CALCULATION OF LONGITUDINAL FIELDS OF HIGH-CURRENT BEAMS WITHIN CONDUCTING CHAMBERS

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Abstract

A technique for computing the longitudinal electric fields of a bunched beam propagating inside a conducting pipe is presented. A beam bunch is represented as a series of discs or slices, and the total electrical field is found by superposition of the fields of individual slices. The results of this technique agree well with other independent algorithms. The primary motivation for developing this technique is to provide efficient space charge calculations for beam dynamics simulation. However, the formalism may be employed in other applications to find electric fields for various beam density distributions in the presence of conducting boundaries.

1 INTRODUCTION

Simulations that include the effects of space charge provide a crucial basis for the design of high-current accelerator systems. In some cases, particularly for unbunched beams, two-dimensional (2-D) beam simulation particle-in-cell (PIC) codes are sufficient. However, for the case of bunched beams, the longitudinal effects can become important, and 3-D or nearly 3-D simulations are required.

The general 3-D PIC methods provide completely self-consistent models, but frequently require very long computational times. As a result, the exploration of possible design space can, as a practical matter, be limited. An alternative approach is the modification of a 2-D PIC formulation to include important aspects of the longitudinal dynamics that will provide a nearly complete physics model and a computational speed that dramatically exceeds the general 3-D approach. A simple linear model of the longitudinal electrical field, corresponding to parabolic line charge density, is valid for ellipsoid-like beams in free space. However, in the presence of a conducting beam chamber with dimensions comparable to the transverse beam size, the longitudinal electric field becomes non-linear due to image charges. The non-linearity is especially apparent for the case of long or longitudinally asymmetric bunches. This paper describes a fast and accurate computational approach for the calculation of the longitudinal electrical field of beams with a relatively arbitrary charge density distribution within a conducting boundary. We believe the proposed method provides a unique strategy for the inclusion of longitudinal dynamics in simulation codes.

2 THE PROBLEM TO BE SOLVED

A common analytical approach to calculate the space charge electric fields of a beam propagating inside a conducting chamber is to find the solution of the Poisson equation. For rectangular or free space regions, the space charge potential may be found via the convolution integral in a rather simple and fast way [1]. However, with the inclusion of a conducting cylindrical surface, the Green's function, satisfying the zero boundary conditions, is expanded via modified Bessel functions. Simple analytical evaluation in this case is possible only after simplifying assumptions [2]. Even for such a simple situation as an ellipsoidal bunch in a cylindrical pipe, numerical methods are required to find the fields. Poisson Solvers using Cartesian or cylindrical grids [3] employing the FFT technique are now in common use in PIC codes. The approach described below is related to a Green's function formalism based on the charge density method [4] appropriate for PIC simulation.

Charge Density Method

For free space, the potential u, produced by the charge density ρ within the volume V is equal to

$$u(\mathbf{r}_0) = \int_V \frac{\rho(\mathbf{r}')}{\mathbf{R}} dV(\mathbf{r}')$$

where $1/\mathbf{R} = 1/|\mathbf{r}_0 - \mathbf{r}'| = 1/\sqrt{(x_0 - x')^2 + (y_0 - y')^2 + (z_0 - z')^2}$ is the Green's function. When V represents a symmetrical bunch, the potential may be rewritten as:

$$u(x_0, y_0, z_0) = \qquad (1)$$

$$= \int_{-Z_b}^{+Z_b} \int_0^{2\pi} \int_0^{R(z,\varphi)} \frac{\rho(r,\varphi,z)\ rdr\ d\varphi\ dz}{\sqrt{(r\cos\varphi - x_0)^2 + (r\sin\varphi - y_0)^2 + (z - z_0)^2}}$$

where for the particular case of an ellipsoid-like bunch $R(z,\varphi) = R_b\sqrt{1-(z/Z_b)^2}$ (with R_b and Z_b the bunch radius and half-length correspondingly). If the charge density is given, then equation (1) determines the corresponding potential u. Conversely, if the potential u is known, then the corresponding ρ may be found from an integral equation (1). The formalism known as the moment method described below is from [5]. A similar technique called the charge density method [6] is commonly used in electron and ion optics.

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3 SLICE FORMALISM

The original charge density method is too slow to be used repeatedly during step-by-step PIC simulation. We have developed a modification of the charge density method, which assumes a discrete representation of the bunch via charged discs or slices (see Figure 1). The total beam field is determined by superposition of individual fields.

