

RESONANCE FAR-FIELD ACCELERATING STRUCTURES AT SHORT WAVELENGTHS

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Abstract

Resonance far-field structures capable to provide high gradient continuous acceleration at millimeter and micron wavelengths are considered. Among such structures are periodically striped open waveguide and oversized waveguide having periodic small perturbations. Main parameters of the structure composed by flat mirrors are estimated on the base of rigorous analytical solution for the eigenmodes.

Some modifications of diffraction-dominated structures based on open waveguides and resonators are proposed.

1 INTRODUCTION

In the previous paper [1] a concept was proposed for laser resonance acceleration of relativistic charged particles. It is based on a resonance interaction of a straightforward charged particle beam interacting with electromagnetic beam having periodic change of wave vector. Closed waveguide having periodical deformation was considered as the accelerating structure. It was shown earlier, that such an overmoded rectangular [2] or circular [3] waveguide can be excited effectively (with dominant single mode) by a properly focused laser beam having linear polarization. However, the stability of the hybrid EH_{11} mode to the transformation into parasitic higher modes requires further consideration.

In this paper we consider another realization of the principle above based on an open periodic waveguide. It can provide stable propagation of the lowest mode due to diffraction resulting in widely-spaced spectrum of eigenmodes.

2 EIGENMODES OF THE PERIODICAL OPEN MIRROR WAVEGUIDE

The open waveguide (or open periodical resonator) composed by chain of mirrors with period λ_w . For simplicity of consideration we assumed here rectangular flat mirrors with dimensions $2a \times 2b$. The principal scheme is shown in the Fig. 1. To consider the fields of the open mirror waveguide we solve first the problem for the equivalent open resonator with rectangular mirrors having dimensions $2a_1 \times 2b_1$ (see Fig. 2). The e.m. fields of the resonator with rectangular mirrors are found by Vainshtein [4] in terms of eigenmodes for Hertz vector potential $\vec{\Pi}$ at $\lambda_s \ll a, b$, where $\lambda_s = 2\pi c/\omega$ is laser

wavelength. The vector potential $\vec{\Pi}$ is defined through the potential function Φ , that satisfies to a Helmholtz wave equation: $\Delta\Phi + k^2\Phi = 0$.

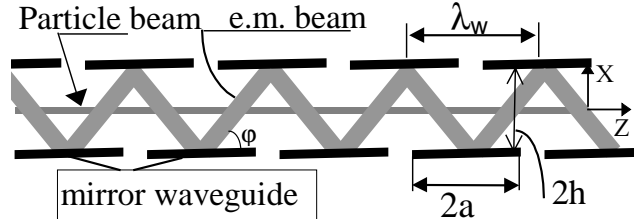


Fig. 1. Schematic drawing of an accelerator with a «wiggling» field propagating in an open mirror waveguide.

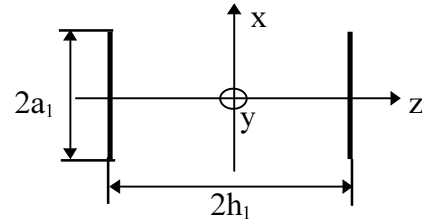


Fig. 2. Equivalent open resonator with rectangular mirrors.

If the fields in the resonator are produced by plain waves propagating with small angles to the axis OZ, the fields of corresponding TE modes can be expressed in the following form:

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \text{Re} \left(\begin{pmatrix} i \text{rot} \vec{\Pi}_{\text{TE}} \\ \text{grad} \text{div} \vec{\Pi}_{\text{TE}} + k^2 \vec{\Pi}_{\text{TE}} \end{pmatrix} e^{-i\omega t} \right), \quad (1)$$

where $\vec{\Pi}_{\text{TE}} = \Phi \mathbf{e}_y$, $k = \omega/c$,

$$\Phi = 2i \sum_{u=0}^1 \sum_{v=0}^1 \sum_{q,m,n} e^{i\pi(q(-1)^v z/2h_1 - p)} i^{2\{(q-1)/2\}} \Phi_{u,v,n},$$

