MEASUREMENTS OF TUNE SHIFTS WITH AMPLITUDE AT LEP

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Abstract

The beam orbit system of the LEP electron-positron collider is able to store the beam position over 1000 turns following a deflection by a horizontal kicker. A precise analysis of such 1000-turn data for many beam position monitors was used to study the dependence of the tune on the horizontal amplitude. The horizontal tune shift with amplitude was determined from the decay of the beam oscillation for various LEP optics. This parameter turned out to be an important issue for the LEP high energy optics.

1 MOTIVATION

During the last years stronger focusing lattices have been developed to improve the LEP performance at high energy [1]. This search for a good high energy optics has revealed the importance of the horizontal detuning with amplitude (anharmonicity) to guarantee a sufficient aperture for the beam. This is due to the fact that with the regular tune working point at LEP (the fractional part of Q_H is about 0.28) a large detuning with amplitude drives particles on the third order resonance.

2 DAMPED COHERENT OSCILLATIONS

The detuning with amplitude can be determined from an analysis of "1000-turn" measurements. A coherent horizontal oscillation is excited by a single kick and the centre-of-charge position of the bunch is observed over 1024 consecutive turns. Figure 1 shows such a 1000-turn measurement for one BPM. The moment of the kick is clearly visible. A phase space representation of the same dataset is shown in figure 2 where the horizontal centre-of-charge position of a bunch at two BPMs with a distance of $\approx 90^o$ in phase is plotted. The Courant-Snyder invariant

$$W \sim x_A^2 + x_B^2$$

can be calculated from the measured positions at the two BPMs. A Fourier spectrum ("Lomb periodogram", [2]) of the time dependence of W shows the tune components (fig. 2). The third order components as well as the change in the horizontal tune Q_x are clearly visible.

A"global" fit to the single BPM 1000-turn data using a damped oscillation with amplitude dependent frequency yields the coherent damping time τ. The coherent damping

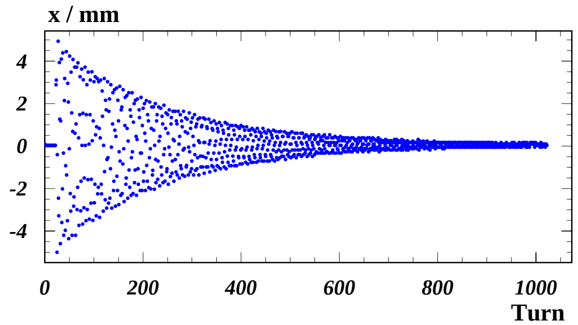

Figure 1: *Centre-of-charge position of a bunch versus turn number at an arc monitor. The kick and the subsequent damped oscillation are clearly visible.*

at LEP is composed of radiation and head-tail damping:

$$1/\tau_{\text{coh}} = 1/\tau_0 + 1/\tau_{\text{head-tail}} \ \text{ with } \ 1/\tau_{\text{head-tail}} \sim \frac{Q'}{E_0} I_{\text{b}}$$

where Q' is the chromaticity, I_{b} the bunch current and E_0 the beam energy.

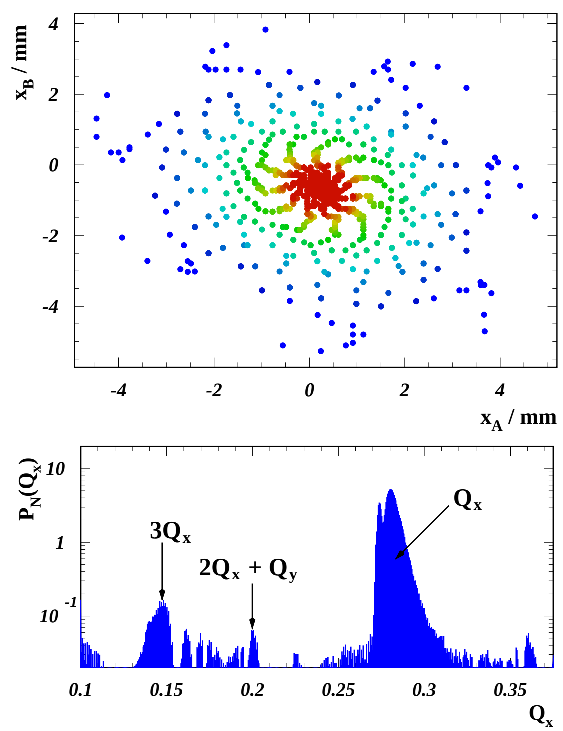

Figure 2: *Upper plot: Centre-of-charge position of a bunch after a kick at two BPMS with a distance of 90^o in phase. Lower plot: Fourier spectrum ("Lomb periodogram") of the Courant-Snyder invariant W. The detuning with amplitude is clearly seen on Q_x and $3\,Q_x$ and $2\,Q_x + Q_y$ are present.*

