# COLLECTIVE INSTABILITIES OF THE 50-50 GeV MUON COLLIDER

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*Abstract*

Single bunch instabilities for the 50-50 GeV muon collider are discussed. An impedance budget for the collider is estimated. An $|\eta| = 1 \times 10^{-6}$ is desired to avoid excessive rf systems. Potential-well distortion can be compensated by rf cavities. Longitudinal microwave growth can be reduced by smoothing the bunch distribution before injection. Transverse microwave instability can be damped by chromaticities and octupoles. Beam breakup can be cured by BNS damping in principle, but is nontrivial in practice. More detailed discussions are given in Ref. 1.

## 1 IMPEDANCE BUDGET

Each of the bunches in the 50-50 GeV muon collider will have an intensity of $N = 4 \times 10^{12}$ muons, rms bunch length $\sigma_\ell = 4$ cm, and rms momentum spread $\sigma_\delta = 1.2 \times 10^{-3}$. There is another mode of operation for precision determination of the Higgs mass, when the rms momentum spread is only $\sigma_\delta = 3.0 \times 10^{-5}$ and rms bunch length $\sigma_\ell = 13$ cm. With such a high intensity and small momentum spread, the study of bunch instabilities becomes an important task.

Because there is about 3.5 cm of tungsten shielding in the superconducting magnets, the physical aperture of the colliding ring will not be large. As a result, careful bunch monitoring becomes essential. Assuming strip-line beam-position monitors like those of the Tevatron, the low-frequency longitudinal and transverse impedances for $M$ pairs of strip lines are [2, 3]

$$\frac{Z_\|}{n} = j2MZ_c \left[\frac{\phi_0}{2\pi}\right]^2 \frac{\ell}{R}, \quad Z_\perp = \left[\frac{4}{\phi_0}\right]^2 \frac{R}{2b^2} \sin^2 \frac{\phi_0}{2}\left[\frac{Z_\|}{n}\right]. \quad (1)$$

Here, each strip-line has a length $\ell \sim 10$ cm subtending a full angle of $\phi_0 \sim 75°$ at the axis of the cylindrically symmetric beam pipe of radius $b$, and forms a transmission line of characteristic impedance $Z_c = 50\,\Omega$ with the vacuum chamber. Note that these impedances are roughly independent of the circumference $C = 2\pi R = 350$ m of the collider ring, because more BPM's will be needed for a larger ring. Assuming $M = 56$ pairs of strip lines, 28 for the horizontal and 28 for the vertical, we obtain $Z_\|/n = j0.44\,\Omega$ and $Z_\perp = j0.066$ MΩ/m (half of the ring with $b = 2$ cm and half with 4 cm). It is worthwhile to point out that the longitudinal impedance is independent of the beam pipe radius.

The impedances of the bellows are roughly proportional to the ring circumference. For the Tevatron, which has a circumference of 6.28 km, the contributions at low frequencies are [3] $Z_\|/n \sim j0.4\,\Omega$ and $Z_\perp \sim j0.4$ MΩ/m. However, there are many more elements per unit length for the muon colliders. We scale them by a factor of 10 to get $Z_\|/n \sim j0.04\,\Omega$ and $Z_\perp \sim j0.04$ MΩ/m. The Tevatron bellows are unshielded. Thus, these impedances will be much smaller if the bellows in the muon collider are shielded.

For the resistive walls of the vacuum chamber, the contributions to the impedances are

$$\frac{Z_\|}{n} = \frac{\text{sgn}(n) + j}{2b} \left[\frac{Z_0\rho C}{\pi|n|}\right]^{1/2}, \quad Z_\perp = \frac{C}{\pi b^2}\left[\frac{Z_\|}{n}\right]. \quad (2)$$

where $Z_\perp$ is to be evaluated at $n + \nu_\beta$ and $\nu_\beta$ is the betatron tune. For an aluminum beam pipe with resistivity $\rho = 0.0265\,\mu\Omega$-m, we obtain $Z_\|/n = [\text{sgn}(n) + j]0.62|n|^{-1/2}\,\Omega$ and $Z_\perp = [\text{sgn}(n) + j]0.13|n + \nu_\beta|^{-1/2}$ MΩ/m, using $b = 2$ and 4 cm for each half of the ring. Thus, at the beam-pipe cutoff frequency, $f_c = 2.405c/(2\pi b) = 5.74$ GHz, $c$ being the light velocity, the resistive-wall contributions become negligible compared with those from the BPM's.

