

# ADIABATIC EXCITATION OF LONGITUDINAL BUNCH SHAPE OSCILLATIONS\*

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## Abstract

By modulating the rf voltage at near twice the synchrotron frequency we are able to modulate the longitudinal bunch shape. We show experimentally that this can be done while preserving the longitudinal emittance when the rf voltage modulation is turned on adiabatically. Experimental measurements will be presented along with theoretical predictions.

## 1 INTRODUCTION

We have studied adiabatic excitation of bunch shape oscillations both theoretically and in experiments performed at Brookhaven's Alternating Gradient Synchrotron (AGS). rf voltage was adiabatically modulated in order to excite parametric resonances. Similar studies are described in [1, 2, 3]. In our studies we focused on the adiabatic creation of a parametric resonance, such that the emittance of the bunch was preserved. In this way we were able to achieve very tight bunches at high proton beam intensities.

This technique was exploited during our fast proton extraction run for the g-2 experiment [4], which needed short muon beam pulses produced by short proton bunches, to most effectively exploit the pulse width of the injection kicker into their storage ring. Beam is extracted in single multiple bunch pulses at a rate of 30 Hz. To synchronize the extraction kicker pulses with the minimum bunch width we modulated the AGS rf gap voltage at 180 Hz, and adjusted the average rf gap voltage such that the synchrotron frequency of the central particle was just under 90 Hz. This allowed the bunch shape to modulate at almost 50% of the normal bunch width. This is more than twice the range achievable by just raising the gap voltage of the AGS rf cavities.

In our experiments we slowly ramped the drive amplitude of the rf voltage. We ramped the drive frequency while ramping the drive amplitude. While ramping the voltage amplitude and while holding it in steady state with fixed frequency at the peak amplitude, the average rf voltage amplitude is kept constant over a full modulation cycle, so that the average synchrotron tune is unchanged. If the drive frequency is just above twice the synchrotron frequency then, as we will show, there is no blow-up of the longitudinal emittance. In experiments we studied the effects of drive frequencies from well above twice the synchrotron

frequency, as well as frequencies well below twice the synchrotron frequency.

We compare tomographic reconstructions of the bunch to our models. The reconstructions required great care, since the rf amplitude varied over a large range. Despite this fact we were able to accomplish very good reconstructions.

## 2 LONGITUDINAL MOTION WITH MODULATED FOCUSING STRENGTH

The synchrotron motion can be described using the conjugate phase space coordinates  $(\phi, \delta = \frac{h\eta}{\nu_s} \frac{\Delta p}{p})$  where  $\phi$  is the particle phase relative to the synchronous particle,  $h$  the harmonic number,  $\eta$  the phase slip factor,  $\nu_s = \sqrt{\frac{h|\eta|eV}{2\pi\beta^2 E}}$  the synchrotron tune at zero amplitude without modulation and  $p$  the particle momentum. The discrete synchrotron equations for a stationary bucket above transition can then be written as [5, 3]

$$\begin{aligned}\phi_{n+1} &= \phi_n + 2\pi\nu_s\delta_{n+1} \\ \delta_{n+1} &= \delta_n + 2\pi\nu_s[1 + \epsilon \sin(\nu_m\theta_{n+1} + \chi)] \sin \phi_n\end{aligned}\quad (1)$$

where  $\epsilon$  is the percentage of rf voltage modulation,  $\nu_m$  is the modulation tune,  $\chi$  the initial modulation phase and the orbital angle  $\theta$  a time-like variable. Eqs. (1) corresponds to the Hamiltonian

$$H = \frac{\nu_s}{2}\delta^2 + \nu_s[1 + \epsilon \sin(\nu_m\theta + \chi)][1 - \cos \phi]. \quad (2)$$

For particles within a bucket the canonical transformation  $\phi = \sqrt{2J} \cos \psi$ ,  $\delta = -\sqrt{2J} \sin \psi$  to action-angle variables can be used, which allows to go into a rotating coordinate frame later. The Hamiltonian can now approximated by [3]

$$\begin{aligned}H &= \nu_s J - \frac{\nu_s}{16} J^2 + \nu_s \epsilon \sin(\nu_m\theta)[1 - J_0(\sqrt{2J})] \\ &\quad + \sum_{k=1}^{\infty} (\Delta H_{2k}^{(+)} + \Delta H_{2k}^{(-)}) \quad \text{with} \\ \Delta H_{2k}^{(\pm)} &= \nu_s \epsilon (-1)^{k+1} J_{2k}(\sqrt{2J}) \sin(\nu_m\theta \pm 2k\psi).\end{aligned}\quad (3)$$

