BUNCH ENERGY LOSS IN CAVITIES: DEPENDENCE ON BEAM VELOCITY *

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Abstract

Proton or H$^-$ linacs accelerate particles through the wide range of the particle velocities. For intense beams, and especially when superconducting cavities are used, a detailed knowledge about how the beam energy loss depends on the beam velocity $\beta = v/c$, and what part of the lost energy remains in the cavity, is required to calculate the cavity heat load. Using a frequency-domain approach, we calculate the bunch energy loss as a function of β for a few particular cases. We are mostly concerned about the cavity modes below the beam-pipe cutoff, because they produce most of the cavity heating. While the loss factor due to an individual mode can show peaks at some particular values of β, the total loss for important particular cases was found smoothly decreasing as β decreases. Comparison with time-domain results for the ultra-relativistic case, $\beta \to 1$, is also presented.

1 INTRODUCTION

Many modern accelerator projects — Accelerator Production of Tritium (APT) [1], Spallation Neutron Source (SNS) [2], and the accelerator-driven transmutation of nuclear waste (ATW) [3] — consider to use linacs to accelerate intense proton or H$^-$ beams providing the final beam power in the range 1–200 MW. Most of them will also rely on superconducting (SC) technology for the accelerating cavities, e.g. [1, 3], to reduce operational costs. With the final beam energies around 1-2 GeV, the ion beam during acceleration changes its velocity from a non-relativistic one to $\beta = v/c = 0.9 - 0.95$. At the same time, the number of types of SC cavities is limited to a few due to cost and production reasons. In the APT linac design [1] there are only two types of SC cavities, optimized for $\beta = 0.64$ and $\beta = 0.82$, respectively, but they are to accelerate protons in the velocity range from $\beta = 0.58$ (proton energy 211 MeV) to 0.94 (1.7 GeV). It is important to know how the amount of the beam energy deposited in the cavities depends on the beam velocity.

It is common to believe that loss factors of a bunch moving along an accelerator structure at velocity $v = \beta c$ with $\beta < 1$ are lower than those for the same bunch in the ultra-relativistic case, $\beta \to 1$. Computations with time-domain codes like MAFIA [4] or ABCI [5] work only for $\beta = 1$, because of difficulties in imposing proper boundary conditions at the open ends of the beam pipe in the numerical time-domain approach for $\beta < 1$. Often the loss factors computed numerically for the ultra-relativistic bunch are considered the upper estimates for $\beta < 1$. We study the

β-dependence of loss factors using the frequency-domain approach and concentrating primarily on the cavity modes below the beam pipe cutoff frequency, since they contribute the major part of the cavity heat load. In particular, we find that the above assumption on the upper estimate from the ultra-relativistic case may be incorrect, especially when only some individual modes of the cavity are concerned. More details can be found in a recent paper [6].

2 CAVITY LOSS FACTORS

Let $\vec{E}_s$, $\vec{H}_s$ be a complete set of eigenfunctions (EFs) for the boundary problem in a closed cavity with perfect walls. The longitudinal impedance can be written as (e.g., [7])

$$Z(\beta, \omega) = -i\omega \sum_s \frac{1}{\omega_s^2 - \omega^2} \frac{|I_s(\beta, \omega)|^2}{2W_s}, \qquad (1)$$

where $I_s(\beta, \omega) = \int_L dz \exp(-i\omega z/\beta c) E_{sz}(0, z)$ is the overlap integral and W_s is the energy stored in the s-th mode. Here $E_{sz}(0, z)$ is the longitudinal component of the s-th mode electric field taken on the chamber axis.

The loss factor $k = \pi^{-1} \int_0^\infty d\omega \, Re \, Z(\beta, \omega)|\lambda(\omega)|^2$, where $\lambda(\omega) = \int ds \exp[i\omega s/(\beta c)]\lambda(s)$ is a harmonic of bunch spectrum, can be expressed as a series by assuming all $Q_s >> 1$ and integrating formally $Re \, Z(\beta, \omega)$ in Eq. (1). For a Gaussian bunch with rms length 2σ, it gives $k(\beta, \sigma) = \sum_s k_s(\beta, \sigma)$, where the loss factors of individual modes k_s are

$$
\begin{aligned}
k_s(\beta, \sigma) &= \exp\left[-(\omega_s'\sigma/\beta c)^2\right]|I_s(\beta, \omega_s')|^2/(4W_s) \\
&= \exp\left[-(\omega_s'\sigma/\beta c)^2\right]\omega_s' R_s(\beta)/(2Q_s) . \quad (2)
\end{aligned}
$$

In the last expression the shunt impedance $R_s(\beta)$ and Q-factor for the s-th cavity mode are used.

