IONIZATION COOLING OF MUON BEAM BY MULTISTAGE SYSTEM OF LITHIUM LENSES WITH MATCHING SECTIONS*

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Abstract

System of lithium lenses with matching sections as the last stage of ionization cooling for muon collider is considered. The matching section including accelerating cavities, straight and bent solenoids, and wedge absorbers provide both acceleration and focusing of the beam as well as transverse-longitudinal emittance exchange. Results of analytical calculations and Monte Carlo simulations with scattering and straggling are presented. Suppression of nonlinear and chromatic effects is discussed.

1 INTRODUCTION

The latest stage of ionization cooling for muon collider requires a multistage lithium lens which can provide a long-distance β - function about 1 cm really unachievable with any other device[1, 2]. Average β - function of matching sections between Li lenses is much more and has more or less significant modulation. Chromatic aberrations are strong in such conditions laying obstacles to focusing of the beam on the next Li lens. Usual way of suppression by sextupole correctors is unusable in this case because of attendant nonlinear effects. But it can be attenuated by taking opposite energy offset ΔE of any particle on the beginning and the end of the matching section what achieved at synchrotron phase advance π per section. It requires rather high momentum of particles ($P \simeq 300$ MeV/c) because chromatic effects are proportional not ΔE but $\Delta p/p$ (another reason is nonlinearity of longitudinal motion, which discussed later).

The 805 MHz, 30 MeV/m accelerating system is considered now as the most perspective[3]. About 5 m of such a linac is required for the phase rotation and approximately the same for 'pure' acceleration to get more or less satisfactory efficiency of the accelerating system. It means that energy gain in the section should be about 150 MeV, and rather long Li lenses (1.5 - 1.8 m) are required.

Emittance exchange is another big problem. Typical energy spread of cooled muon beam is about ± 50 MeV on 3σ level which should be decreased approximately on 20% by wedge absorber to compensate straggling in Li lens. It means that energy loss in the center of the absorber is about 10 MeV requiring 7 - 8 cm of LiH. It gives a scale of average β - function in the absorber and means that scattering in it may be comparable with scattering in Li lens of 1 cm β - function. Therefore a using of wedge absorbers in very end of the cooling channel is at least undesirable.

Another important effect is violation of balance of chromaticity by wedge absorber which changes energy distribution. Therefore it can be placed only at the end of matching section to avoid accumulation of chromatic distortions. It means also that only 1 wedge per section may be used providing exchange of longitudinal emittance with one transverse direction. The exchange with other direction is available in practice only in the following matching section.

2 SCHEMATIC AND PARAMETERS OF THE COOLER

A system of 5 Li lenses with 4 matching sections is considered to meet these conditions (Fig.1). It includes 2 sections with and 2 ones without wedge absorbers. Main parameters of the cooler are listed in Table 1.

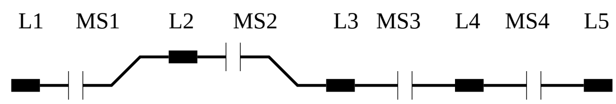

Figure 1: Schematic of the cooler.

Table 1: Parameters of the cooler

Muon momentum, MeV/c	298.9 - 463.6
Total length, m	49.69
Length of Li lens	1.686 and 1.807
Length of matching section, m	9.875 - 10.667
Accelerating frequency, MHz	805
Accelerating gradient, MV/m	30

Scheme of a matching section is shown on Fig.2. 1st short solenoid matches strongly divergent beam going out the Li lens with a long central solenoid. By this, there is no beam envelop modulation in the central solenoid what weaken chromatic effects. Short solenoids in the end of the section focus the beam on the center of next Li lens. Sections 3 - 4 without emittance exchange include 1 straight solenoid whereas 1st and 2nd sections include additionally 2 bent solenoids to excite and suppress dispersion. Bent solenoids placed inside of combine function magnets with field index 0.5 to prevent vertical drift. Such a system has almost the same transfer matrix as usual solenoid and creates easy controlled dispersion function. All front and back

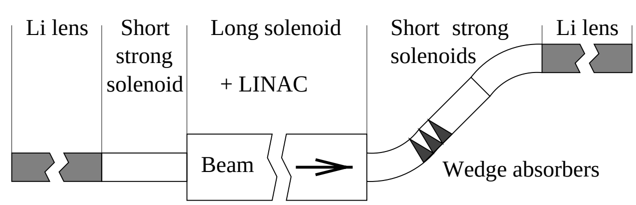

Figure 2: Matching section with emittance exchange.

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solenoids have a length $\lambda_{Larmor}/2$, and central solenoid – $4\lambda_{Larmor}$. Parameters of the matching sections are listed in Table 2.