The  $J_n$  are Bessel functions of the order  $n$  and we have set  $\chi = 0$  for convenience. If the modulation tune is only slightly different from twice the synchrotron tune the term  $\Delta H_{2k}^{(-)}$  in Eq. (3) is only slowly changing with time while all other terms  $\Delta H_{2k}^{(\pm)}$  are rapidly oscillating and

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are therefore neglected. We now go into a coordinate system that rotates around the origin with half the modulation tune by using the canonical transformation  $J = \tilde{J}$ ,  $\tilde{\psi} = \psi - \frac{\nu_m}{2}\theta - \frac{3\pi}{4}$ . The new Hamiltonian has terms oscillating at  $\nu_m, 2\nu_m, \dots$  which average to zero over time and the time averaged Hamiltonian in this coordinate system is [3]

$$\langle \tilde{H} \rangle = (\nu_s - \frac{\nu_m}{2})\tilde{J} - \frac{\nu_s}{16}\tilde{J}^2 + \frac{\nu_s}{4}\epsilon\tilde{J}\cos 2\tilde{\psi} \quad (4)$$

The investigation of fixed points of the Hamiltonian (4) leads to the following result:

- If  $\nu_s(2 + \frac{\epsilon}{2}) < \nu_m \Rightarrow$  The origin  $J = 0$  is the only fixed point, a stable one.
- If  $\nu_s(2 - \frac{\epsilon}{2}) < \nu_m < \nu_s(2 + \frac{\epsilon}{2}) \Rightarrow$  There is one unstable fixed point at the origin and two stable fixed points at  $J = 8(1 - \frac{\nu_m}{2\nu_s}) + 2\epsilon$  and  $\tilde{\psi} = 0, \pi$ .
- If  $\nu_m < \nu_s(2 - \frac{\epsilon}{2}) \Rightarrow$  There is a stable fixed point at the origin, two more stable fixed points at  $J = 8(1 - \frac{\nu_m}{2\nu_s}) + 2\epsilon$  and  $\tilde{\psi} = 0, \pi$  and two unstable fixed points at  $J = 8(1 - \frac{\nu_m}{2\nu_s}) - 2\epsilon$  and  $\tilde{\psi} = \frac{\pi}{2}, \frac{3\pi}{4}$ .

We work in the regime (b) where particles are pushed away from the origin and held in two islands. This situation is depicted in Fig. 1 and known as a parametric resonance [6].

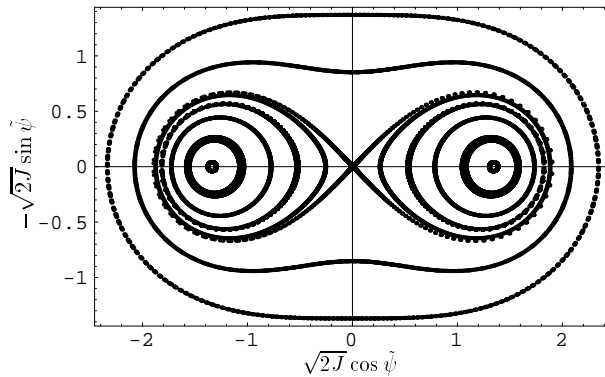


Figure 1: Particle trajectories in a surface of section of the Hamiltonian (4) for case (b).

To reach the state depicted in Fig. 1 the modulation depth  $\epsilon$  is increased adiabatically, typically over 25 synchrotron periods.

### 3 EXPERIMENTAL RESULTS

The experiments of adiabatically modulating the rf cavity were conducted with the AGS high intensity proton beam at 24 GeV.

To achieve the adiabatic excitation of the parametric resonance, the amplitude of the rf voltage modulation was slowly ramped up in about 10 synchrotron oscillations, kept fixed for about 30 synchrotron oscillations and then slowly

ramped down to zero. Fig. 2 shows typical scope signals of this process: the modulated rf voltage, the beam peak detector signal and the beam current signal.

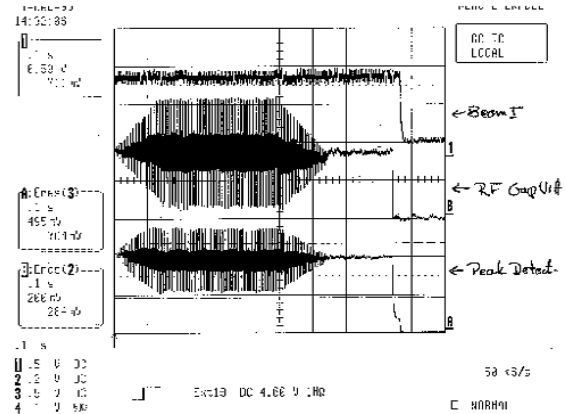


Figure 2: Typical scope signals with the modulated rf voltage signal, the beam peak detector signal and the beam current signal. The time scale on the horizontal axis is 0.1 sec/div.

