STUDIES FOR MUON COLLIDER PARAMETERS AT CENTER-OF-MASS ENERGIES OF 10 TEV AND 100 TEV

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Abstract

Parameter lists are presented for speculative muon colliders at center-of-mass energies of 10 TeV and 100 TeV. The technological advances required to achieve the given parameters are itemized and discussed, and a discussion is given of the design goals and constraints. An important constraint for multi-TeV muon colliders is the need to minimize neutrino radiation from the collider ring.

1 INTRODUCTION

The main motivation for research and development efforts on muon collider technology is the assertion that affordably priced muon colliders might provide lepton-lepton collisions at much higher center of mass (CoM) energies than is feasible for electron colliders, and perhaps eventually explore the spectrum of elementary particles at mass scales inaccessible even to hadron colliders.

This paper attempts a first look at these assertions through discussion and evaluation of the self-consistent muon collider parameter sets given in table 1, at CoM energies of 10 TeV and 100 TeV. These parameter sets have the purpose of pin-pointing the challenges of very high energy muon colliders and they have not been studied or discussed in detail within the Muon Collider Collaboration (MCC) or elsewhere.

The 10 TeV parameter set was presented previously [1] and the parameter values appear to be internally consistent. In contrast, the 100 TeV parameter set represents work in progress to improve in luminosity and other parameters over the self-consistent 100 TeV parameter set given in reference [1] and it's parameters are not yet fully consistent, as discussed below.

2 GENERATION OF PARAMETER SETS

As described previously [1], the parameter sets in table 1 were generated through iterative runs of a stand-alone FORTRAN program, LUMCALC. The parameter sets are calculated from the input values for several input parameters – namely, the CoM energy (E_{CoM}), the collider ring circumference (C) and depth below the Earth's surface (D), the beam momentum spread (δ) and 6-dimensional invariant emittance (ϵ_{6N}), the reference pole-tip magnetic field for the final focus quadrupoles ($B_{4\sigma}$), and the time until the beams are dumped (t_D) – and from the input of maximum allowable values for several other parameters – namely, the bunch repetition frequency (f_b), the initial number of

muons per bunch (N_0), the beam-beam tune disruption parameter ($\Delta\nu$), the beam divergence at the interaction point (σ_θ), the maximum aperture for the final focus quadrupoles ($A_{\pm4\sigma}$), and maximum allowable neutrino radiation where the plane of the collider ring cuts the Earth's surface.

As a preliminary stage of calculation, LUMCALC makes any parameter adjustments that may be required to satisfy the input constraints. These are, in order: 1) reducing σ_θ to the limit imposed by $A_{\pm4\sigma}$ (based on scaling to existing final focus designs at 0.1 TeV and 4 TeV[1]), 2) reducing N_0 to attain an acceptable $\Delta\nu$, and 3) reducing f_b until the neutrino radiation is acceptable.

3 DISCUSSION

The physics motivation for each of the parameter sets in table 1 is discussed in [2]. Briefly, the number of $\mu\mu \to$ ee events gives a benchmark estimate of the discovery potential for elementary particles at the full CoM energy of the collider, while the production of hypothesized 100 GeV Higgs particles indicates roughly how the colliders would perform in studying physics at this fixed energy scale.

Both parameter sets give exciting luminosities with good potential to explore the physics processes at and below their respective CoM energy scales.

The output luminosity may be derived in terms of the input parameters as:

$$
\begin{aligned}
\mathcal{L}[\text{cm}^{-2}.\text{s}^{-1}] \quad = \quad & 2.11 \times 10^{33} \times H_B \times (1 - e^{-2t_D[\gamma\tau_\mu]}) \\
& \times \frac{f_b[\text{s}^{-1}](N_0[10^{12}])^2(E_{CoM}[\text{TeV}])^3}{C[\text{km}]} \\
& \times \left(\frac{\sigma_\theta[\text{mr}].\delta[10^{-3}]}{\epsilon_{6N}[10^{-12}]} \right)^{2/3}. \quad (1)
\end{aligned}
$$

This formula uses the standard MCC assumption[1] that the ratio of transverse to longitudinal emittances can be chosen in the muon cooling channel to maximize the luminosity for a given ϵ_{6N}. The pinch enhancement factor, H_B, is very close to unity (see table 1), and the numerical coefficient in equation 1 includes a geometric correction factor of 0.76 for the non-zero bunch length, $\sigma_z = \beta^*$ (the "hourglass effect") .