Table 2: Parameters of matching section

No of MS	**1**	**2**	**3**	**4**
Length, m: Total	10.657	10.544	9.933	9.875
Front solenoid	.255	.228	.197	.171
Central solenoid	9.500	9.500	9.500	9.500
Any back solenoid	.304	.272	.236	.204
Field, T: Front s.	12.29	13.74	15.86	18.32
Central solenoid	3.157	3.154	3.145	3.142
Any back solenoid	15.97	17,86	20.62	23.81

Traveling wave linacs are placed inside central solenoids. Each of them is divided on 3 subsections with different stable phases to get synchrotron phase advance π and avoid an inadmissible adiabatic increase of energy spread of the beam.

Table 3: Length (m)/Stable phase (deg) of subsections

MS	**1**	**2**	**3**	**4**
SS1	3.776/11.6	3.821/13.7	3.944/12.0	3.968/12.0
SS2	2.498/90	2.391/90	2.048/90	1.966/90
SS3	3.225/38.9	3.288/40.0	3.508/43.5	3.567/44.4

3 NONLINEAR AND CHROMATIC DISTORTIONS

Fig.3 shows nonlinear distortion of longitudinal motion. Nonlinearity of accelerating field as well as nonlinear dependence of flying time on energy are responsible for this. The latest effect become very stronger at lower energy which is another (besides chromaticity) reason to take high energy muons for the cooling.

Fig.4 demonstrates chromatic effects in 4th matching section. It is assumed that phase ellipse in the beginning is canonical corresponding to the beam boundary on the level 3σ. It is seen that chromatic effects is suppressed almost totally at $-\sigma_E < \Delta E < 3\sigma_E$ and not very strong at $-3\sigma_E < \Delta E < -2\sigma_E$. Effect depends both on particles energy and *distribution of synchronous phases* which probably can be optimized more. 1st and 2nd sections give more chromaticity because of wedge absorbers.

Fig.5 demonstrates transverse nonlinear effects in 4th section for equilibrium particles. Initial phase ellipses are canonical corresponding to transverse emittance on the levels 1,2,3 σ. These distortions are tolerable, but it is necessary to remember that effect substantially enhanced for non-equilibrium particles.

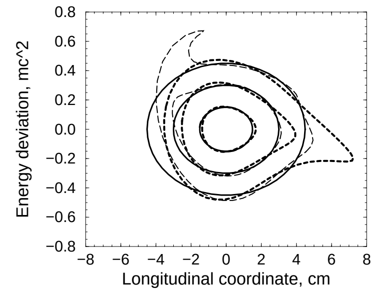

Figure 3: Effect of longitudinal nonlinearity. Solid lines – initial phase ellipses (levels 1,2 3 σ), dashed lines – after 1 cell, long dashed – after 2 cells.

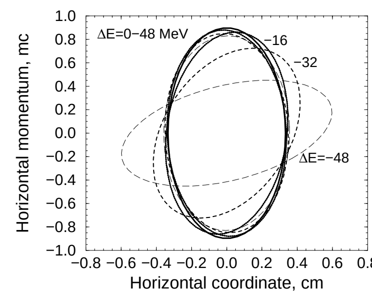

Figure 4: Phase ellipse after 4th section at different energy deviations. Initial phase ellipse is canonical, $\sigma_E = 16$ MeV.

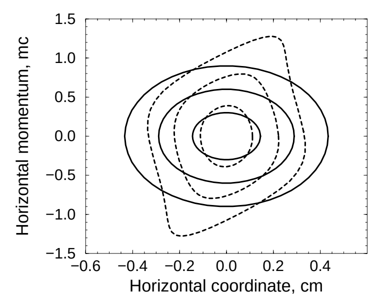

Figure 5: Nonlinear distortion in 4th section. Solid lines – phase ellipses before the section (1,2,3 σ - levels), dashed lines – phase ellipses after the section.

4 LITHIUM LENSES

Any section provides cooling of transverse emittance by factor $\lambda = \varepsilon_{out}/\varepsilon_{in}$,

$$\lambda = \frac{p_{min}}{p_{max}} \left(1 + \frac{\beta_{max}}{2\epsilon_{in}} \int \sqrt{\frac{p}{p_{max}}} \frac{d\langle\theta^2\rangle}{ds} ds \right), \quad (1)$$

where p is momentum, ϵ – geometrical emittance, β_{max} – β - function in the beginning of Li lens, $d\langle\theta^2\rangle/ds$ – r.m.s. angle of multiple scattering per unit of length. Calculation of the integral gives 0.0015 at given energy.

The cooling goes down when emittance decreases. To avoid this, it is necessary to decrease β - functions of Li lenses in the series to keep constant angle spread of the beam x'[4]; then $\epsilon_{in} = \beta_{max} x'^2$. At $x' = 1/15$ scattering increases the cooling factor on 20% and gives $\lambda = 0.75$ in good agreement with simulation. Therefore transverse momentum spread decreases by factor 0.77 what should be compensated in matching sections to have constant spread in any Li lens (it achieved by appropriate ratio of fields of front and back solenoids). As a result, beam radius decreases from lens to lens by factor 0.75 but it grows up to 0.8 in 1st and 2nd sections because of emittance exchange.