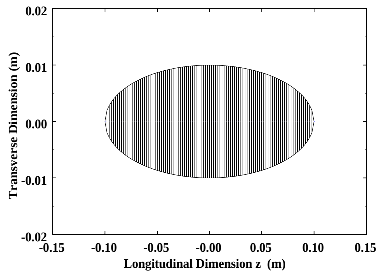

Figure 1. A uniformly populated ellipsoidal beam bunch modeled as 100 discrete slices. The bunch radius is R_b=0.01 m and the half bunch length is Z_b =0.1 m.

For a single slice of radius R^{slice} and charge density $\sigma^{slice}(r)$ the potential in the free space can be expressed:

$$u^{slice}_{free}(x_0, y_0, z_0) = \int_0^{2\pi}\int_0^{R^{slice}} \frac{\sigma^{slice}(r)\, rdr\, d\varphi}{\sqrt{(r\cos\varphi - x_0)^2 + (r\sin\varphi - y_0)^2 + z_0^{\ 2}}} \qquad (2)$$

(The integration over z is not required for the infinitesimally thin slice). If the point (x_0, y_0, z_0) is on the cylindrical surface, then $x_0^{\ 2} + y_0^{\ 2} = R_{cyl}^{\ 2}$ and $u(x_0, y_0, z_0)$ defines the potential due to a single slice on the cylindrical surface at longitudinal position z_0. The same potential evaluated at the position of that boundary ($r \equiv R_{cyl}$) with opposite sign, is used to find the unknown surface image charge density σ^{image} on the cylinder from a single slice:

$$-u(x_0, y_0, z_0) = \int_0^{2\pi}\int_{-Z_L}^{+Z_L} \frac{R_{cyl}\sigma^{image}(r)\, dz\, d\varphi}{\sqrt{(R_{cyl}\cos\varphi - x_0)^2 + (R_{cyl}\sin\varphi - y_0)^2 + (z - z_0)^2}} \qquad (3)$$

with $z_0 \in [-Z_L, +Z_L]$ ($Z_L = 4Z_b$ was found empirically). Reapplication of (3) for $x_0^{\ 2} + y_0^{\ 2} = r^2$ ($0 \le r \le R_{cyl}$) and for the determined σ^{image} provides $u^{slice}_{image}(r,z)$. The total potential: $u^{slice}_{total}(r,z) = u^{slice}_{free}(r,z) + u^{slice}_{image}(r,z)$ will then automatically satisfy the zero boundary conditions. The longitudinal on-axis potentials ($r=0$) for the middle slice are given in Figure 2.

When this procedure is applied in a PIC code, it is proposed that the potentials and electrical field values be calculated once for different slices and stored. During the simulation, these values with interpolation would be used to provide an accurate model of the longitudinal dynamics without the penalty of long computational time [4].

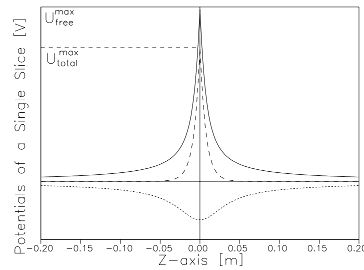

Figure 2. The free space (solid curve), image charge (dotted) and total (dashed) potential for a single slice.

4 APPLICATIONS OF THE SLICE TECHNIQUE

In the numerical examples below, we consider a specific case of an elliptical bunch inside a conducting cylindrical pipe of radius 0.02 m as shown in Figure 1. The half bunch length, is Z_b =10 cm, the maximum transverse radius is R_b =1 cm, and the total charge is $Q_{total}=10^{-11}$ C. The transverse dimensions of the beam were assumed circular with a uniform radial charge distribution. The longitudinal dimension was assumed parabolic thus resulting in a parabolic line charge density $\lambda(z)$. In this example, the bunch was modeled as 100 individual slices.

For the full bunch in the absence of the conducting chamber, superposition of potentials for all slices results in parabolic potential and linear field. With the inclusion of a conducting cylinder, the total field (by superposition) is given in Figure 3. Note that, for this case, the $E_z(z)$ within the bunch is strongly non-linear. The potential and fields, calculated for Z_b / R_b =10, 5, 1 are in very good agreement [4] with reference [7] page 407.

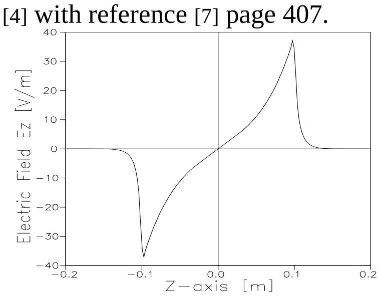

Figure 3. The longitudinal electric field $E_z(z)$ [V/m] along the beam axis, for the case of a bunch with m and the half bunch length is Z_b / R_b =10.