$$\Phi_{u,v,n} = A e^{ik_x m (-1)^u x} f_{u,m} f_{v,q} f_n(y),$$

$$f_{u,m} = -(1-u - (-1)^m u) (-1)^{[(n-1)/2]} / 2i^{n+1},$$

$$f_n(y) = \begin{cases} \cos(nk_y y), & n = 1, 3, 5, \dots \\ \sin(nk_y y), & n = 2, 4, 6, \dots \end{cases}, \quad M_{a,b} = \sqrt{\frac{2k}{h_1} (a_1^2, b^2)},$$

$$k_{a,b} = \frac{\pi/2}{(a_1, b)(1 + \eta(1+i)/M_{a,b})}, \quad \eta = -\zeta(-0.5)\pi^{0.5} \approx 0.824,$$

$$p_{a,b} = \frac{(\pi/4)(m^2, n^2)}{(M_{a,b} + \eta(1+i))^2}, \quad p = p_a + p_b, \quad \zeta(z) \text{ is Riemann's}$$

Zeta function.

To find the potential function for the waveguide it is necessary to make a transition from the standing wave to the traveling one, to make a transformation from local frame xoz (see Fig. 2 corresponding to each half a period) to the waveguide frame XOZ (Fig. 1) and to impose Floquet condition of periodicity. The transformation is valid under the following conditions:

$$\lambda_s \ll \lambda_w, \quad \pi p' \ll 1 \text{ and } h_1 = h/\sin\varphi, \quad a_1 = a \sin\varphi, \quad (2)$$

where $p = p' - ip''$.

After Fourier expanding of the potential function on the period λ_w one can derive:

$$\Phi = A e^{ik_z Z} \sum_{q_w} C_{q_w} e^{iq_w k_w Z}, \quad C_{q_w} = \sum_{u,m,n} f_n(y) f_{u,m} B_{u,m,n}^{(q)} \quad (3)$$

where $k_z = (k - 2k_w p' \cos\varphi) \cos\varphi - ik_w p''$, $q_w = 0, 1, 2, 3, \dots$,

$$B_{u,m,n}^{(q)} = \sum_{\pm} \left\{ e^{i(k_{cu} X - p' 3\pi/2)} S\left(\frac{\pi}{2} \left(\frac{k_{su}}{k_w} \mp q_w\right)\right) \times \right. \\ \left. \exp\left[\pm i \left(k'_Z X \operatorname{tg}\varphi + \frac{\pi}{2} \left(p' + \frac{k_{su}}{k_w} \mp q_w\right)\right)\right] \right\}$$

$$k_{cu} = k_{am}(-1)^{u+1} \cos\varphi, \quad k_{su} = k_{am}(-1)^{u+1} \sin\varphi, \\ S(x) = \sin(x)/x, \quad k_w = 2\pi/\lambda_w.$$

3 BASIC RELATIONSHIPS

One can see from (3), that the Fourier coefficient C_{q_w} is a function of coordinate X due to wave front inclination. With taking into account condition $\lambda_s \ll a, b$ the dependency is weak at small glancing angles $\varphi \ll 1$:

$$(k'_Z X \operatorname{tg}\varphi)^2 \ll 1, \text{ or } (\varphi X k)^2 \ll 1 \quad (4)$$

If the angle φ is small, substantial contribution in Φ give the following terms: $q_w = 0$, $q_w = \text{odd}$. We consider below only resonance acceleration, i.e. $q_w = 1, 3, 5, \dots$.

