THE BEAM LIFETIME FROM ELASTIC SCATTERING ON NUCLEI OF RESIDUAL GAS IN ELECTRON STORAGE RING WITH THE VARIOUS SHAPE OF THE VACUUM CHAMBER.

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Abstract

In report the process of elastic scattering of the stored beam on nuclei of residual gas with take into account of size and shape of the vacuum chamber of a storage ring is considered. The expressions permitting to calculate a cross section of elastic scattering of electrons on nuclei of residual gas and partial lifetime of a beam, stipulated by this process for the various shape of a cross of the vacuum chamber (ellipse, cylinder, rectangle) are obtained.

1 INTRODUCTION

The total lifetime of an electron beam in storage ring τ_Σ is determined as the sum of inverse partial lifetimes stipulated by various processes. As main processes, defining lifetime of a beam in a storage ring it is possible to consider the following: elastic scattering of electrons on nucleuses of residual gas, bremsstrahlung on nucleuses of residual gas, inelastic scattering of electrons on nucleuses of residual gas, quantum fluctuations of a radiation, collective effects. Experimentally and theoretically is shown [1,2] that for small beam currents the lifetime is determined by elastic scattering of electrons on nuclei of residual gas. There is a number of diagnostic techniques [3], based on a measurement of lifetime of a beam for "zero currents" and definition through it of other parameters of a beam in particular of dynamic aperture. In the present work the technique of deriving of dependence of lifetime of a beam stipulated by process of the elastic scattering for various shapes of the vacuum chamber from characteristic geometric sizes of the chamber is indicated and the expressions obtained for the elliptic, round and rectangular chamber are indicated.

2 LIFETIME OF ELECTRONS STIPULATED BY SCATTERING ON RESIDUAL GAS.

The loss rate of electron beam in storage ring is given by:

$$\frac{1}{\tau} = -\frac{1}{n_0}\frac{\mathrm{d}n}{\mathrm{d}t}, \qquad (1)$$

where n_0 – the number of electrons, $\mathrm{d}n/\mathrm{d}t$ - velocity of the lost.

In case, when the main channel of losses of electrons is the scattering on residual gas, electron beam, after passing through a volume with residual gas with number of atoms N and thickness dx loses dn of particles:

$$\mathrm{d}n = -n_0 \sigma N \,\mathrm{d}x, \qquad (2)$$

where σ - the cross section of scattering of electrons causing to it to loss.

Then the time of loss of electrons by a relativistic beam will make:

$$\frac{1}{\tau} = -\frac{1}{n_0}\frac{\mathrm{d}n}{\mathrm{d}t} = c\,\sigma\,N, \qquad (3)$$

where c – velocity of a light.

Defining the lifetime as the time it takes for the initial particle intensity to be reduced by *1/e* and considering residual gas distributed is uniform in the chamber of a storage and number of atoms N_0 in unit of volume to constants, the lifetime is determined by the equation [4]:

$$\frac{1}{\tau_{SC}} = c\sigma\,N_0. \qquad (4)$$

Thus for definition of lifetime of a beam stipulated by elastic scattering of electrons on atoms of residual gas, it is necessary to determine a cross section of scattering of an electron causing to it to loss. Obviously, that the magnitude of this cross section will depend on the shape and sizes of the vacuum chamber.

3 CROSS SECTION OF ELASTIC SCATTERING.

Differential cross section of elastic scattering on nuclei of residual gas has the form [4]:

$$\frac{d\sigma}{d\Omega} = \frac{Z^2}{4} r_0^2 \left(\frac{m_e c}{\beta\ p}\right)^2 \frac{\left(1-\beta^2 \sin^2\frac{\theta}{2}\right)}{\sin^4\frac{\theta}{2}}, \qquad (5)$$

where dΩ=sinθ dθ dφ - the solid angle in which electron is scattered,

φ, θ - axial and polar angle of scattering,
Z – charge of nuclei of residual gas,
r_0 - radius of an electron,
βc – velocity of an electron,
p – momentum of an electron,

It can rewrite as:

$$d\sigma(\theta,\varphi) = \frac{Z^2}{4} r_0^2 \left(\frac{m_e c}{\beta\ p}\right)^2 \frac{\left(1-\beta^2 \sin^2\frac{\theta}{2}\right)}{\sin^4\frac{\theta}{2}} \sin\theta\, d\theta\, d\varphi\ . \qquad (6)$$

For small angle of scattering θ <<1 and relativistic electrons γ>>1 the equation (6) we can write in the form:

$$d\sigma(\theta,\varphi)=\frac{Z^2}{2}r_0^2\frac{1}{\gamma^2}\frac{\cos^3\left(\theta/2\right)}{\sin^3\left(\theta/2\right)}\,\mathrm{d}\theta\,\mathrm{d}\varphi = \frac{Z^2}{2}r_0^2\frac{1}{\gamma^2}\frac{8\,\mathrm{d}\theta\,\mathrm{d}\varphi}{\theta^3} \quad (7)$$

Thus the cross section of elastic scattering in (4) is possible to receive by an integration of expression (6) on θ from θ_{min} – minimum angle, the scattering on which results in loss of an electron up to π and on φ from 0 up to 2π. After an integration on θ we find:

$$d\sigma(\varphi)=\frac{2Z^2r_0^2}{\gamma^2}\frac{\mathrm{d}\varphi}{\theta_{\min}^2}\ . \quad (8)$$

For the various shape of the vacuum chamber expression for θ_{min} will differ, besides for case is axial of the asymmetrical chamber θ_{min} will be function from φ.