In practice, the muon beam power and current are limited so the optimization of parameters actually involves the "specific luminosity"

$$
l \equiv \frac{\mathcal{L}}{f_b \times N_0}. \quad (2)
$$

Further, the parameter sets at these high energies are always limited by $\Delta\nu$ and it is useful and easy to rewrite

equations 1 and 2 in the form

$$l \propto \frac{\Delta\nu}{\beta*}, \tag{3}$$

with no explicit dependence on emittance for a given energy. This is misleading, however, since smaller emittances mean smaller magnet apertures and hence allow the design of lattices with smaller $\beta*$'s and, in practice, one can almost always gain in specific luminosity by reoptimizing parameter sets with smaller emittances.

The assumed 6-dimensional emittances are factors of 3.5 (10 TeV) and 50 (100 TeV) smaller than the value 170×10^{-12} m^3 that is normally used in MCC scenarios for first generation muon colliders, in anticipation that the muon cooling channel may eventually be improved through further design optimization, stronger magnets, higher gradient rf cavities and other technological advancements and innovations.

The final focus region presumably presents the most difficult design challenge that is relatively specific to high energy muon colliders. (The muon cooling channel is, of course, a formidable challenge for all muon colliders.) Progressively stronger focusing is required at higher energies to generate the smaller spot sizes necessary to increase the luminosity. The strength of the focusing can be gauged from the overall beam demagnification, defined as $M \equiv \sqrt{\beta_{\max}/\beta*}$. This is a dimensionless parameter that should be closely correlated with fractional tolerances in magnet uniformity, residual chromaticity etc. Hence, it might be prudent to decrease the fractional momentum spread of the beams, δ, to compensate for an increasing M. In the absence of final focus designs for these parameter sets the difficulty of the chromatic correction for the final focus has simplistically been assessed by the value of a single parameter: the "chromaticity quality factor" $q \equiv M\delta$ [3].

In the absence of "proof by example" lattice designs, the next generation of proposed linear e+e- colliders (LC's) may well provide the best benchmarks for heuristically assessing the difficulty of the final focus parameters. For the 10 TeV parameter set, each of $\beta*$, M and q are roughly comparable to those proposed for these LC's. The 100 TeV parameter set necessarily has a much larger $\beta*$ and M but the value of δ was forced down to $\delta = 8 \times 10^{-5}$ to nearly hold steady the value of q. Unfortunately, this is clearly inconsistent with the rising energy loss due to beamstrahlung at the ip. Following the lead of the LC's, the next iteration of the 100 TeV parameter set will use colliding *flat beams* to make the parameter sets fully consistent. The spot size – clearly indicative of vibration and alignment tolerances – also falls with energy, but even at 100 TeV it remains an order of magnitude above the spot size in the y coordinate for future LC design parameters.

Neutrino radiation is an extremely serious problem for many-TeV muon colliders and further information and discussion on this can be found in these proceedings [4]. The radiation levels are predicted to rise rapidly with the collider energy [4] and beyond CoM energies of a few TeV it may well be necessary to build the colliders at isolated sites where the public would not be exposed to the neutrino radiation disk.

4   CONCLUSIONS

Muon colliders from 10 to 100 TeV CoM energies may offer an exciting long-term future to experimental HEP and are not obviously infeasible to this author. However, they clearly introduce some daunting new challenges beyond those common to all muon colliders and the parameter sets in table 1 will require some technological extrapolations.