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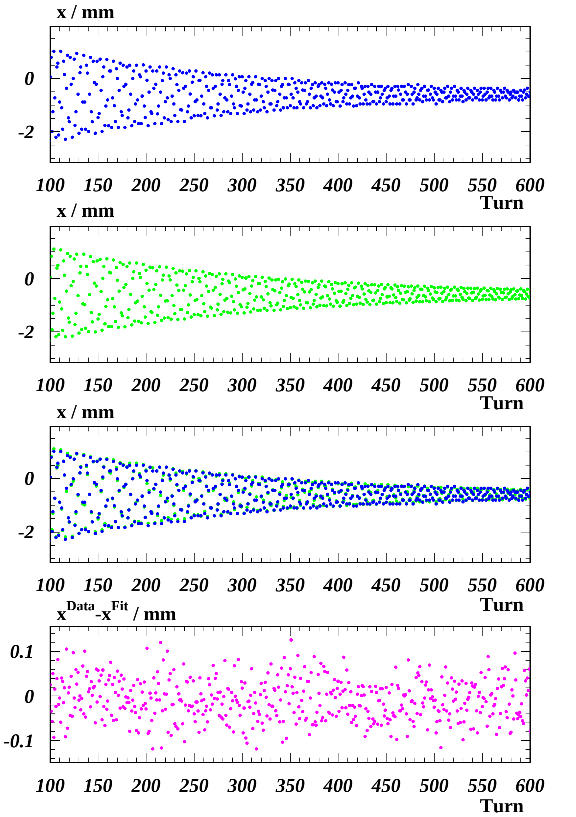

Figure 3: *Centre-of-charge position of a bunch versus turn number at an arc monitor. The upper two plots show data and the results of a "global" fit to the oscillation. The third plot is an overlay of data and fit results and the fourth shows the difference.*

This relation holds down to very low bunch currents. No filamentation effect is observed down to the smallest measurable currents. In general the head-tail effect is the dominating damping mechanism. Figure 3 shows such a global fit. The first plot shows the measured data, the second one the results of the fit and the third an overlay of measurement and fit results. The last plot of fig. 3 shows the fit residuals. Obviously the damping behaviour is very well reproduced by the fit model.

3 EXTRACTING THE HORIZONTAL DETUNING WITH AMPLITUDE

A series of fits is used to extract the horizontal detuning with amplitude from the 1000-turn measurements. Using the damping time from the previously mentioned "global" fit, the tune evolution with time is extracted by fitting damped harmonic oscillations to subsamples of several turns. The results of such fits to the data of fig. 1 are shown in fig. 4 where the resulting tune is plotted as a function of the turn number. The horizontal detuning with amplitude ($\partial Q_x/\partial W_x$) is given by the dependence of the tune Q on the Courant-Snyder invariant W. The latter is easily

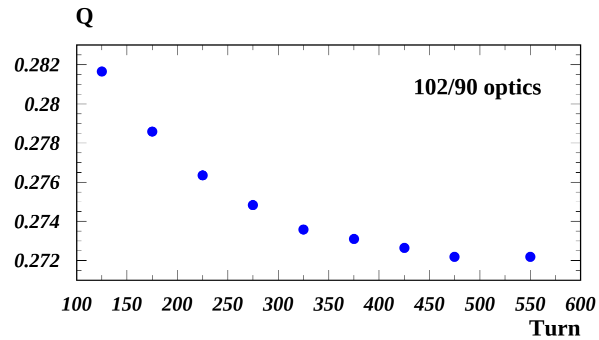

Figure 4: *Tune evolution corresponding to the damped bunch oscillation shown in figure 1.*

calculated from

$$W = \frac{1+\alpha^2}{\beta}x^2 + 2\alpha x x' + \beta x'^2 \quad \approx \quad A^2/\beta$$

with

$$A = A_0 e^{-t/\tau}$$

where τ is the damping time and A_0 the zero turn amplitude. x stands for a single particle position and A denotes the measured amplitude of the centre-of-charge oscillation of a bunch. Figure 5 shows the relation between tune Q and Courant-Snyder invariant W for one BPM. The detuning is given by the slope using:

$$Q = Q_0 + \frac{\partial Q_x}{\partial W_x}\, W$$

This analysis is applied to all 240 beam position monitors of the arcs in order to improve the statistics for a reliable measurement. Averaging over all arcs also cancels periodic perturbations like β-beating. The statistical error is given by the standard deviation of all single measurements

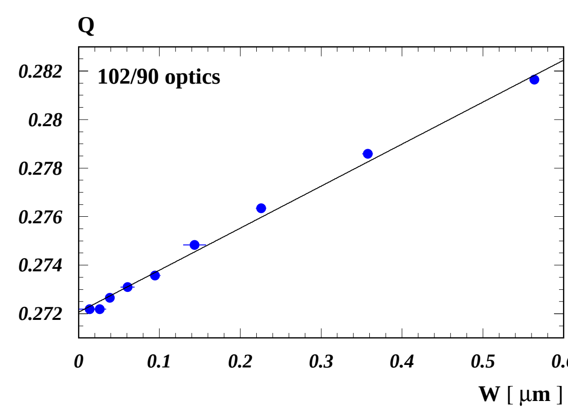

Figure 5: *Tune as function of Courant-Snyder invariant for the data sample shown in fig. 1 and fig. 4 (the straight line is a fit to the data).*