The cross section of elastic scattering, causing to loss of an electron is possible to note as:

$$\sigma=\frac{2Z^2r_0^2}{\gamma^2}\int_0^{2\pi}\frac{\mathrm{d}\varphi}{\theta_{\min}^2(\varphi)}= \frac{2Z^2r_0^2}{\gamma^2}F\left(b,a,\beta_x,\langle\beta_x\rangle,\beta_z,\langle\beta_z\rangle\right), \quad (9)$$

where a and b geometric parameters of the vacuum chamber on a vertical and horizontal plane,

β_x, $<\beta_x>$, β_z, $<\beta_z>$ - maximum and average value of amplitude functions on a horizontal and vertical plane.

After collision with atom of residual gas on azimuth s_0 the particle deviates on angles $\dot{x}=\sin\theta\cos\varphi$ and $\dot{z}=\sin\theta\sin\varphi$. Then on some azimuth s_1 this particle will have coordinates:

$$\dot{x}=x\sqrt{\beta_x(s_0)\beta_x(s_1)}\sin\left(\int_{s_0}^{s_1}\frac{\mathrm{d}s}{\beta_x(s)}\right)= \sin\theta\cos\varphi\sqrt{\beta_x(s_0)\beta_x(s_1)}\sin\left(\int_{s_0}^{s_1}\frac{\mathrm{d}s}{\beta_x(s)}\right)$$

$$\dot{z}=z\sqrt{\beta_z(s_0)\beta_z(s_1)}\sin\left(\int_{s_0}^{s_1}\frac{\mathrm{d}s}{\beta_z(s)}\right)= \sin\theta\sin\varphi\sqrt{\beta_z(s_0)\beta_z(s_1)}\sin\left(\int_{s_0}^{s_1}\frac{\mathrm{d}s}{\beta_z(s)}\right).$$

The radius vector of this particle will be noted as $r^2=x^2+z^2$.

Considering, that on azimuth s_1 the particle reaches the boundary of the vacuum chamber r=ρ and that phase advance $\int_{s_0}^{s_1}\frac{\mathrm{d}s}{\beta_x(s)}=\int_{s_0}^{s_1}\frac{\mathrm{d}s}{\beta_z(s)}=0$ On this azimuth, we can write expression for a determination θ_{min}. For it we need only to make out a radius vector ρ of the boundary of the vacuum chamber. For example, for the elliptic vacuum chamber:

$$\frac{z^2}{a^2}+\frac{x^2}{b^2}=1,\ z=\rho\sin\varphi,\ x=\rho\cos\varphi\,,$$

from here

$$\rho^2=\frac{1}{\left(\dfrac{\sin^2\varphi}{a^2}+\dfrac{\cos^2\varphi}{b^2}\right)}.$$

Then we get:

$$\frac{1}{\theta_{\min}^2}=\left(\frac{\sin^2\varphi}{a^2}+\frac{\cos^2\varphi}{b^2}\right)\left[\beta_z(s_0)\beta_z(s_1)\sin^2\varphi+\beta_x(s_0)\beta_x(s_1)\cos^2\varphi\right] \quad (10)$$

Integrating (10) from 0 up to 2π and averaging on all azimuths is received expression for function F for case of the elliptic vacuum chamber. Substituting it in (9), and then (9) substituting in (3), we receive lifetime stipulated by elastic scattering on atoms of residual gas in the chamber of a storage ring the elliptic vacuum chamber.

4 LIFETIME FOR CASES OF THE VARIOUS SHAPE OF THE VACUUM CHAMBER.

We obtained expressions for function F for several most typical variants of the vacuum chamber and disposition in it of the diagnostic equipment bounding the geometric aperture of the chamber. For a simplicity all devices bounding the aperture of the vacuum chamber, we shall name as "shutters". Is clear, that from expression for the vacuum chamber with a shutter it is easy to proceed to expression for the vacuum chamber without a shutter.

1. The rectangular vacuum chamber, rectangular shutter.

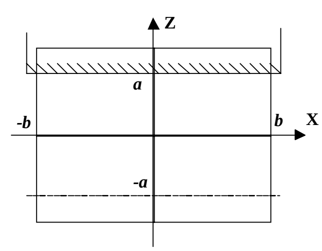

Figure 1.

$$F=\left\{\beta_z\langle\beta_z\rangle\left[\frac{3}{4}\frac{\pi}{a^2}+\frac{a^2-3b^2}{2a^2b^2}\mathrm{arctg}\frac{a}{b}+\frac{a^2-b^2}{2ab\left(a^2+b^2\right)}+\frac{2b}{a\left(a^2+b^2\right)}\right]+\beta_x\langle\beta_x\rangle\left[\frac{1}{4}\frac{\pi}{a^2}-\frac{b^2-3a^2}{2a^2b^2}\mathrm{arctg}\frac{a}{b}+\frac{b^2-a^2}{2ab\left(a^2+b^2\right)}+\frac{2a}{b\left(a^2+b^2\right)}\right]\right\}$$

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2. The elliptic vacuum chamber, elliptic shutter.