Phase velocity related to the speed of light follows simply from (3):

$$\beta_{ph} = \left(q_w \frac{k_w}{k} + \left(1 - 2 \frac{k_w}{k} p' \cos\varphi\right) \cos\varphi \right)^{-1}.$$

Then for $\gamma \gg 1$ we have the following condition of resonance:

$$\frac{\lambda_w}{\lambda_s} = \frac{q_w + 4p'}{2 \sin^2(\varphi/2) + \gamma^{-2}/2} \quad (5)$$

One can obtain from (5) for $\gamma^2 \varphi^2 \gg 1$, $\varphi \ll 1$ and $M_a, M_b \gg 1$:

$$\lambda_w/\lambda_s \approx 2q_w/\varphi^2 \quad (6)$$

Note, that (6) is equivalent to analogous condition (2) in [1] when $\varphi \rightarrow \pi r_w \sqrt{2/\lambda_w} \ll 1$. From (6) one can estimate the tolerance on the most critical parameter φ : $\Delta\varphi/\varphi \ll 1/2q_w$.

For the practical situation, when $a - \lambda_w/4 \ll a$, we have from (6) and (2) the following important rule:

$$N_{fx} = q_w/2,$$

where $N_{fx} = M_a^2/8\pi$ is the Fresnel number for XOZ plane.

If $M_a, M_b \gg 1$ and waist of the incident laser beam is matched with the mirror dimensions a, b (analogously to [2,3]), we can assume, that the dominant propagating mode is $m=1, n=1$. Then the power flow through the waveguide is:

$$P = P_0 e^{-2k''_Z Z}, \quad \text{where} \quad (7)$$

$$P_0 = |A|^2 \left(k^2 - \frac{k_a^2 + k_b^2}{2}\right) a_1 b \frac{k^2 - k_b^2}{2Z_0}, \quad k''_Z = k_w p'', \quad Z_0 = 120\pi\Omega.$$

For the lowest dominant mode we obtain from (1,3) the following expression for the accelerating field amplitude:

$$E_a = -iAB_{1,1,1}^{(q)} k'_Z \operatorname{tg}\varphi \cos(k_{s1}) \cos(k'_Z \operatorname{tg}\varphi \Delta X), \quad (8)$$

$$\text{where } \Delta X = X - X_0, \quad X_0 = \frac{\pi(1-p')}{2k'_Z \operatorname{tg}\varphi} \approx \frac{1}{4} \sqrt{\frac{\lambda_w \lambda_s}{2q_w}}$$

the optimal transverse position for the particle beam centroid.

In order to remove the details of the specific accelerator mechanism let us define the coupling strength g_c as the ratio between the local acceleration gradient E_a and vacuum electric field $\sqrt{2Z_0 P_0/S_0}$ of the focused laser beam (here $S_0 \approx 4a_1 b$). Under the conditions above we have from (7,8):

$$g_c \approx 4\varphi / q_w = 4\sqrt{2\lambda_s / q_w \lambda_w}. \quad (9)$$

Note, for our scheme g_c is equal approximately to E_a/E_c defining the ratio between accelerating field and surface field.

Power attenuation length L_{att} can be defined from (7) provided ohmic losses are negligible:

$$L_{att} = \frac{1}{2k_w p''} \approx \frac{\lambda_w}{2\pi^2 \eta} \frac{M_a^3 M_b^3}{M_a^3 + M_b^3}. \quad (10)$$

4 PERFORMANCE ESTIMATIONS

It is seen from (10), that power losses are proportional to $\lambda_w^{-1} q_w^{-3/2}$, whereas the coupling strength scales as

$\lambda_w^{-1} q_w^{-1}$. Hence, to use the advantage of long resonance acceleration we should choose higher harmonics. However, as q_w increases tolerances becomes more stringent. Estimated performance parameters of the scheme are presented in the Table 1 for $a_1 \approx b/3$, $a \approx \lambda_w/4$, wavelength $\lambda_s = 10\mu\text{m}$ and surface field $E_c = 5 \text{ GV/m}$.

5 DISCUSSION

Since the angle φ is small for short wavelengths, ohmic losses of the TE modes can be considerably reduced with the help of special corrugations made on the mirror surface (having period $\leq \lambda_s/2$ and depth $(0.2 \pm 0.5)\lambda_s$) or proper dielectric coating (see [2,5]). Besides, the radiation losses defined in (10) on the basis of diffraction in resonator (Fig. 2) for waveguide are actually less due to interference effect [4]. The diffraction losses can be reduced further by enhancement of both the effect with

proper choose of the relationships between λ_s , λ_w , a , ϕ and figure of merit Q for single cell due to mirror curvature optimization.

