BEAM ECHO MEASUREMENTS*

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Abstract

Beam echo measurements provide a sensitive way to obtain the diffusion rate in storage rings. Often intrabeam scattering is the dominant diffusion mechanism degrading a beam. The analytical formalism of beam echoes will be reviewed, followed by a summary of current experimental data and their analysis. A specific case study of scattering rates below transition in the Fermilab antiproton storage ring will be presented.

1 INTRODUCTION

A beam echo is a weakly nonlinear wave mixing phenomenon, whereby a large, coherent response arises at the difference frequency of two previous excitations. Often the large beam motions due directly to the prior excitations have long since damped away, so that the echo seemingly comes out of nowhere. Although the response to each initial excitation has Landau damped, the particles remain correlated, the 'memory' of the kick remaining in the beam. The recoherence which is the echo is made possible by the mixing of the two sets of correlations at different frequencies.

Echoes arise at a specific time which is directly proportional to the time separation of the excitations. In the case of echo measurements, these kicks are externally applied so that the time of the echo can be well controlled. It is this feature which allows echoes to be exploited as a means of measuring the diffusion coefficient in a beam. Any source of particle collisions has the effect of degrading particle correlations. Once particle correlations from the applied kicks have been sufficiently destroyed, echo reconstruction is no longer possible. Scattering rates within a beam may be measured by examining the degradation of echo amplitude as a function of the time at which the echo occurs. This method of determining diffusion rates is very sensitive and requires little machine time compared with more conventional techniques. Scattering rate measurements using longitudinal echoes in unbunched beams have been done successfully at both Fermilab and CERN [1, 2].

The potential of echoes as a diagnostic is in the early stages. The use of echoes has been explored experimentally primarily in the longitudinal degree of freedom in unbunched beams. However, longitudinal beam echoes in bunched beams have been observed, and a corresponding theory developed [5]. Transverse beam echoes have been theoretically described [3, 4]. There is also a wealth of information in the shape of each individual echo. Echo shape is dependent on the beam distribution, and as such, can be used to determine the beam profile or related information. For example, longitudinal echoes in an unbunched Gaussian beam have been used to measure its energy spread [6]. There is much to be gained from the continued study of echoes.

2 THEORY AND MEASUREMENT

An expression for the longitudinal echo current in an unbunched beam can be found analytically. In the absence of wakefields, but allowing for scattering processes, the current has the following form,

$$I_{echo} = AJ_1(k_1\delta\Delta t)\exp\left(-c\nu t^3\right) \times \int d\varepsilon \frac{df_0(\varepsilon)}{d\varepsilon}\exp\left(ig(\varepsilon)[t-\frac{h_2}{h_2-h_1}\Delta t\right) \quad (1)$$

where A, k_1, and c are constants depending on various machine parameters, and the definition of the rest of Eq. 1 will follow.

The CERN group of Brüning, et al. [2] have coined the second line of Eq. 1 as the form factor of the echo response, because it determines the shape of an individual echo. The sinusoidal term in the integrand determines the temporal location of the echo. Its average causes the integral to go to zero, except at the time $t_{echo} = [h_2/(h_2-h_1)]\Delta t$, where h_1 is the harmonic number of the frequency of the first applied kick, h_2 is the harmonic number of the frequency of the second applied kick, and Δt is the time separation between the two kicks. The derivative of the unperturbed beam distribution, $f_0(\varepsilon)$, with respect to the energy deviation ε is what determines the shape of the echo. A Gaussian beam will thus have a two-lobed echo with a notch that goes to zero in the center. Such echoes are typical of the ones seen in the Fermilab Accumulator, an example of a single echo is shown in Fig. 1. The CERN group has sometimes observed four-lobed echoes in their SPS machine, and have successfully modeled it using a parabolic function for the beam distribution [2].

Since the echo occurs at a time dependent on the applied kick separation, it is possible to do scans of echo amplitude versus time-to-echo, by systematically varying kick separation. The superposition of echoes from such a scan is shown in Fig. 2. The envelope function of an echo scan is

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given by line one in Eq. 1 and consists of a Bessel function multiplied with an exponential having a time cubed dependence. The argument of the Bessel function depends on the kick strength, δ, and time ($\Delta t \propto t_{echo}$). The exponential comes from including diffusion into the beam description, ν is the collision rate. In the echo scan of Fig. 2, the beam parameters were such, and the diffusion rate high enough, so as to prevent echo reconstruction before reaching the first zero of the Bessel function. In contrast, echo scans in the CERN SPS clearly showed the first several lobes of the Bessel function dependence.