From a physical viewpoint, the loss factor (2) of a given mode includes two velocity-related effects. The exponent factor shows that the bunch length σ effectively increases to σ/β. The β-dependence of R_s is essentially due to that of the cavity transit-time factor for this resonance mode: the effective cavity length scales as $1/\beta$. Although Eqs. (2) give us the dependence of the loss factor on β, it is only practical when the number of strong resonances is small, since this dependence varies from one resonance to another:

$$\frac{k_s(\beta, \sigma)}{k_s(1, \sigma)} = \exp\left[-\left(\frac{\omega_s'\sigma}{c\beta\gamma}\right)^2\right]\frac{R_s(\beta)}{R_s(1)}, \qquad (3)$$

where $\gamma = 1/\sqrt{1 - \beta^2}$.

It follows from Eq. (3) that for long bunches loss factors will decrease rapidly with β decrease, as $\exp\left(-\beta^{-2}\right)$. The impedance ratio dependence on β is generally more complicated, and we consider below a few typical examples.

3 EXAMPLES

3.1 Cylindrical Pill-Box

For a cylindrical pill-box cavity of length L and radius d, in the limit of vanishing radius of the beam pipes, the ratio of loss factors (3) for the lowest E_{010}-mode is

$$\frac{k_{010}(\beta,\sigma)}{k_{010}(1,\sigma)} = \exp\left[-\left(\frac{\mu_{01}\sigma}{d\beta\gamma}\right)^2\right]\left(\beta\frac{\sin\frac{\mu_{01}L}{2\beta d}}{\sin\frac{\mu_{01}L}{2d}}\right)^2 , \quad (4)$$

where μ_{mn} is the n-th zero of the first-kind Bessel function $J_m(x)$. The ratio is almost independent of β when the bunch is short, $\sigma \ll d$, and the cavity is short compared to its radius, $L \ll d$. For longer cavities, however, the ratio oscillates and can exceed 1 many times, see [6]. Obviously, a similar behavior is expected for any other individual resonance mode in this case.

3.2 Small Discontinuity

Consider now the case with a small number of modes below the pipe cutoff. The simplest example is a small discontinuity on a smooth beam pipe, like a small axisymmetric cavity or a hole. Let the area of the longitudinal cross section of the cavity be A, and its length and depth be small compared to the pipe radius b, i.e. $A \ll b^2$. In this case there exists a trapped mode [8] with the frequency slightly below the pipe cutoff frequency $f_c = \mu_{01}c/(2\pi b)$. The on-axis longitudinal electric field of the mode is given by a simple expression $E_z(z) = E_z(0)\exp(-|z|/L)$, where $L = b^3/(\mu_{01}^2 A) \gg b$ is the characteristic length occupied by the trapped mode in the pipe.

The overlap integral is easily calculated analytically, and the ratio of the shunt impedances Eq. (3) is

$$\frac{R_s(\beta)}{R_s(1)} = \beta^4\left[\frac{(\omega L/c)^2+1}{(\omega L/c)^2+\beta^2}\right]^2 \simeq \beta^4 , \quad (5)$$

where $\omega \simeq 2\pi f_c = \mu_{01}c/b$ is the resonance frequency. The last expression in the rhs of Eq. (5) is due to the fact that $\omega L/c \gg 1$, since $\omega b/c \simeq \mu_{01}$ and $L \gg b$. The above results hold for a small hole in the pipe wall: one just has to substitute $A = \alpha_m/(4\pi b)$, where α_m is the magnetic susceptibility of the hole, in all expressions, see [8]. For comparison with a small but finite-size cavity, see [6].