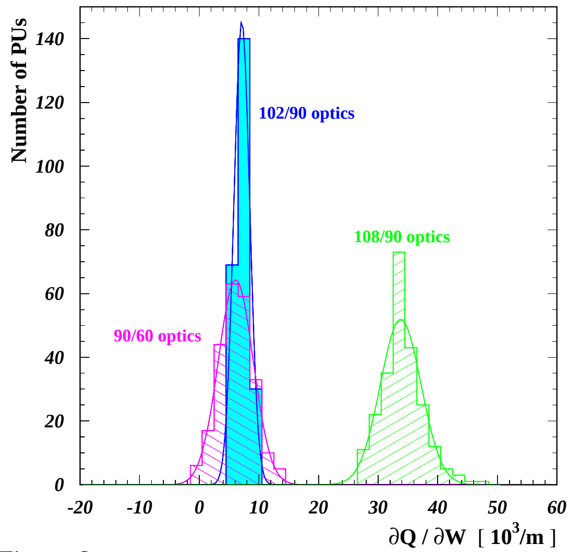

Figure 6: *Horizontal detuning with amplitude for the LEP physics optics with horizontal/vertical phase advance in the arcs of 90/60, 102/90 and 108/90 degrees.*

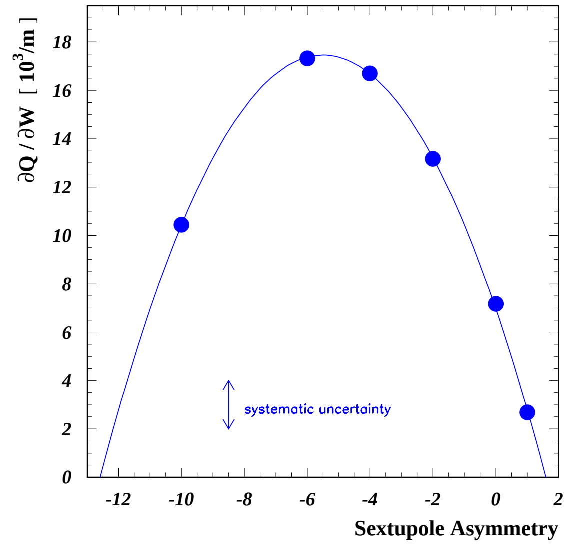

Figure 7: *Measured horizontal detuning with amplitude for different sextupole settings. Measurement errors are drawn. The size of the systematic error is given.*

$(\sigma/\sqrt{N})$. Simulation studies of the systematic error with the MAD simulation program [3] show that the method is reliable.

Datasets for several optics have been analysed. The results are shown in tab. 1 and fig. 6. The histograms of fig. 6 represent the distributions of measurements for all arc monitors. The distributions are consistent with a Gaussian shape and are clearly separated. Although the measured detuning is usually larger than the predictions for a perfect machine there is a reasonable agreement between measurements and predictions. A more detailed description of this topic can be found in [4].

4 DETUNING OPTIMISATION

Three horizontal sextupole families are used to optimise the chromaticity and tune shifts with amplitude. The detuning can be trimmed around its nominal value by changing the strength K of sextupole families in opposite directions while keeping the chromaticity constant ($n_1\, K_1^s + n_3\, K_3^s = \mathrm{const}$ where n_1 and n_3 are the number of sextupoles in the families). The horizontal detuning with amplitude has been measured for several such settings of the sextupoles. Figure 7 shows that the detuning can be changed in a controlled way. The measurement errors are drawn and the systematic uncertainty is shown.

Optics	$\partial Q_x/\partial W_x$ $[10^3/\mathrm{m}]$	
	MAD	Measured
90/60	1.7	6.2 ± 0.8
102/90	7.0	7.2 ± 1.0
108/90	25.0	34.0 ± 1.0

Table 1: *Measured and predicted horizontal detuning with amplitude for three LEP physics optics. The errors include statistical and systematic uncertainties.*

5 SUMMARY

Measurements of damped coherent oscillations following a horizontal kick have been used to study the horizontal tune shift with amplitude. The horizontal detuning can be extracted in a precise and robust way and the measured tune shifts with amplitude confirm predictions of the LEP model.

6 ACKNOWLEDGEMENTS

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7 REFERENCES

[1] Y. Alexahin et al. Low emittances lattices for LEP2. In *Proceedings of the 17th Particle Accelerator Conference*, 1997.

[2] N.R. Lomb. In *Astrophysics and Space Science*, volume 39, pages 447–462. 1976.

[3] H. Grote and F. C. Iselin. The MAD Program, User's Reference Manual. SL Note 90-13 (AP) (Rev. 4), CERN, March 1995.

[4] A.-S. Müller and J. Wenninger. Measurements of Coherent Damping and Tune Shifts with Amplitude at LEP. In *Proceedings of the 6th European Particle Accelerator Conference*, pages 1377–1379, 1998.