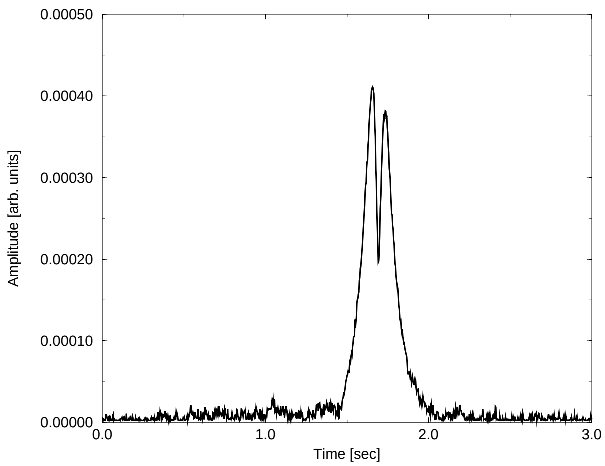

Figure 1: Amplitude of a single echo versus time, as seen in the Fermilab Accumulator.

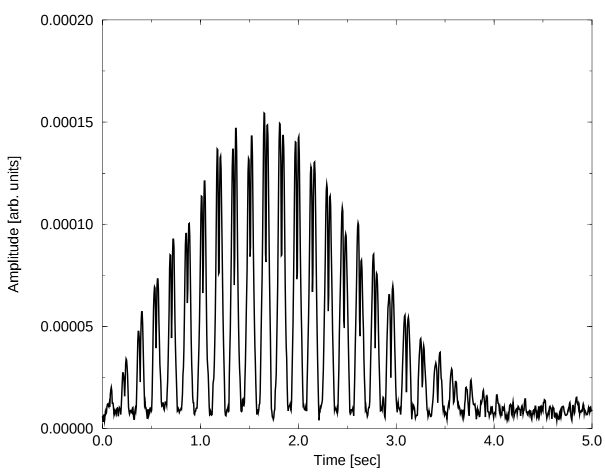

Figure 2: Results of an echo scan done in the Fermilab Accumulator. The data shows echo amplitude versus time-to-echo.

3 SIMULATION

The effect of diffusion on beam echo response has been studied with simulation at Fermilab. The simulation is a particle tracking code using the difference equations for unbunched longitudinal motion. The single frequency excitations are applied as short kicks which modify the energies of the individual particles in accordance with their phase space coordinates.

The available outputs of the program show the time evolution of the phase space coordinates of each particle, the projection of the phase space onto the spatial axis (the current modulation of the beam around the machine), and the amplitude of oscillation of selected Fourier components in the beam. The amplitude of oscillation at the echo frequency may thus be viewed, and mirrors the experimentally observed time development of an echo.

An echo scan may be simulated by superposing the outputs of the amplitude of motion at the echo frequency from successive runs in which the time separation of the applied kicks is varied. This mimics the actual experimental procedure of an echo scan, and the results of the simulation agree well with experimental results. A simulated echo scan, without the intentional introduction of a diffusional term is shown in Fig. 3. There is a noise floor apparent in Fig. 3, which is due to the finite number of particles in the tracking code. This is essentially Schottky noise, and does not affect the echo decay.

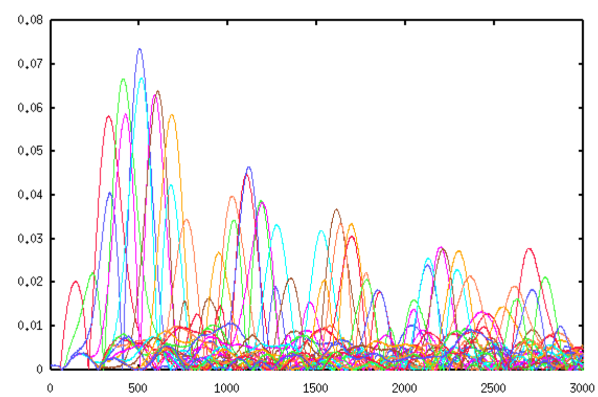

Figure 3: Simulated echo scan, the case of no diffusion.

A simulated echo scan with the intentional introduction of a diffusional term is shown in Fig. 4. Here, a 1% noise level has been injected into the particle dynamics in order to model a random scattering process. The amplitude of the echoes becomes degraded, with the echoes suffering more as they become later in time. The clearly visible Bessel function dependence of Fig. 3 has been eroded by the randomizing process. It is worthwhile to note that echoes can be used in this manner to determine true random processes in numerical simulations.