The lattice of the collider ring entails large variations of the betatron functions, $\sim 1500$ m in the final focusing quadrupoles, $\sim 100$ m in the local chromaticity-correction region, and $\lesssim 25$ m in the arcs. For this reason, there will be plenty of transitions in beam-pipe cross section, leading to significant contributions to the coupling impedances. If a broad-band impedance model with quality factor $Q = 1$ is assumed, it will be difficult to have $\mathcal{I}mZ_\|/n \lesssim 0.5\,\Omega$, $\mathcal{I}mZ_\perp \lesssim 0.1$ MΩ/m at low frequencies. The resonant angular frequency is chosen as $\omega_r = 50$ GHz, $\sim 38\%$ above cutoff, because $|Z_\|/n|$ does not roll off up to $\omega = 60$ GHz in both the CERN LEP and the SLAC damping rings.

## 2 CHOICE OF SLIPPAGE FACTOR

Longitudinally, the worst fast collective growth is the microwave instability, which has a stability limit of [4]

$$\left|\frac{Z_\|}{n}\right| \leq \frac{2\pi|\eta|E\sigma_\delta^2}{eI_{\text{pk}}}, \quad (3)$$

where $I_{\text{pk}}$ is the peak current, $E$ the muon energy, and $e$ its charge. Taking the $\sigma_\ell = 4$ cm bunch, stability can be assured only if the slippage factor $|\eta| > 0.0021$. Neglecting the influence of coupling impedance, to keep such a bunch in an rf bucket, the synchrotron tune will be $\nu_s = |\eta|R\sigma_\delta/(2\pi\sigma_\ell) = 3.54 \times 10^{-3}$, and the rf voltage will be $V_{\text{rf}} = 2\pi E\nu_s^2/|\eta| = 1.86$ GV, which is very large. Another choice is to make $|\eta|$ as small as possible. Although we have to give up Landau damping, hopefully, the growth rate, which is proportional to $\sqrt{|\eta|}$, will be slow enough and cause insignificant harm. Here, we are talking about the total spread in $\eta$, since the latter is a function of momentum spread. Because $\eta$ is related to the momentum-compaction factor, to reduce $\eta$ and its spread, we need to reduce the contribution of the higher-order momentum compaction also. Roughly speaking, sextupoles can be used to reduce $\eta_1$, octupoles reduce $\eta_2$, etc. The experience with the 2-2 TeV muon collider lattice [5] tells us that it will be hard to reduce the spread of $|\eta|$ to below $1 \times 10^{-6}$, and this value will be used in our discussion below. A particle at an energy spread of $3\sigma_\delta$ will drift by 4.2 ps (0.13 cm) only in 1000 turns. For this reason, it appears that a bunching rf will not be necessary. However, this is not true in the presence of the coupling impedance.
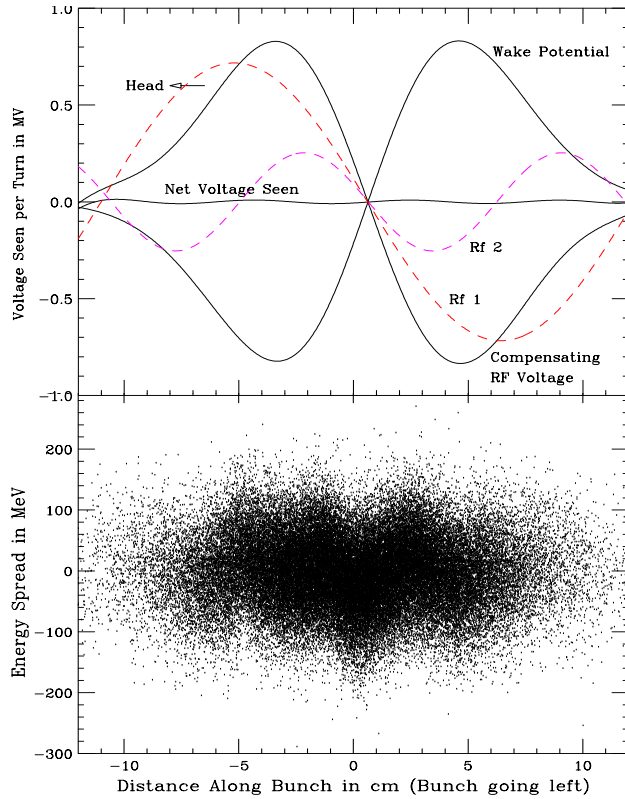
Figure 1: Top: Wake potential, compensating rf voltages, and net voltage seen by particles in the 4-cm bunch at injection. Bottom: Bunch distribution after 1000 turns.