5   REFERENCES

[1] The Muon Collider Collaboration, "Status of Muon Collider Research and Development and Future Plans", to be submitted to Phys. Rev. E.

[2] B.J. King, "Discussion on Muon Collider Parameters at Center of Mass Energies from 0.1 TeV to 100 TeV", 19 June, 1998, Submitted to Proc. Sixth European Particle Accelerator Conference (EPAC'98), Stockholm, Sweden, 22-26 June, 1998. Available at http://pubweb.bnl.gov/people/bking/.

[3] The idea to use this parameter emerged through discussions with Frank Zimmermann and Carol Johnstone .

[4] B.J. King, "Potential Hazards from Neutrino Radiation at Muon Colliders", these proceedings.

Table 1: Example parameter sets for 10 TeV and 100 TeV muon colliders. The generation of these parameter sets is discussed in the text. These parameters represent speculation by the author on how muon colliders might evolve with energy. The beam parameters at the interaction point are defined to be equal in the horizontal (x) and vertical (y) transverse coordinates.

center of mass energy, E_{CoM}	10 TeV	100 TeV
collider physics parameters:		
luminosity, $\mathcal{L}$ [cm^{-2}.s^{-1}]	1.0×10^{36}	3.1×10^{37}
$\int \mathcal{L} dt$ [fb^{-1}/det/year]	10 000	310 000
No. of $\mu\mu \to$ ee events/det/year	8700	2700
No. of 100 GeV SM Higgs/det/year	1.4×10^{7}	6.5×10^{8}
fract. CoM energy spread, σ_E/E [10^{-3}]	1.0	0.08
collider ring parameters:		
circumference, C [km]	15	100
ave. bending B field [T]	7.0	10.5
beam parameters:		
(μ^- or) μ^+/bunch,N_0[10^{12}]	2.4	2.0
(μ^- or) μ^+ bunch rep. rate, f_b [Hz]	15	10
6-dim. norm. emittance, ϵ_{6N}[10^{-12}m^3]	50	3.5
x,y emit. (unnorm.) [$\pi.\mu$m.mrad]	0.55	0.0046
x,y normalized emit. [π.mm.mrad]	26	22
fract. mom. spread, δ [10^{-3}]	1.4	0.084
relativistic γ factor, E_μ/m_μ	47 322	473 220
ave. current [mA]	24	7.9
beam power [MW]	58	320
decay power into magnet liner [kW/m]	1.4	2.4
time to beam dump, $t_D[\gamma\tau_\mu]$	no dump	0.5
effective turns/bunch	1039	985
interaction point parameters:		
spot size, $\sigma_x = \sigma_y$[nm]	780	93
bunch length, σ_z [mm]	1.1	0.185
β^* [mm]	1.1	0.185
ang. divergence, σ_θ [mrad]	0.71	0.5
beam-beam tune disruption parameter, $\Delta\nu$	0.100	0.100
pinch enhancement factor, H_B	1.108	1.130
beamstrahlung fract. E loss/collision	2.3×10^{-7}	6.5×10^{-4}
final focus lattice parameters:		
max. poletip field of quads., $B_{4\sigma}$ [T]	15	20
max. full aperture of quad., $A_{\pm 4\sigma}$[cm]	20	88
β_{max}[km]	1100	260 000
final focus demagnification, $M \equiv \sqrt{\beta_{max}/\beta^*}$	31 000	1.2×10^6
chrom. quality factor, $Q \equiv M\delta$	43	100
synchrotron radiation parameters:		
syn. E loss/turn [MeV]	17	25 000
syn. rad. power [MW]	0.4	200
syn. critical E [keV]	12	1700
neutrino radiation parameters:		
collider reference depth, D[m]	300	300
ν beam distance to surface [km]	62	62
ν beam radius at surface [m]	1.3	0.13
str. sect. length for 10x ave. rad., L_{x10}[m]	1.0	2.4
ave. rad. dose in plane [mSv/yr]	0.66	12.6