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OBTAINING THE BUNCH SHAPE IN A LINAC FROM BEAM SPECTRUM MEASUREMENTS

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1 INTRODUCTION

In linacs with high single–bunch charge, and tight tolerances for energy spread and emittance growth, controlling the short–range wakefield effects becomes extremely important. The effects of the wakefields, in turn, depend on the bunch length and also on the bunch shape. It was shown in the linac of the Stanford Linear Collider (SLC), for example, that by shaping the bunch, the final rms emergy spread could be greatly reduced, compared to for the standard Gaussian bunch shape[1]. Therefore, in machines with high single–bunch charge, a method of measuring bunch shape can be an important beam diagnostic.

In a linac with low single–bunch charge, the longitudinal bunch shape can be obtained relatively easily from a single measurement of the beam's final energy spectrum, provided that the final to initial energy ratio is large. One merely shifts the average phase of the beam, so that it rides off–crest sufficiently to induce an energy variation that is monotonic with longitudinal position. Then, by knowing the initial and final energies, the rf wave number, and the average beam phase, one can directly map the spectrum into the bunch shape. In a linac with high single–bunch charge, however, due to the effect of the longitudinal wakefield, this method either does not work at all, or it requires such a large shift in beam phase as to become impractical.

In earlier work[2],[3] it was shown that, even when wakefields are important, if one measures the final beam spectrum for two different (properly chosen) values of beam phase, then one can again obtain the bunch shape, and—as a by–product—also the form of the wakefield induced voltage; this method was then illustrated using data from the linac of the SLC. These SLC measurements, however, had been performed with the machine in a special configuration, where the current was low; in addition, the noise in the data was low and the measured spectra were smooth distributions. Under normal SLC conditions, however, the currents were higher, and it was difficult to get the required separation in phase for the two measurements (the required separation increases with current); and the measured spectra were not smooth functions. Under such conditions, the above method works poorly or fails.

If we know the Green function wake of the linac, however, we can still obtain the bunch shape from beam spectrum measurements. In this report, we present two such methods. One requires one spectrum measurement and involves the solution of a Volterra integral equation. The other requires a knowledge of upstream beam and transport properties and involves a least squares minimization to simulated spectra. We then apply these methods to data from the SLC.

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2 THEORY

Consider a bunch of charged particles that are accelerated in a linac from initial energy E_0 to final energy E_f. Let us assume that $E_f/E_0 \gg 1$, so that we can ignore the component of energy variation that is uncorrelated with longitudinal position. Then the relative energy of a particle at position z within the bunch (we take the convention that a more negative value of z is more to the front) at the end of the linac becomes

$$\delta(z) = [E_0 + E_a \cos(kz+\phi) + eV_{ind}(z)]/E_f - 1 \ , \quad (1)$$

with E_a the total peak energy gain, k the rf wave number, and ϕ the average beam phase; with $V_{ind}(z)$ the induced voltage, given by

$$V_{ind}(z) = -eNL \int_0^\infty W_z(z')\lambda_z(z-z')\,dz' \quad , \quad (2)$$

where N is the bunch population, L the total length of accelerating structure in the linac, W_z the Green function wakefield, and λ_z the bunch shape. If the final energy E_f is fixed, as in the SLC with energy feedback on, E_a is also an unknown given by

$$E_a = [E_f - E_0 + eNLk_{tot}]/\langle\cos(kz+\phi)\rangle \quad , \quad (3)$$

with $k_{tot} = -\langle V_{ind}\rangle/(NL)$ the loss factor†.

By knowing both $\lambda_z(z)$ and $\delta(z)$ over the bunch length we can compute the energy distribution $\lambda_\delta(\delta)$. Conversely, if we know $\lambda_\delta(\delta)$ and $\delta(z)$, we can calculate $\lambda_z(z)$, provided $\delta(z)$ is monotonic over the bunch. Let us assume that this is the case. Then

$$\lambda_z(z) = \lambda_\delta(\delta(z))\,|\delta'(z)| \quad . \quad (4)$$

Suppose now that we know E_0, E_f, k, and ϕ. Without knowing the induced voltage we cannot, in general, obtain λ_z from λ_δ, since δ depends also on V_{ind}. Only if eV'_{ind} is small compared to $\delta' E_f$ over the bunch does a single measurement of λ_δ suffice to give λ_z.