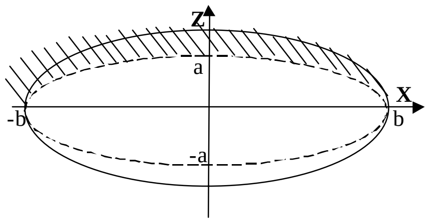

Figure 2.

$$F=\left\{\beta_z\langle\beta_z\rangle\left[\frac{3}{4}\frac{\pi}{a^2}\left(1+\frac{a^2}{3b^2}\right)\right]+\beta_x\langle\beta_x\rangle\left[\frac{3}{4}\frac{\pi}{a^2}\left(1+\frac{b^2}{3a^2}\right)\right]\right\}$$

3. The elliptic vacuum chamber, rectangular shutter..

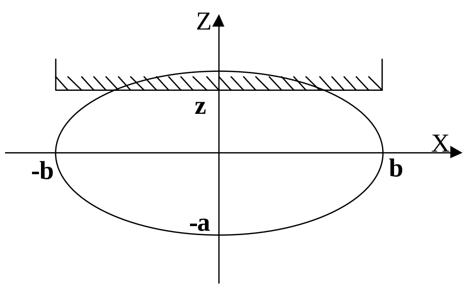

Figure 3.

$$F=\left\{\beta_z\langle\beta_z\rangle\left[\frac{3}{4}\frac{\pi}{z^2}+\frac{1}{2}\left(\frac{3}{a^2}+\frac{1}{b^2}-\frac{3}{z^2}\right)\operatorname{arctg}\frac{az}{b\sqrt{a^2-z^2}}+\right.\right.$$
$$\left(1-\frac{z^2}{a^2}\right)\frac{2ab\sqrt{a^2-z^2}}{z\left[b^2\left(a^2-z^2\right)+a^2z^2\right]}+$$
$$\left.\left(\frac{3}{a^2}-\frac{1}{b^2}-\frac{1}{z^2}\right)\frac{abz\sqrt{a^2-z^2}\left[b^2\left(a^2-z^2\right)-a^2z^2\right]}{2\left[b^2\left(a^2-z^2\right)+a^2z^2\right]^2}\right]+$$
$$\beta_x\langle\beta_x\rangle\left[\frac{1}{4}\frac{\pi}{z^2}+\frac{1}{2}\left(\frac{3}{b^2}+\frac{1}{a^2}-\frac{1}{z^2}\right)\operatorname{arctg}\frac{az}{b\sqrt{a^2-z^2}}+\right.$$
$$\frac{2az\sqrt{a^2-z^2}}{b\left[b^2\left(a^2-z^2\right)+a^2z^2\right]}+$$
$$\left.\left.\left(\frac{1}{b^2}-\frac{1}{a^2}+\frac{1}{z^2}\right)\frac{abz\sqrt{a^2-z^2}\left[b^2\left(a^2-z^2\right)-a^2z^2\right]}{2\left[b^2\left(a^2-z^2\right)+a^2z^2\right]^2}\right]\right\}$$

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4. The round vacuum chamber, rectangular shutter.

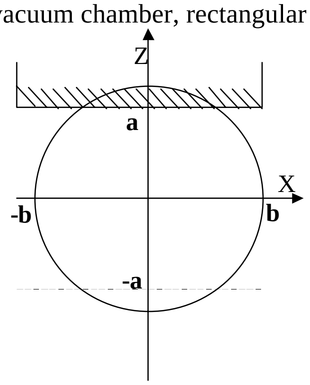

Figure 4.

$$F=\left\{\frac{\beta_z\langle\beta_z\rangle}{a^2}\left[\frac{3\pi}{4}+\frac{4a^2-3b^2}{2b^2}\arcsin\frac{a}{b}+\frac{a\sqrt{b^2-a^2}}{b^4}\left(3b^2-2a^2\right)\right]+\right.$$
$$\left.\frac{\beta_x\langle\beta_x\rangle}{b^2}\left[\frac{\pi b^2}{4a^2}+\frac{4a^2-b^2}{2a^2}\arcsin\frac{a}{b}+\frac{\sqrt{b^2-a^2}}{2ab^2}\left(2a^2-b^2\right)\right]\right\}$$

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5 CONCLUSION

The obtained expressions allow analytically to connect lifetime of a beam stipulated by elastic scattering on residual gas, to amplitude functions of a storage ring and geometric sizes of transversal area of circulation of a beam. Besides that it allows to evaluate influence of the shape of the vacuum chamber to lifetime of a beam, the installation of such relation allows by a measurement of lifetime for " zero currents " to determine sizes of area of stable motion of a beam (dynamic aperture).

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