In general case input coupling can give a combination of TE and TM mode [3]. Note, for the TM mode one can derive from (3), that coupling strength is less ($g_c \approx 4\phi^2/q_w$) and $X_o = 0$. To solve this multi-mode problem further study is necessary.

The main disadvantage of the scheme compared to an Open Iris Loaded Waveguide (OILW, [6]) is dependence of the acceleration rate on the particle transverse coordinate (3). This imposes rather stringent condition for the both particle beam radius and beam alignment tolerance that practically should not exceed several wavelengths (see (4)).

Table 1. Performance parameters for the rectangular mirror waveguide.

q_w	3	5
λ_w , cm	1.2	4
ϕ , mrad	70	50
$2h$, mm	0.43	1.0
g_c	0.06	0.025
M_a	6.1	7.9
L_{int} , cm	12.2	80
$I_c = P_o/S_o$, W/cm ²	$4.5 \cdot 10^{16}$	$1.3 \cdot 10^{17}$
P_o , W	$2.5 \cdot 10^{10}$	$4 \cdot 10^{11}$
R_s , M Ω /m	0.6	0.12
Q	$7.6 \cdot 10^4$	$5 \cdot 10^5$
E_s , GeV/m	0.35	0.25

6 SOME VARIANTS

To reduce the diffraction losses limiting the acceleration section length one can use cylindrical or spherical mirrors (see Fig. 3).

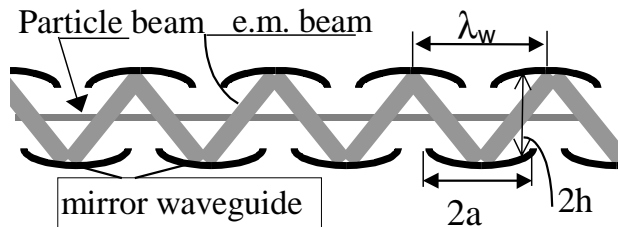


Fig. 3. Accelerating waveguide with reduced losses.

One can provide a resonance mechanism in OILW by means of special sections, where the interaction between particle beam and field is negligible (see Fig. 4). In this sections the propagating mode is converted into A-mode of spheroidal cavity [4]. In the scheme $\lambda_w/2$ lies between $L_s/2$ and L_s (slippage length). An average acceleration rate is less then that local value for OILW (up to twofold).

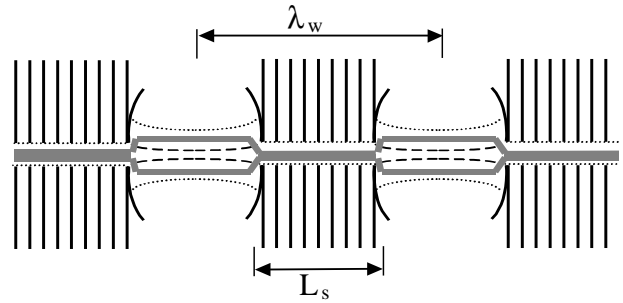


Fig. 4. Modified OILW to provide resonance acceleration. Gray thick lines show flow of primary power. Dotted lines show caustics.

7 CONCLUSION

One can outline the following attractable features of the acceleration scheme considered:

- Relative simplicity for manufacturing of the waveguide composed by flat mirrors and suitability for acceleration of flat beams.
- The final energy gain is limited by attenuation length rather than slippage length, because it is resonance acceleration.
- Tapering is not necessary to provide synchronism between relativistic particles and accelerating harmonic.
- Axicon scheme is not required to couple the laser power in the structure.
- Synchrotron radiation losses are very low due to low equivalent undulator strength $K_w \leq 1$ [1].

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