We now describe three possible solution strategies for the case when the wakefields cannot be neglected. The most suitable one in a given situation depends upon what is known and on properties of the data.

† In Ref. [3], the term $\langle\cos(kz+\phi)\rangle$ in Eq. 3 was replaced by $\cos(\phi)$, an approximation that did not affect the result greatly, except that the area under λ_z in the solution was found not to equal 1.

2.1 Both W_z and λ_z are Unknown

Suppose we measure the bunch spectrum twice: with the beam at phase ϕ^a we obtain λ_δ^a, and then with the beam at ϕ^b we obtain λ_δ^b. We assume the phases are chosen so that $\delta(z)$ is monotonic for both measurements. For the first measurement Eq. 4 becomes

$$\lambda_z(z) = \lambda_\delta^a \left|E_a^a k \sin(kz+\phi^a) - eV'_{ind}(z)\right| / E_f \ , \quad (5)$$

and a similar equation, with superscript b replacing superscript a, holds for the second measurement. Combining these two equations we obtain

$$eV'_{ind}(z) = k\frac{\left[E_a^a\lambda_\delta^a \sin(kz+\phi^a) \pm E_a^b\lambda_\delta^b \sin(kz+\phi^b)\right]}{\lambda_\delta^a \pm \lambda_\delta^b} \ . \quad (6)$$

In Eq. 6 (and below) the upper symbol of $\pm$ applies if the sign of $\delta'(z)$ is different for the two measurements, otherwise the lower symbol applies. The right hand side of Eq. 6 is a function both of z and—through the argument of λ_δ—of $V_{ind}(z)$. Eq. 6 is therefore a first order non-linear differential equation which we can solve numerically for the unknown $V_{ind}(z)$. As initial condition we take V_{ind} at the front of the bunch to be zero. Once V_{ind} is known we obtain λ_z using

$$\lambda_z(z) = k\lambda_\delta^a\lambda_\delta^b \frac{\left|E_a^a \sin(kz+\phi^a) - E_a^b \sin(kz+\phi^b)\right|}{E_f\left|\lambda_\delta^a \pm \lambda_\delta^b\right|} \ . \quad (7)$$

We begin by setting $V_{ind} = 0$, $k_{tot} = 0$, and $\langle\cos(kz+\phi)\rangle = \cos(\phi)$. Then we solve, in order, Eqs. 3, 1, 6, and 7, and then iterate. We have the correct answer when the area under λ_z is 1. We see from Eq. 7, that for accuracy in λ_z we want two measurements from opposite sides of the rf crest. One problem with this, especially at higher currents, however, is that for the $\phi > 0$ measurement one may need to go way off–crest (which may not be possible) in order that λ_δ at the front of the beam—where the calculation begins—be monotonic.

2.2 W_z is Known, λ_z is Unknown

If we assume that $W_z(z)$ is known, and again that the total voltage is monotonic, then one measurement $\lambda_\delta(\delta)$ suffices for obtaining $\lambda_z(z)$. Eq. 5 can be written as

$$\lambda_z(z) = \frac{\pm\lambda_\delta}{E_f \mp e^2NLW_z(0)\lambda_\delta}\Big[E_a k \sin(kz+\phi) + e^2NL\int_{-\infty}^{z} W'_z(z-z')\lambda_z(z')\,dz'\Big] \ , \quad (8)$$

where the upper(lower) symbols represent the case of $\phi >(<)\ 0$. Eq. 8 is a Volterra integral equation of the second kind, which can be readily solved numerically.

