BEAM HALO STUDIES USING A 3-DIMENSIONAL PARTICLE-CORE MODEL*

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Abstract

In this paper we present a study of beam halo based on a three-dimensional particle-core model of an ellipsoidal bunched beam in a constant focusing channel. For an initially mismatched beam, three linear envelope modes – a high frequency mode, a low frequency mode and a quadrupole mode – are identified. Stroboscopic plots are obtained for particle motion in the three modes. With higher focusing strength ratio, a 1:2 transverse parametric resonance between the test particle and core oscillation is observed for all three modes. The particle-high mode resonance has the largest amplitude and presents potentially the most dangerous beam halo in machine design and operation. For the longitudinal dynamics of a test particle, a 1:2 resonance is observed only between the particle and high mode oscillation, which suggests that the particle-high mode resonance will also be responsible for longitudinal beam halo formation.

1 INTRODUCTION

The physics of beam halo has been extensively studied through analytical theory and multi-particle simulations[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. In these studies, the so-called particle-core model has been frequently used. This model provides insight into the essential mechanism of halo formation and enables estimates of the extent of beam halo. In this paper, we will use a three-dimensional particle-core model with a nonlinear rf field to study beam halo formation in a mismatched ellipsoidal bunched beam. Three envelope modes will be identified and their effects on the formation of beam halo through a parametric resonance with test particles will also be studied for the given physical parameters.

Bunch Current (A)	0.1
Protron Energy (MeV)	471.4
Synchronous Phase (degrees)	-30
rf Frequency (M Hz)	700
Accelerating gradient (MV/m)	5.246
Transverse Phase Advance (degrees)	81
Lattice Period (m)	8.54
Transverse RMS Emittance (π-mm-mrad)	0.2319
Longitudinal RMS Emittance (π-deg-MeV)	0.42

The physical parameters of the beam and the accelerator are given in Table 1[11]. The organization of this paper is as follows: the particle-core model is described in Section 2, the linear envelope modes are discussed in Section 3, the test particle dynamics under three envelope modes is presented in Section 4, and the conclusions are drawn in Section 5.

2 THREE-DIMENSIONAL PARTICLE-CORE MODEL

In the three-dimensional particle-core model, the beam consists of a core and test particles. The core, which contains most particles, is modeled by the rms envelope equations. The test particles contain a small fraction of the beam and are subject to the effects of external forces and space charge forces due to the core. The effects of test particles on the core and the mutual Coulomb interactions among test particles are neglected. The bunched core is assumed to have a uniform charge density distribution. Under the smooth approximation, the envelope equations for the bunched beam including nonlinear rf focusing are:

$$r_x'' + k_{x0}^2 r_x' - I_x(r_x, r_y, r_z, 0) r_x - \frac{\epsilon_x^2}{r_x^3} = 0 \quad (1)$$

$$r_y'' + k_{y0}^2 r_y' - I_y(r_x, r_y, r_z, 0) r_y - \frac{\epsilon_y^2}{r_y^3} = 0 \quad (2)$$

$$r_z'' + k_{z0}^2 f(r_z) r_z' - I_z(r_x, r_y, r_z, 0) r_z - \frac{\epsilon_z^2}{r_z^3} = 0 \quad (3)$$

with

$$I_i(r_x, r_y, r_z, s) = C \int_s^\infty \frac{dt}{(e_i^2 + t)\sqrt{(r_x^2 + t)(r_y^2 + t)(\gamma^2 r_z^2 + t)}} \quad (4)$$

where the semi-axes r_i are related to the RMS beam sizes a_i by $r_i = \sqrt{5} a_i$, $e_i = r_x, r_y, \gamma r_z$ $(i = x, y, z)$, and $C = \frac{1}{2}\frac{3}{4\pi\epsilon_0}\frac{q}{mc^2}\frac{I}{f_{rf}\beta^2\gamma^2}$. Here, ϵ_0 is the vacuum permeability, q is the charge, mc^2 is the rest energy of the particles, c is the vacuum light speed, I is the beam average current, f_{rf} is the rf bunch frequency, $\beta = v/c$, v is the bunch speed, and $\gamma = 1/\sqrt{1-\beta^2}$. The quantities k_{x0} and k_{y0} are the transverse betatron wave numbers at zero current, which are defined as $k_{i0} = \sigma_{i0}/L$, $i = x, y$, under the smooth approximation for a periodic quadrupole focusing element. Here, σ_{i0} is the zero-current transverse phase advance per focusing period L. The longitudinal synchrotron wave number at zero current, k_{z0}, is defined as $k_{z0} = \sqrt{2\pi q E_0 T sin(-\phi_s)/\gamma^3\beta^3 mc^2\lambda}$, where E_0T is the accelerating gradient, ϕ_s is the synchronous phase, and λ is the rf wavelength. The function $f(r_z)$ in the envelope

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equation is a nonlinear rf focusing factor defined in Ref. 9. In the above envelope equations, we have used a continuous sinusoidal wave to represent the average effect of the synchronous rf space harmonics in the rf gap and neglected the acceleration of the rf field. The emittances, ϵ_x, ϵ_y, and ϵ_z, are five times the corresponding RMS emittances.

