:

$$\begin{aligned}
&\frac{d}{d\xi}\left(\frac{a_1}{\sqrt{\alpha_1}}\right) = \sqrt{\alpha_1}\left\langle e^{-j\chi_{i,1}} I_0(\bar{\Gamma}_1 \bar{r}_i)\right\rangle, \\
&\frac{d}{d\xi}\left(\frac{a_2}{\sqrt{\alpha_2}}\right) = \sqrt{\alpha_2}\left\langle e^{-j\chi_{i,2}} I_1(\bar{\Gamma}_2 \bar{r}_i)\right\rangle, \\
&\frac{d}{d\xi}\chi_{i,1} = \frac{\Omega_1}{\beta_i} - K_1, \quad \frac{d}{d\xi}\chi_{i,2} = \frac{\Omega_2}{\beta_i} - K_2, \\
&\frac{d}{d\xi}\gamma_i = -\frac{1}{2}\left[a_1 e^{j\chi_{i,1}} I_0(\bar{\Gamma}_1 \bar{r}_i) + a_2 e^{j\chi_{i,2} - j\phi_i} I_1(\bar{\Gamma}_2 \bar{r}_i) + c.c\right].
\end{aligned} \tag{4}$$

The first two are amplitude dynamics equations, followed by the phase dynamics equations and in the last line we have a single particle energy conservation; 1 represents the TM_{01} mode and 2 the HEM_{11L} mode; $\langle \ldots \rangle$ represents averaging over entire ensemble of particles. The other definitions used here are: $\xi \equiv z/d$, d is the total interaction length, $\Omega \equiv \omega d/c, K \equiv kd, a \equiv e\mathsf{E}_{n=0} d/mc^2$, $\gamma_i^{-1} = (1-\beta^2)^{1/2}$, $\chi_{i,1}$ is the phase of the i'th particle relative to the TM_{01} mode whereas $\chi_{i,2}$ is the phase of the same particle relative to the HEM_{11L} mode; ϕ_i is the azimuthal location of the i'th particle; α_1, α_2 are the coupling coefficients defined as $\alpha_\mu \equiv (eIZ_{\mathrm{int}}^{\mu}/mc^2)(d^2/\pi R_{\mathrm{int}}^2)$, $\mu = 1,2$; $\bar{\Gamma} \equiv \Gamma R_{\mathrm{int}}$ and $\bar{r} \equiv r/R_{\mathrm{int}}$. Based on (4) the spatial growth of the system may be evaluated and the result is:

$$S_{\pm}^3 = -\frac{1}{2}(S_1^3 + S_2^3) \pm \frac{1}{2}\sqrt{(S_1^3 - S_2^3)^2 + 4 S_1^3 S_2^3 \bar{u}^2}\ ; \tag{5}$$

where $S_\mu^3 = \frac{1}{2} p_\mu \alpha_\mu \Omega_\mu$, $p_1 = \left\langle I_0^2(\bar{\Gamma}_1 \bar{r}_i)(\gamma_i\beta_i)^{-3}\right\rangle$, $p_2 = \left\langle I_1^2(\bar{\Gamma}_2 \bar{r}_i)(\gamma_i\beta_i)^{-3}\right\rangle$ and the real parameter describing the coupling between the modes $\bar{u}$ is given by:

$$\bar{u} \equiv \frac{\left|\left\langle \dfrac{e^{-j(\chi_{i,1} - \chi_{i,2} + \phi_i)}}{(\gamma_i \beta_i)^3} I_0(\bar{\Gamma}_1 \bar{r}_i) I_1(\bar{\Gamma}_2 \bar{r}_i)\right\rangle\right|}{\sqrt{p_1 p_2}} \tag{6}$$

The solution S_+ corresponds to the "HEM_{11L}"-like solution since at the limit $\bar{u} = 0$, $S_+ = S_2$ whereas S_- corresponds to the "TM_{01}"-like solution. Figure 3 illustrates the value of $\bar{u}$ as a function of the angular spread of the beam $-\phi_0 < \phi < \phi_0$ with other parameters chosen as follows: $\bar{\Gamma}_1 = 3$, $\bar{\Gamma}_2 = 4.5$, $-\pi/2 < \chi_{i,1} < \pi/2$, $\chi_{i,2} = 1.5\chi_{i,1}$, $0 < \bar{r}_i < 0.6$ and $2.4 < \gamma_i \beta_i < 2.5$. It shows that the coupling is maximum when the azimuthal spread of particles is minimal and evidently in the case of a symmetric beam the coupling vanishes.