The rf voltage was driven by an amplitude modulated sinusoidal signal generated by a waveform generator (WaveTek 296). The average of the rf voltage was kept unperturbed. The beam peak detector measures the peak of the beam pulse obtained from a wall current monitor. Assuming the beam current is constant, the narrower the bunch length, the higher the beam peak detector signal.

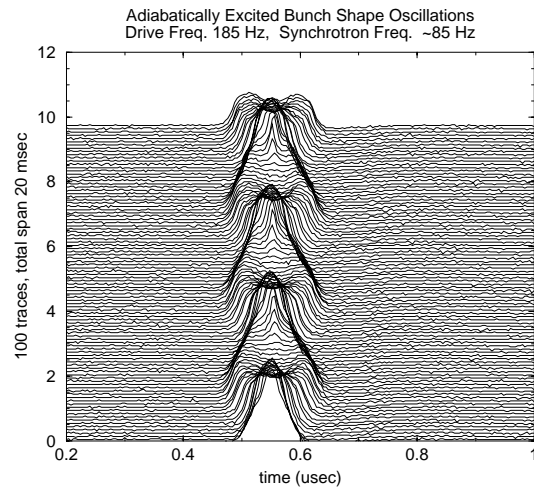


Figure 3: Driven bunch shape oscillations seen with the wall current monitor.

The average synchrotron frequency in Fig. 2 is 85 Hz, and the rf voltage drive amplitude  $\epsilon = 0.8$ . The rf voltage signal and the beam peak detector signal shows that the bunch shape oscillations nicely follow the rf voltage modulation. The stronger the rf voltage is driven, the stronger the bunch shape oscillates. As the rf voltage modulation is

ramped down to zero, the bunch shape oscillation is also diminished. This indicates that the parametric resonance was excited adiabatically.

The corresponding beam longitudinal profile evolution is shown in Fig. 3 in the mountain range fashion. It clearly shows the bunch shape oscillation when the rf voltage was driven at 185 Hz, twice the synchrotron oscillation frequency.

Since  $\nu_s = 0.23$ ,  $\nu_m = 0.5$  and  $\epsilon = 0.8$  satisfy

$$\nu_s(2 - \frac{\epsilon}{2}) = 0.368 < \nu_m < \nu_s(2 + \frac{\epsilon}{2}) = 0.552, \quad (5)$$

two stable fixed points (SFTs) were developed. The beam was stretched into the two islands around these two SFTs and shaped like a “dumb-bell”, as shown in the Tomographic reconstruction of the bunch Fig. 4.

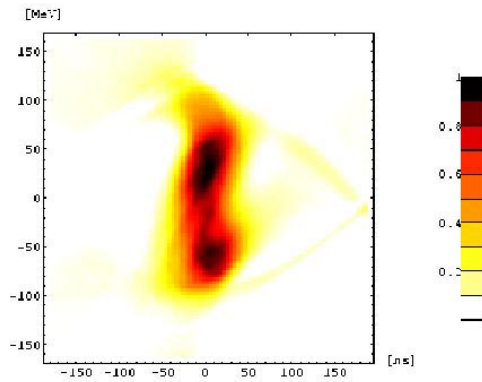


Figure 4: Phase space reconstruction of the bunch.

The bunch length was also measured with different modulation frequencies while the rf voltage drive amplitude was kept fixed. Fig. 5 shows that the minimum bunch length becomes shorter when the rf voltage is driven closer the resonance frequency

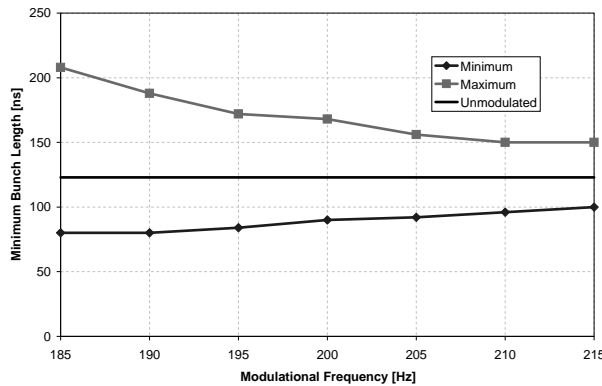


Figure 5: Measured bunch length versus modulation frequency.

## 4 CONCLUSIONS

By modulating the rf voltage at twice the synchrotron frequency, a parametric resonance can be excited. The longitudinal beam emittance can be preserved if the excitation is adiabatic. This technique can allow us to obtain very short bunches and was successfully applied in the AGS fast extraction of high intensity proton beam for the g-2 experiment, which requires proton beam with short bunch length.

## 5 ACKNOWLEDGMENTS

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