## 3 POTENTIAL-WELL DISTORTION AND MICROWAVE INSTABILITY

Particles in the high-intensity bunch will be affected by the wake from the particles ahead. Assume a linear Gaussian distribution and a broad-band with $Q = 1$ and $\mathcal{R}e\, Z_\parallel/n = 0.5\ \Omega$ at the angular resonance frequency $\omega_r = 50$ GHz. At such a high resonant frequency, the wake potential seen by a bunch particle is roughly equal to the derivative of the Gaussian with a maximum and minimum of $\sim \pm 0.83$ MV, as shown in the top plot of Fig. 1. Therefore, in 1000 turns some particles will gain and some will lose as much as $\sim 499$ MeV, taking into consideration the reduction in intensity due to muon decay, while the designed rms energy spread is only 60 MeV. With such a large energy spread, there will be some drift in time, especially if $|\eta|$ is not too small. Thus, there may be bunch lengthening as well as particle loss due to the limited physical aperture of the vacuum chamber. If the linear distribution is parabolic, a sinusoidal rf of wavelength longer than the bunch length will compensate for this bunch distortion due to wake potential. If the linear distribution is cosine-square, a sinusoidal rf with wavelength exactly equal to the bunch length will do the job. For a Gaussian distribution, one needs a combination of sinusoids [6]. For example, to compensate up to $\pm 3\sigma_\ell$, we need at injection two sinusoidal rf's of frequencies 1.290 and 2.673 GHz, of voltages 717.4 and 253.4 kV, and of phases $170.55°$ and $159.33°$, as shown in the top
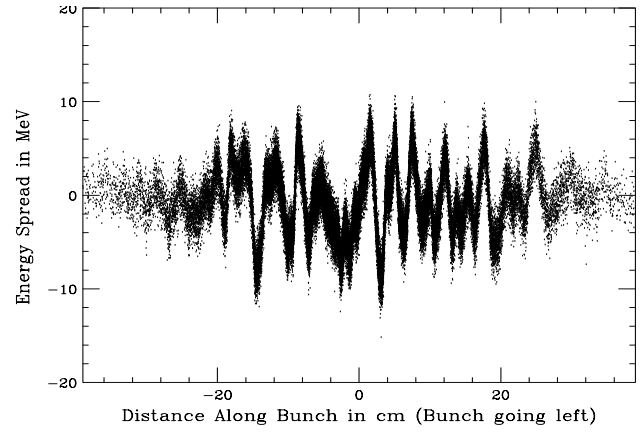