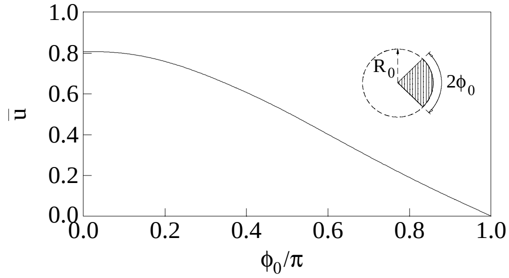

Figure 3: The value of $\bar{u}$ as a function of the angular distribution of the beam.

Finally, the 3D approach that due to space limitations will not be described her, enables to examine the development of the beam expansion. Figure 4 shows the radius of the envelope, $R_e / R_{\text{int}} \equiv 2\langle \bar{r} \rangle$, for several initial HEM_{11L} power levels at the input (1,10,100,500,1000kW); the TM_{01} mode is generated by a modulated $|\chi_{i,1}| < \pi/4$ beam. The other parameters of the simulation are as follows: I=300A, V=0.85MV, R_{int}=8mm, R_b=3mm, d=3.11cm, $|h_0|$=0.94, $f_{\text{TM}_{01}} = 9\text{GHz}$, $f_{\text{HEM}_{11L}} = 11\text{GHz}$, $Z_{\text{int}}^{\text{TM}_{01}} = 1.5\text{k}\Omega$, $Z_{\text{int}}^{\text{HEM}_{11L}} = 3.8\text{k}\Omega$, $\chi_{i,2}^{(\text{input})} = \chi_{i,1}^{(\text{input})} f_{\text{TM}_{01}} / f_{\text{HEM}_{11L}}$.

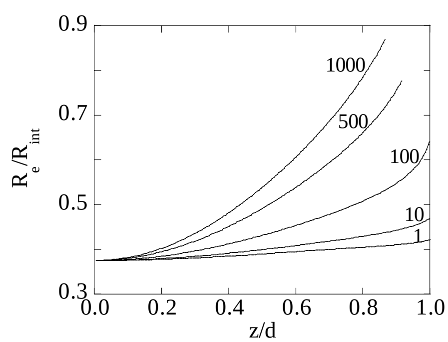

Figure 4: The radius of the envelope for several HEM_{11L} power level at the input (kW).

The increase of beam's envelope is directly correlated with the efficiency of HEM_{11L} mode as illustrated in Figure 5. At the same time, the interaction of the TM_{01} is very efficient reaching the 80% level due to initially bunched beam that drives the system. This efficiency is virtually not affected by the HEM_{11L} mode and all the curves overlap. The efficiencies of the TM_{01} and HEM_{11L} modes are defined as follows:

$$\eta_{TM_{01}}(\%) = \frac{100}{2\alpha_1} \frac{|a_1(\xi)|^2 - |a_1^{in}|^2}{\langle \gamma_i^{in} \rangle - 1 + \frac{1}{2\alpha_1}|a_1^{in}|^2 + \frac{1}{2\alpha_2}|a_2^{in}|^2};$$
$$\eta_{HEM_{11L}}(\%) = \frac{100}{2\alpha_2} \frac{|a_2(\xi)|^2 - |a_2^{in}|^2}{\langle \gamma_i^{in} \rangle - 1 + \frac{1}{2\alpha_1}|a_1^{in}|^2 + \frac{1}{2\alpha_2}|a_2^{in}|^2}. \qquad (7)$$

For $P_{\text{in}}^{(\text{HEM}_{11L})} = 0.5\text{MW}$ there are particles that hit the structure and for this reason the interaction is terminated.

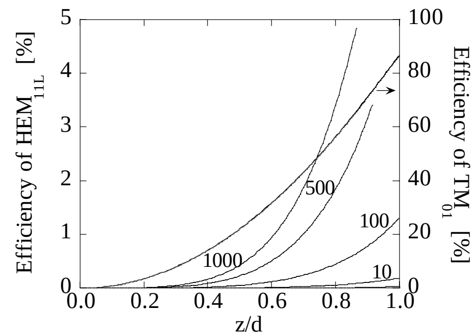

Figure 5: The way the efficiency of both modes develops for several HEM_{11L} power levels at the input (kW).

3 CONCLUSIONS

The design of a slow wave traveling wave structure has to take into consideration the effect of the asymmetric modes that the beam may interact with; the coupling between the symmetric and asymmetric modes was shown to be determined by a single parameter. When substantial power is associated with the HEM_{11L} mode it may cause deflection of the beam to the wall.

4 ACKNOWLEDMENTS

This study was supported by the US DoE and the Bi-National Science Foundation US – Israel.

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