3.3 APT 1-cell Cavities

As a more realistic example, consider an APT SC 1-cell cavity, e.g. [9]. Direct time-domain simulations with the codes MAFIA [4] and ABCI [5] show the existence of only 2 longitudinal modes below the cutoff for the $\beta = 0.64$

cavity, and only 1 such mode for $\beta = 0.82$, in both cases including the fundamental mode at $f_0 = 700$ MHz. The contributions from these lowest resonance modes for a Gaussian bunch with $\sigma = 3.5$ mm for $\beta = 0.64$ and $\sigma = 4.5$ mm for $\beta = 0.82$ are about 1/3 of the total loss factor.

The on-axis longitudinal field of the fundamental mode is fitted very well by $E_z(z) = E_z(0)\exp\left[-(z/a)^2\right]$, with $a = 0.079$ m for $\beta = 0.64$ and $a = 0.10$ m for $\beta = 0.82$. The ratio of the shunt impedances in Eq. (3) is then easy to get analytically: $R_s(\beta)/R_s(1) = \exp\left[-\left(\omega a/\beta\gamma c\right)^2/2\right]$, where $\omega = 2\pi f_0$. The resulting loss factor for the lowest mode for the cavity design value of $\beta = 0.64$ is 0.378 times that with $\beta \to 1$, and for $\beta = 0.82$ is 0.591 times the corresponding $\beta \to 1$ loss factor.

3.4 APT 5-cell Cavities

For 5-cell APT SC cavities the lowest resonances are split into 5 modes which differ by a phase advance per cell, $\Delta\Phi$, and their frequencies are a few percent apart [9]. The calculated on-axis fields of all five modes in the TM_{010}-band, with $\Delta\Phi$ from $\pi/5$ to π — the last one is the accelerating mode of the cavity — are shown for the cavity with $\beta = 0.82$ in Fig. 1, left column. Using MAFIA results for the fields of the modes to calculate overlapping integrals in Eq. (2) with an arbitrary β, we find the β-dependencies of the loss factors for the five TM_{010}-modes, see the right column of Fig. 1. Obviously, they are strongly influenced by the mode field pattern.

When we sum up all five contributions to the loss factor for this band, the resulting dependence on β is smooth, and the total loss factor decreases monotonically as β decreases, see Fig. 2. As seen in Fig. 2, the loss factor of the accelerating mode is maximal near the design β, while for all other modes it is almost zero in that region. This is not surprising, since the cavity design is optimized for that value of β to provide a strong interaction of the accelerating mode with the beam.

A similar picture holds for the $\beta = 0.64$ APT cavity: loss factors of individual modes in the given band show a rather irregular β-dependence, with peaks at different values of β, but their sum smoothly decreases with the β decrease. It works for both bands below the cutoff frequency of the pipe, TM_{010} and TM_{020}. The total loss factor for all longitudinal modes below the cutoff is shown versus β in Fig. 3, as well as the separate contributions of both bands. The contribution of the TM_{010}-band is certainly larger, about 0.5 V/pC for $\beta = 1$, compared to less than 0.1 V/pC from TM_{020}-band. These $\beta = 1$ results are in a good agreement with direct time-domain simulations using ABCI [5], see [6]. In the velocity range near the design value of $\beta = 0.64$, TM_{010} contribution dominates, and the total loss factor is mostly due to the accelerating π-mode, cf. Fig. 3. The total loss factors for a given resonance band are lower for the design β than at $\beta = 1$, except the TM_{020} band for the $\beta = 0.82$ cavity. This band, however, includes some propagating modes, and for those the
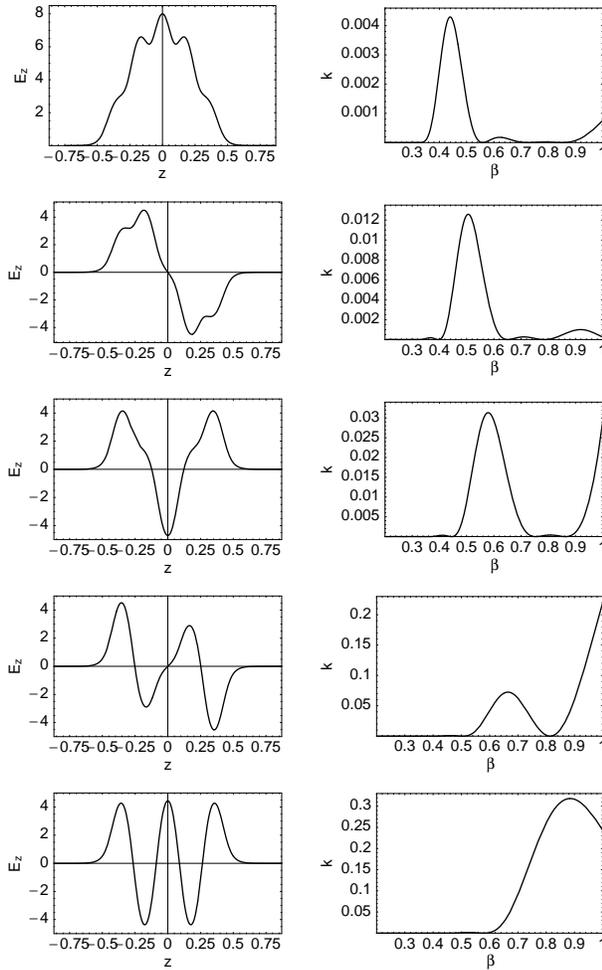
Figure 1: Longitudinal component of the on-axis electric field (arbitrary units) for the TM_{010}-modes in the 5-cell APT $\beta = 0.82$ cavity versus z (m) (left column), and their loss factors (V/pC) versus β (right) .