Figure 2: Distribution of 13-cm bunch after encountering the wake potential and the compensating rf's for 1000 turns

plot of Fig. 1, with the voltages decreasing turn by turn according to the decay of the muons. Tracking was done for 1000 turns using $2 \times 10^6$ macro-particles with bin size 15 ps (0.45 cm). The result is shown in lower plot of Fig. 1. We see that the compensation rf's do a good job by keeping the muons bunched without any increase in bunch length. However, there are some wigglings along the bunch. They are not due to inexact compensation, because the top plot of Fig. 1 shows that the net voltage seen has a wiggling with wavelength about 210 ps and $\sim \pm 0.008$ MV at injection or $\sim \pm 4.8$ MV for 1000 turns. On the other hand, the lower plot in Fig. 1 shows a wiggling of $\sim \pm 40$ MeV with wavelength of $\sim 125$ ps corresponding to the resonant frequency of the coupling impedance. These are signs of microwave instability. Figure 2 shows the 1000-turn tracking simulation of a rms 13-cm bunch using the same number of macro-particles with the same bin width and subject to the same broad-band impedance. The best fitted compensation rf's used at injection are at 0.3854 and 0.7964 GHz, 65.40 and 24.74 kV, and $177.20°$ and $174.28°$. Note that the rf's can only compensate for potential-well distortion and *cannot* cure microwave instability.

The Keil-Schnell criterion can be rewritten in the form

$$n_r \omega_0 \sqrt{\frac{|\eta||I_{\text{peak}}|Z_\parallel/n|}{2\pi eE}} \leq 2\omega_0|\eta|\sqrt{\frac{n_r \ln 2}{\pi}}\ , \qquad (4)$$

$\omega_0/(2\pi)$ being the revolution frequency and $n_r$ the resonant harmonic, where the left side is the raw microwave growth rate without damping and the right side the Landau damping rate, which is a few orders of magnitude smaller than the left side for both bunches and can be neglected. We therefore obtain the growth rates as $4.91 \times 10^4$ and $2.72 \times 10^4$ s$^{-1}$, respectively, for the 4-cm and 13-cm bunches. The 4-cm bunch should have a larger growth rate because the peak current is larger and momentum spread plays no role since the Landau damping rate is too small. However, Fig. 1 actually shows a less violent growth than Fig. 2. One reason is that the same number of macro-particles and same bin widths have been used in the two situations. The 4-cm distribution will have more macro-particles per bin than the 13-cm bunch and therefore smaller fluctuations, thus providing smaller seeds for the growth. The other reason is

that the impedance wavelength is comparable to the rms bunch length of the 4-cm bunch, over which the local current drops to $\sim e^{-1/2} = 0.6$ of its peak value, thus reducing the growth rate. It is important to point out that Figs. 1 and 2 *do not* indicate the actual particle distributions after 1000 turns. The actual growth depends on the initial linear bunch shape. If the initial bunch shape is extremely smooth, the total growth in 1000 turns may be very minimal. On the other hand, if the initial bunch distribution is very rugged, it will provide a large seed and the final distribution after 1000 turn can be much more violent than those depicted in Figs. 1 and 2. Therefore, to prevent excessive microwave instability, methods must be devised to smooth the bunch distribution after ionization cooling and linac acceleration. Further reduction in impedance and $|\eta|$ will lower the growth. Since the growth rate is proportional to frequency, one must try to smooth the vacuum chamber so that the impedance contribution at high frequencies will be reduced to a minimum.

## 4 TRANSVERSE MICROWAVE INSTABILITY

The Keil-Schnell-like stability criterion for transverse microwave instability can be written at chromaticity $\xi$ as [7]

$$\frac{eI_{\mathrm{pk}}|Z_\perp|c}{4\pi E\nu_\beta} \leq \frac{4\sqrt{2}\omega_0}{\pi}|(n_r-\nu_\beta)\eta + \xi|\sigma_\delta , \quad (5)$$

where the left side is the raw growth rate without damping and the right side the damping rate. With $\nu_\beta \approx 8$, the raw growth rates are $11.4\times10^3$ and $3.51\times10^3$ s$^{-1}$, respectively, for the 4-cm bunch and 13-cm bunch. For stabilization, one requires $|(n_r - \nu_\beta)\eta + \xi| > 1.7$ and 21, respectively, for the two bunches. Although our choice of $\eta$ is small, stability can still be maintained if the chromaticities are less than 1.7 and 21, if octupoles are installed to provide additional amplitude-dependent tune spread. Also, the presence of a small growth may not be serious for only 1000 turns.