More complicated situations, such as when the bunch has a non-symmetrical form in the longitudinal direction, can also be accommodated by the algorithm. Shown in Figure 4, is a possible asymmetrical bunch within a 4 cm diameter conducting pipe again modeled as 100 individual slices with the calculated $E_z(z)$ given in Figure 5. Though the geometrical shape of the bunch of Figure 4 is not dramatically different from that of Figure 1, the electric potential and field have significantly different profiles. The $E_z(z)$ is more non-linear, than that of Figure 3, changing sign four times. In this case, the g-factor method [7] would lead to the wrong result.

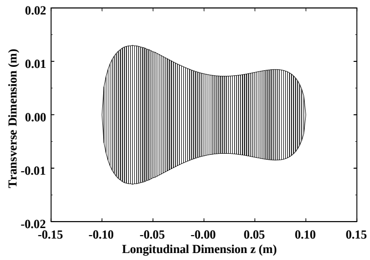

Figure 4. Asymmetrical bunch.

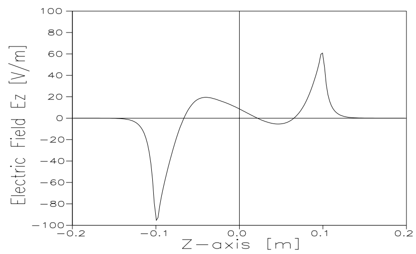

Figure 5. Longitudinal field $E_z(z)$ [V/m] found by slice superposition for the asymmetrical bunch of Figure 4.

Off-axis Electrical Field

So far we have computed only on-axis field $E_z(z)$ - at zero radial position. Shown in Figure 6 are the potentials and fields for different radial positions. From Figure 6 (all dimensions are the same as those of Figure 1), the approximation of simply using the on-axis value for $E_z(z)$ will cause significant errors only near the bunch ends. However, the maximum $Z_{max}(r)$ values for the example shown are $Z_{max}\{r=(0, 0.4, 0.8)\times R_b\}$= 0.1, 0.09, 0.06 m. (The symbols are plotted at Z_{max} values in Figure 6) I.E., there are few or no particles in the area of significant deviation from E_z on-axis value.

Note that the off-axis potential $u(r,z)$ as a function of r may be used to determine the transverse field E_r for all (r,z) that additionally could be employed, under some circumstances, to produce a very fast algorithm.

5 DISCUSSION AND CONCLUSIONS

For the examples given, a cylindrical vacuum pipe was assumed. However, the procedure can be used to accommodate more general chamber boundaries and beam shapes and this generalization is planned. To reduce simulation time, a range of slice geometries (in the simplest case – a range of discs of different radii) will be calculated and tabulated.

The required number of these "template" slices is an order of magnitude less than the number of slices representing the bunch. In the examples presented, 15 different slice configurations were used requiring a computational time of about 5 min (on a multi-user 433 MHz DEC Alpha). The example bunch configurations were modeled as 100 distinct slices with the potential for each slice derived from appropriate interpolation of the tabulated data and scaling for charge density. The construction of the total bunch potential required only about 0.2 s. Since the pre-simulation calculations (about 5 min) need only be done once while the accelerator simulations require many computations of the total bunch potential (about 0.2 s) as the particles are tracked through the system, the decrease in overall computational time will be enormous.

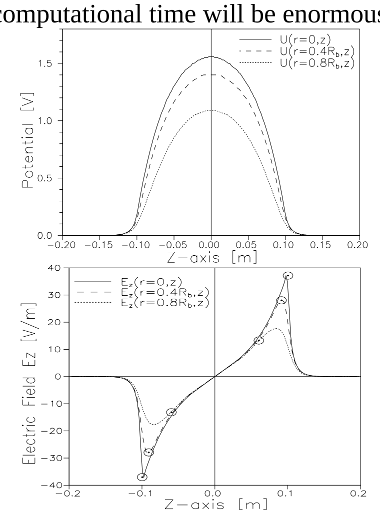

Figure 6. The dependence of $u(r,z)$[V] and the field $E_z(r,z)$ [V/m] as a function of z for r = (0, 0.4 and 0.8)×R_b [m].

We are aware that the proposed sub-3-D PIC code will not be completely self-consistent since the planned pre-simulation calculational procedure will be unable to reflect all possible evolutions of the particle density. However, preliminary studies of the radial dependence of E_z field for different distributions suggests that the effect of variations in the transverse charge distribution is minor, and therefore, the analysis will be nearly self-consistent. Comparisons with the general 3-D codes are planned.

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