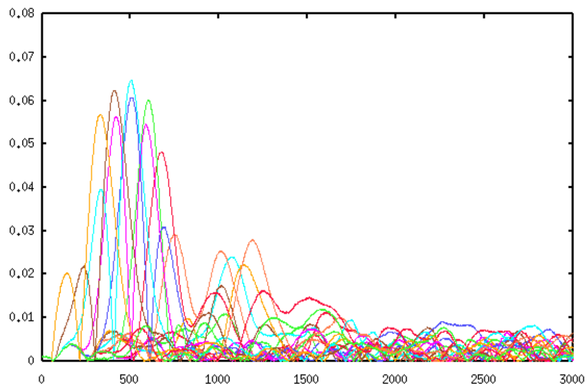

Figure 4: Simulated echo scan, with diffusion introduced.

4 DIFFUSION COEFFICIENT MEASUREMENTS

Through the use of echo scans, scattering rate measurements have been done in the FNAL Accumulator and the CERN SPS. The scattering rate (or diffusion coefficient, $D = \nu(\Delta\varepsilon/\varepsilon_0)$) may be extracted by fitting the amplitude envelope of a scan with the function given in the first line of Eq. 1. It is difficult to know the kick strength δ, as seen by the beam, so there are two free parameters in the fit. The constant c in the exponent depends on known quantities, and can be written as,

$$c = (2\pi\Delta f)^2 \frac{1}{3}\left(\frac{(h_2 - h_1)h_1}{h_2}\right)^2$$

where Δf is the sigma of the beam distribution in frequency (this can be measured with a Schottky pickup), and where h_1 and h_2 are the harmonic numbers of the first and second applied excitations.

The CERN group found that a typical diffusion coefficient in the SPS was $D = 10^{-13}s^{-1}$. In addition, they measured the diffusion coefficient as a function of externally applied noise amplitude, and found that echo measurements had two orders of magnitude greater sensitivity than did Schottky measurements [2].

At Fermilab, a series of scattering rate measurements was undertaken after the beam was decelerated below the transition energy in the Accumulator ring [7]. The purpose was to determine whether the scattering rate was consistent with the prediction for intrabeam scattering in a ring below transition [8, 9]. The results are shown in Fig. 5.

Conveniently, the Accumulator has a number of stochastic cooling systems, for both longitudinal and transverse cooling. Once they are turned off, the beam emittances will grow in free expansion. The diffusion coefficients can be determined either from the growth rates during the expansions, or by using the echo scan measurement technique.

A comparison between experiment and intrabeam scattering theory below transition is done in Fig. 5 by plotting the measured scattering rate versus the theoretically predicted scattering rate for various beam emittances. If there were no errors in the measured parameters, the points would fall on a straight line with a slope of one. However, there is a systematic error due to the uncertainty in the beta functions below transition which could be as high as 20%. Therefore, as long as the measured points fall along a straight line, having a slope consistent with the systematic error, the data can be considered to be in good agreement with the theory. The open circles in Fig. 5 correspond to free expansion data, and the filled circles to the echo measurement data. Not only is there good agreement with the theory, but once again the echo measurement data is seen to have greater sensitivity than is possible with the more conventional methods. The free expansion data lies along the theoretical curve, but begins to fall off at low scattering rates. In contrast, the echo measurements show good agreement with theory even at the lowest measured scattering rates.

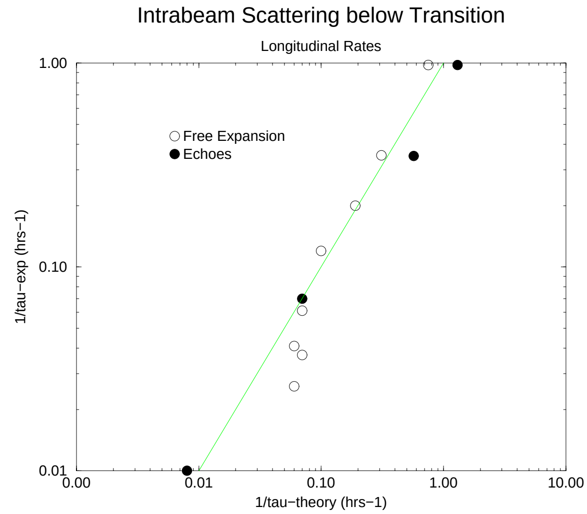

Figure 5: Comparison of the theory to experiment for longitudinal scattering rates. The dashed lines represents agreement.

5 DISCUSSION

Longitudinal beam echo scans with an unbunched beam have been shown to be a useful way of measuring the scattering rate. Both at CERN and at FNAL, echo scans were able to measure smaller scattering rates than other methods. Echo scans are fast as well as sensitive, the measurement itself taking on the order of minutes. Even with this success, the potential use of echoes is still largely unexplored. Full application of diffusion rate measurements in bunched beams and in the transverse plane has yet to be done. There are also possibilities of using echoes in other ways, since the echo shape is dependent on the beam profile.

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