Choosing initial transverse emittance of the beam as 1π mm, it is possible to pre-estimate r.m.s. beam radius in any Li lens. The lens radius should be 3.5 times more to avoid excessive particle loss (see below). It allows to calculate all the lenses parameters which are listed in Table 4. The first lenses are shorter because wedge absorbers additionally decrease the energy on 10.6 MeV. Current of all the lenses is 440 kA.

Table 4: Parameters of Li lenses

Lens No	**1**	**2**	**3**	**4**	**5**
Length, m	1.686	1.686	1.686	1.807	1.807
Radius, cm	1.166	0.932	0.700	0.524	0.393
Grad, T/cm	6.476	10.12	17.99	31.98	56.86
Max field, T	7.551	9.432	12.59	16.76	22.35

5 SIMULATION

Gaussian distribution with parameters: $\sigma_x = \sigma_y = 3.33$ mm, $\sigma_{p_x} = \sigma_{p_y} = 32$ MeV/c, $\sigma_z = 1.5$ cm, $\sigma_E = 16$ MeV was taken for simulation. The distribution was cut on the level 3σ according to radius of 1st Li lens.

Fig.6 gives distributions of injected (left) and cooled beams on transverse phase plane. Very similar picture is obtained in linear approximation. It means that nonlinear and chromatic transverse distortions are suppressed rather good. Longitudinal distortions are more significant as shown on Fig.7. Both straggling and nonlinearity cause a beam halo and large additional loss of particles. The distributions have different slops because reside in different ends of the lenses and have canonical form in centers.

Fig.7 shows dependence of normalized r.m.s. emittance on distance along the cooler. Filled and unfilled symbols are related to nonlinear and linear consideration of the matching sections. Lower lines give a behavior of transverse emittance and confirm that distortions are rather small in this direction. Upper lines concerning to longitudinal motion are less regular because longitudinal perturbations in any section partially compensated by nonlinearity of the next one (see Fig.3). Because of aperture restriction, final 6-dimensional emittance is almost the same in both considerations (0.20 and 0.23 mm^3) but loss of particles is very different: 8% and 32%. Distribution of the losses ate: decay about 3%, scattering/straggling in Li lens and wedge absorber 5%, imperfection of the matching sections 24%.

6 CONCLUSION

Li lenses with surface field 20-25 T can provide 6-dimension emittance required for muon collider but particle loss in matching sections is large still and an improvement of the system is necessary. A good result gives decrease of accelerating frequency. Emittance exchange is unsolved problem for a beam with ultimate small transverse emittance, too. Interesting idea is the exchange inside Li lens[5] but it is very very difficult technical problem.

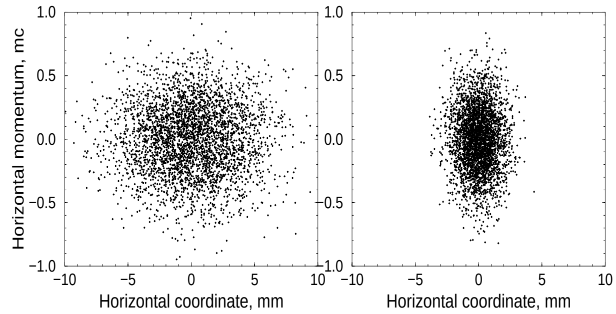

Figure 6: Initial and final transverse distributions.

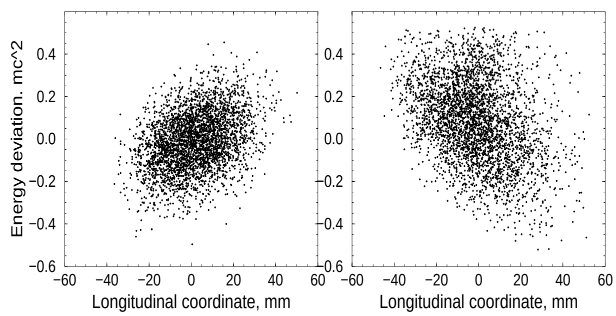

Figure 7: Initial and final longitudinal distributions.

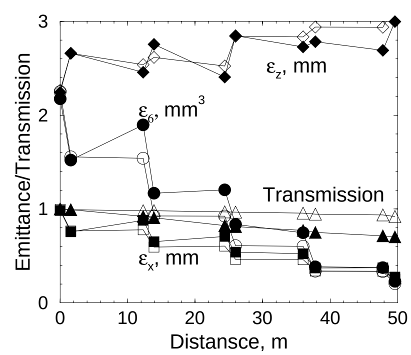

Figure 8: Dependence of emittance and transmission on distance. Unfilled symbols – linear approximation

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