## 5 TRANSVERSE BEAM BREAKUP

Since bunch particles do not move much longitudinally with respect to the bunch center during their lifetime, any off-axis particle will affect its followers constantly leading to beam breakup. Take the simple two-particle model, by which the bunch is represented by two macro-particles of charge $\frac{1}{2}eN$ separated by a distance $\hat{z} \sim \sigma_\ell$. The transverse displacements of the head, $y_1$, and the tail, $y_2$, satisfy

$$y_1'' + \frac{\nu_\beta^2}{R^2}y_1 = 0 , \quad y_2'' + \frac{\nu_\beta^2}{R^2}y_2 = \frac{e^2NW_1(\hat{z})}{2CE}y_1 . \quad (6)$$

In a length $L$, the displacement of the tail will grow $\Upsilon = e^2NW_1(\hat{z})L/(8\pi E\nu_\beta)$ times [8]. For a broad-band impedance with $Z_\perp = 0.1$ M$\Omega$/m at $\omega_r = 50$ GHz and $Q = 1$, the wake function has a maximum of $W_1 = 2\omega_rZ_\perp\cot\phi\sin\phi\,e^{-\phi\cot\phi}$ at $z=c\phi/(\omega_r\sin\phi)<\hat{z}$, where $\cos\phi = 1/(2Q)$, indicating that the two-particle model is not suitable for long bunches and high resonant frequencies. As an estimate, with this maximum value of $W_1$, the displacements of some particles along the bunch will be doubled in $\sim 27$ turns.
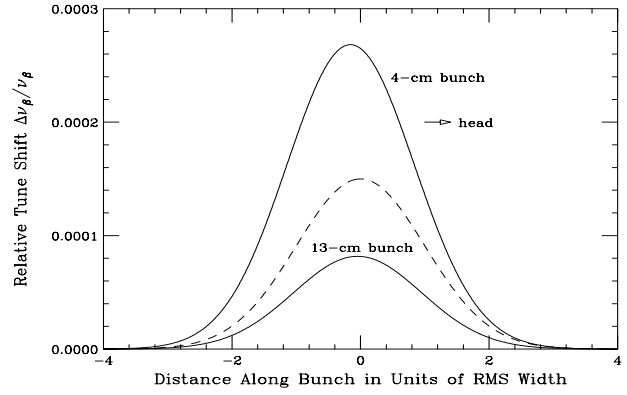


Figure 3: Relative tune shift compensation as functions of distance along the bunch to cure beam breakup. The bunch profile is plotted in dashes as a reference.

This beam breakup can be cured by varying the betatron tune of the beam particles along the bunch, so that resonant growth can be avoided. The is known as BNS damping [9]. In order that all particles in the bunch will perform betatron oscillation with the same frequency and same phase, special focusing force is required to compensate for the variation of betatron tune along the bunch according to [8]

$$\frac{\Delta\nu_\beta(z)}{\nu_\beta} = \frac{e^2NR}{4\pi\nu_\beta^2E} \int_z^\infty dz'\rho(z')W_1(z' - z) , \quad (7)$$

If the linear bunch distribution $\rho$ is a Gaussian interacting with a broad-band impedance, the integration simplifies to

$$\frac{\Delta\nu_\beta(z)}{\nu_\beta} \approx \frac{e^2N\omega_rZ_\perp R}{2(2\pi)^{3/2}\nu_\beta^2QvE} \left[1 - \frac{z}{vQ\sigma_\ell}\right]e^{-z^2/(2\sigma_\ell^2)} , \quad (8)$$

where use has been made of $v = \omega_r\sigma_\ell/c \gg Q$, which is certainly satisfied for both bunches. The relative tune shifts required for the two bunches, shown in Fig. 3, are small. To cure beam breakup, however, an rf quadrupole must be installed and pulsed according to the compensation curve for each bunch as the bunch is passing through it. Note that the compensations are only slightly shifted backward instead of linearly increasing towards the tail. These are the results of long bunches and high resonant frequencies, as was indicated in the two-particle model discussion. The shift will be more towards the tail as $v$ decreases. In practice, compensations matching these tune-shift curves are nontrivial.

## 6 REFERENCES

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