Figure 2: The loss factor for all TM_{010}-modes in the 5-cell APT $\beta = 0.82$ cavity versus β (solid). The contribution of the accelerating (π) mode is short-dashed, and that of all others is long-dashed.

Figure 3: The total loss factor for all modes below the cutoff in the 5-cell APT $\beta = 0.64$ cavity versus β. The contribution of the TM_{010}-band is long-dashed and that of TM_{020}-band is short-dashed. The thin solid line shows the loss factor for the accelerating mode, TM_{010} π-mode.

frequency-domain results can not be trusted. Its contribution is certainly very small anyway, see [6] for detail.

Of course, the contribution of higher-frequency modes to the bunch loss factor can also be significant. However, the beam energy transferred to the higher modes, which have frequencies above the cutoff and propagate out of the cavity into the beam pipes, will be deposited elsewhere outside the cavity. For the SC cavities, we are mostly concerned about the lowest resonance modes, below the cutoff, since they contribute to the heat load on the cavity itself.

4 DISCUSSION

The bunch loss factors were calculated with the frequency-domain approach as functions of the beam velocity β. The approach is useful when we know the fields of all modes that contribute significantly into the bunch energy loss. Calculating modes above the cutoff presents most difficulties, since there is no well-established numerical method, except for periodic structures. Nevertheless, for many prac-
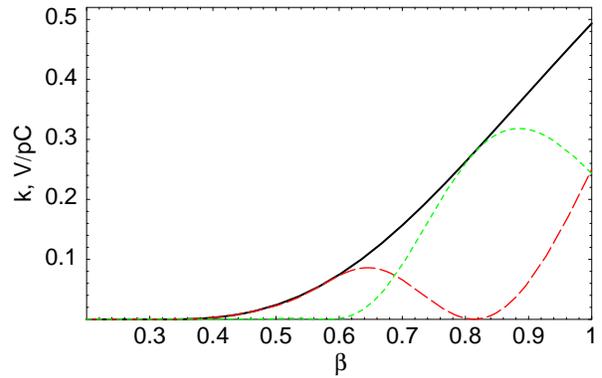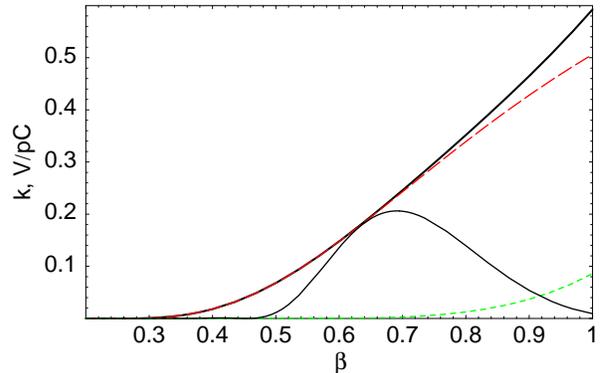
tical applications, especially in SC cavities, the contribution of the lowest modes is a major concern, because the above-cutoff modes travel out of the cavity and deposit their energy far away from the structure cold parts, where the heat removal is not a big problem.

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