The equations of motion for a test particle in the presence of a uniformly charged core and external focusing fields are

$$x'' + k_{x0}^2 x - I_x(r_x, r_y, r_z, s)x = 0 \quad (5)$$

$$y'' + k_{y0}^2 y - I_y(r_x, r_y, r_z, s)y = 0 \quad (6)$$

$$\Delta z'' + k_\xi(cos(k_\eta \Delta z + \phi_s) - cos(\phi_s)) - I_z(r_x, r_y, r_z, s)\Delta z = 0 \quad (7)$$

where $k_\xi = qE_0T/mc^2\beta^2\gamma^3$, and the parameter s is zero for a particle inside the core and is determined from the root of the equation

$$\frac{x^2}{r_x^2 + s} + \frac{y^2}{r_y^2 + s} + \frac{\Delta z^2}{r_z^2 + s} = 1 \quad (8)$$

for a particle outside the core. These coupled nonlinear ordinary differential equations are solved numerically using a leap-frog algorithm.

3 LINEAR ENVELOPE MODES

The steady state solution of the envelope equations has three components which define the stationary core size. For a mismatched beam three linear eigenmodes of the core envelope will be excited. From linear perturbation theory, we can find the eigenmodes of a mismatched core oscillation. For the physical parameters given in Table 1, we get the normalized wave number 1.945 for the high frequency mode, 1.641 for the low frequency mode, and 1.456 for the quadrupole mode.

To investigate the possible resonance between the test particle and the mismatched core oscillation, we calculated the evolution of the ratio of the possible test particle wave numbers to the mismatched mode wave number as a function of current with all the other physical parameters given in the Table 1 fixed. The results for the transverse betatron motion is given in Fig. 1. It shows that a 1:2 resonance between the test particle and low mode and quadrupole mode is always excited. For the high mode, the 1:2 resonance is excited when the current exceeds $40\ mA$. At a current of $100\ mA$, as in the present APT design, the 1:2 resonance between the betatron motion of test particles and all three mismatch-modes is excited. Fig. 2 shows the evolution of the ratio between the wave number of the particle synchrotron motion and the wave number of the high mode and low mode. Here, the 1:2 resonance between the test particle and high mode is excited with current greater than $60\ mA$. The 1:2 resonance between the particle and the low mode is only excited with a current greater than $230\ mA$.

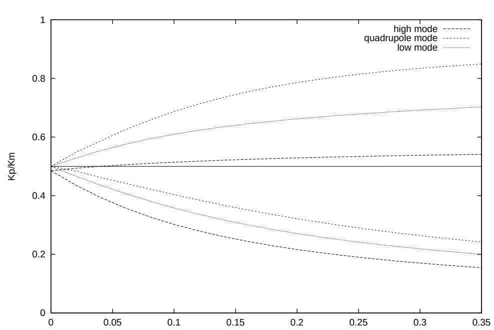

Figure 1: The ratio of particle betatron wave number to mode wave number as a function current

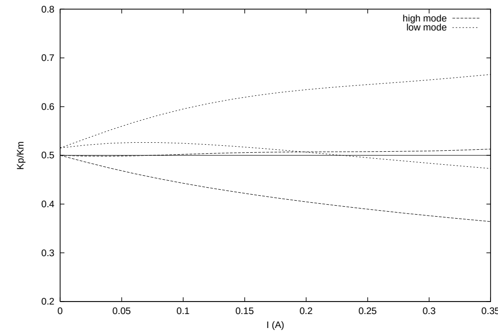

Figure 2: The ratio of particle synchrotron wave number to mode wave number as a function current