As a measurement, we prefer one with $\phi < 0$, where the front of the beam is not near the rf crest. Our method of solving for λ_z is as follows: Initially we let $V_{ind} = 0$, $k_{tot} = 0$, and $\langle\cos(kz+\phi)\rangle = \cos(\phi)$. We solve Eq. 3, then Eq. 1, then Eq. 8. Having obtained a first estimate of $\lambda_z(z)$, we can then obtain a first estimate of $V_{ind}(z)$ (using Eq. 2), k_{tot}, and $\langle\cos(kz+\phi)\rangle$. The process is then iterated until the area under λ_z equals 1.

2.3 Least Squares Fitting Method

If $\delta(z)$ is not monotonic the above methods fail. In such a case, however, we can use a least squares approach. In many linacs the longitudinal distributions of the beam at some position upstream of the linac are known, and the transport from this position to the linac is also well known. In the example of the SLC linac, the bunch shape and energy distribution in the damping rings is fairly well known, as are properties of the compressor section leading from the ring to the linac. We can simulate the development of longitudinal phase space from the known position to the end of the linac. Note that to do this we need to know W_z in the linac. We can define an objective function:

$$y = \int \left[(\lambda_\delta)_{meas} - (\lambda_\delta)_{calc}\right]^2\, d\delta \quad , \quad (9)$$

with $(\lambda_\delta)_{meas}$ and $(\lambda_\delta)_{calc}$, respectively, the measured and calculated energy distributions. We minimize the objective function by varying parameters in the system that we know imperfectly. If the fit is good, we can believe the calculated bunch shape. This method can work well if there are few unknowns, and if these unknowns have orthogonal effects (a subject which we have not systematically studied). Note that even though the least squares method does not require a monotonic $\delta(z)$, to obtain accurate results we still need a widened spectrum measurement.

3 APPLICATION

The measurements that we analyze come from the SLC linac. They were performed on July 7, 1997 with a wire monitor (with a dispersion of 70 mm) in the BSY region on the north side (the electron side). The parameters were $N = 1.9\times10^{10}$, $E_0 = 1.19$ GeV, $E_f = 46$ GeV, $k_{rf} = 60$ m^{-1}, and $L = 2733$ m; the peak rf voltage of the damping ring was 800 kV; the bunch compressor voltage was set to $V_c = 41.8$ MV[‡]. The linac phase knob was first calibrated. Then for several phase settings the beam spectrum was measured, all the while keeping the feedback in final energy on. The measured rms energy spread, σ_δ, and the full–width–at–.2–max/3.59, $\tilde\sigma_\delta$, are shown in Fig. 1, and six representative spectra are shown in Fig. 2 (the plotting symbols).

First we apply the least squares method. In the SLC the beam leaves a damping ring, goes into a bunch compressor, and then enters the linac. In the damping ring the bunch shape can be described as a tilted Gaussian in z, given by

$$\lambda_z \approx \frac{1}{\sqrt{2\pi}\sigma_z}\exp\left[\frac{-z^2}{2\sigma_z^2(1\pm\epsilon)^2}\right] \quad , \quad (10)$$

[‡] The reading was actually 5% higher, but historically this compressor has been known to read low by this amount.

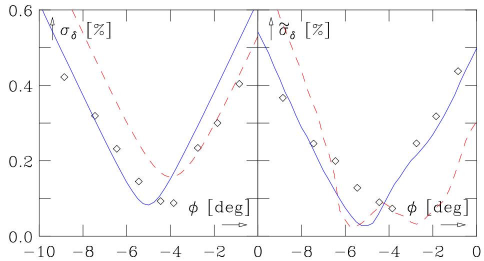

Figure 1: The rms energy spread (left) and full width at 0.2max/3.59 (right) of the measured spectra (the plotting symbols). Simulation results for $\phi_c = 90°$ (dashes) and $\phi_c = 102°$ (solid curves) are also given.