4 TEST PARTICLE DYNAMICS UNDER THREE ENVELOPE MODES

For the mismatched core, three linear modes can be excited. When the ratio of the test particle wave number to the core envelope mode wave number is rational, the resonance between the test particle and core will be excited. Among these resonances, the 1:2 resonance is what we are most interested in. This is because this low order resonance will have the large oscillation amplitude and is generally believed to be responsible for the presence of beam halo. To understand the potential effects of these envelope modes on the test particle dynamics and beam halo formation, we use stroboscopic map to study the test particle dynamics with only one envelope mode excited each time. Fig. 3 (a) shows the stroboscopic plot of test particle dynamics in the $x - p_x$ plane under high mode envelope oscillation with 20% initial transverse mismatch. Fig. 3 (b) shows a similar plot of test particle dynamics in the $x - p_x$ plane under the low mode. Fig. 3 (c) shows the same plot for the quadrupole envelope oscillation. The peanut structure in the $x - p_x$ plane suggests that the 1:2 resonance between the test particles and the core envelope oscillation could be excited for all three modes. In Fig. 4 (a) and (b), we show the stroboscopic plots of the resonance between the synchrotron particle motion and the high envelope mode and low envelope mode in the longitudinal $\Delta z - \Delta p_z$ plane. In this case, the 1:2 resonance is present only for the high mode in the

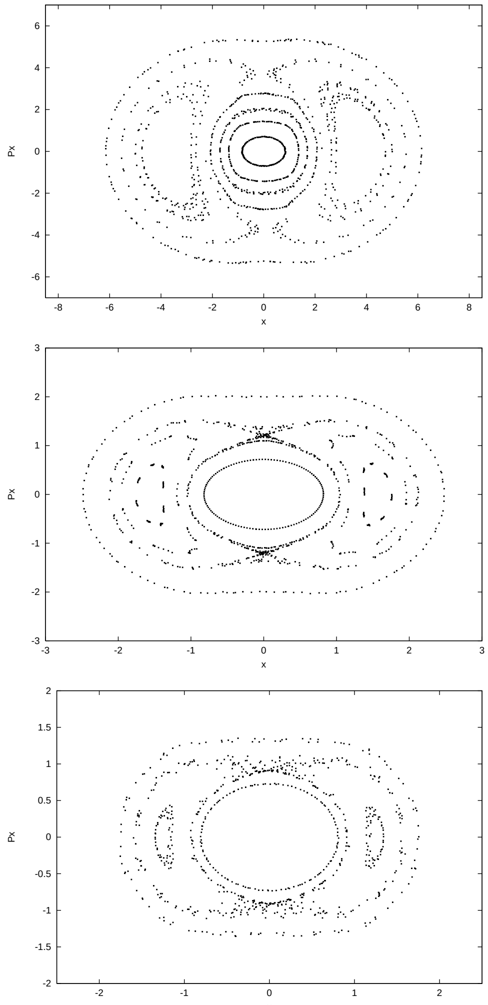

Figure 3: Stroboscopic plot of $x - p_x$ for high mode, low mode and quadrupole mode with 0.2 mismatch.

longitudinal phase space.

5 CONCLUSIONS

In the above study, we have used a three-dimensional particle-core model to study beam halo in a mismatched ellipsoidal bunched beam. Three linear envelope modes, a high frequency mode, a low mode and quadrupole mode, are identified. The 1:2 transverse resonances are present between the test particle and all three envelope modes. Among the three transverse particle-core resonances, the high mode presents the greatest potential danger for the machine design and operation due to its large transverse resonance amplitude. The high mode resonance also dominates the longitudinal 1:2 resonance which contributes to the formation of longitudinal beam halo.

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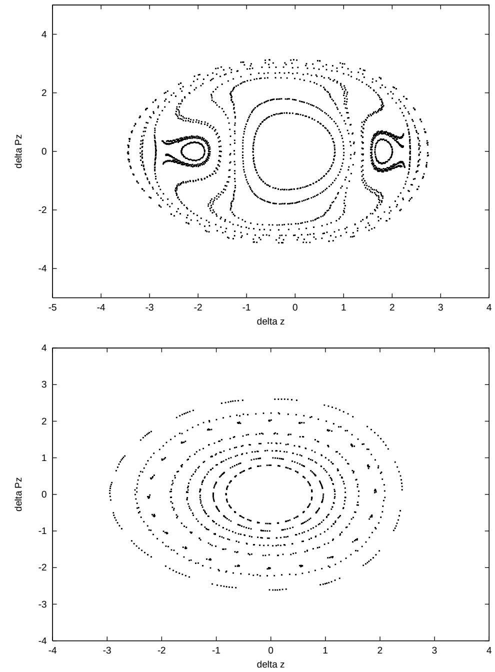

Figure 4: Stroboscopic plot of $\Delta z - \Delta p_z$ for high mode and low mode with 0.2 mismatch.

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