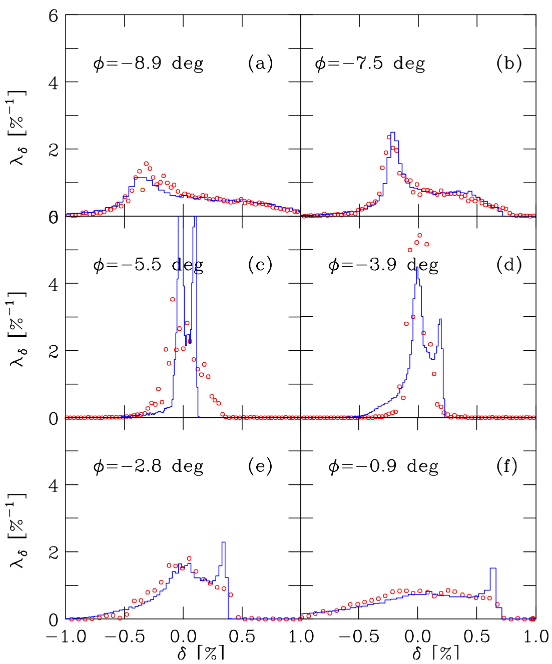

Figure 2: Six representative measured spectra (symbols); and simulation results for $\phi_c = 102°$ (histograms).

with parameters rms length σ_z and asymmetry factor ϵ; and a Gaussian in δ, with rms σ_δ. In this case we take $\sigma_z = 5.75$ mm, $\epsilon = .27$, and $\sigma_\delta = .085\%$[4]. The wakefield for the SLAC linac is found in Ref. [5]. In the transport to the end of the linac, we know E_0 and E_f well, which leaves us with three parameters: the compressor voltage V_c, the compressor phase ϕ_c, and the linac phase ϕ. In our analysis we let these three parameters vary to minimize the objective function, Eq. 9. We obtained the expected result for V_c and ϕ; however, the optimal ϕ_c was $102°$, not the expected $90°$. Simulation results for $\phi_c = 102°$ and $90°$ are given by the curves in Fig. 1; and for the case $\phi_c = 102°$, by the histograms in Fig. 2. Since these fits are good, we can believe the calculated bunch shape. The calculated bunch shape and spectrum for the optimum case are given in Fig. 3 (the solid curves) and compared with those for the case $\phi_c = 90°$. We see that the latter case cannot possibly be correct (compare with Fig. 2b). For the optimized case, $z_{rms} = 1.2$ mm and $z_{FWHM} = 2.8$ mm.

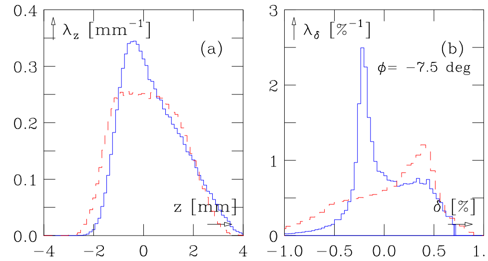

Figure 3: Bunch shape and spectrum obtained by the least squares method, with $\phi_c = 102°$ (the solid curves). The results with $\phi_c = 90°$ are also shown (the dashes).

Finally, we apply the Volterra integral equation method to the data of Fig. 2a and 2b (see Fig. 4). Note that the two results give almost the same bunch shape and induced voltage, and that the bunch shape agrees well with the result of the least squares method (Fig. 3a, the solid curve).

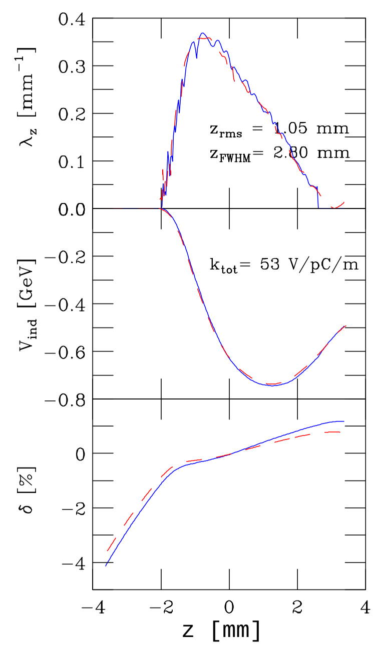

Figure 4: Results of the Volterra integral equation method, using data of Fig. 3a (solid curves) and 3b (dashes).

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