CORRECTION OF THE BETATRON COUPLING BY LOCAL ORBIT CROSS TALK

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Abstract

Analytical expressions, in presence of skew quadrupoles errors, have been derived to estimate the betatron coupling contribution to the vertical beam size and emittance [1]. The treatment of the betatron coupling is based on the matrix transport perturbation and related eigen vectors. The advantage of this approach is to be free of the usual resonant approximation and to allows a compact formulation of the vertical emittance. In addition, this approach gives a fast analytical statistical analysis. From this analytical expressions, we propose a new scheme of the betatron coupling correction also based on the closed cross orbit correction [2]. This method, called local cross orbit correction, improve the correction of the betatron coupling with few judicious measurements [3]. It allows also to fully uncoupled the beam locally on the lattice.

1 INTRODUCTION

The third generation synchrotron machine are designed with a very small beam emittance to produce very high brilliance beams of synchrotron radiation in the VUV and soft X-ray regions. This requires relatively strong focusing, and magnet errors will therefore introduce large distortions in closed orbit (C.O.) as well as residual dispersion and large betatron coupling. To achieve these performances we need to control the coupling value, define by the ratio of the vertical emittance ε_z by the horizontal emittance ε_x, in a range from few 0.1 % to about 10 %. We have studied the correction of the spurious vertical dispersion and betatron coupling on the SOLEIL [4] and ESRF [5] machine from an analytical and statistical point of view, using the BETA-LNS code.

2 COUPLING SOURCES

The vertical emittance is generated in two ways, directly when a photon is radiated in a region where there is nonzero vertical dispersion, or by betatron coupling between the two planes horizontal and vertical. This two sources of vertical emittance are induced by the presence of misalignment errors and by the resulting closed orbit.

Noting Δ_{zj} the vertical kick in j, $\Delta\phi_q$ the tilt of the quadrupoles and Δz_s the vertical misalignment of the sextupoles, we have for the vertical corrected closed orbit and the vertical spurious dispersion in i :

$$z_i^{CO} = \sum_j C_{ij}^z \Delta_{zj} \quad \text{with} \quad C_{ij}^z = c_{ij}^z + \sum_{cor} c_{ic}^z d_{ic}$$

$$D_i^z = \sum_{quad} c_{iq}^z \left[(2KLD)_q \Delta\Phi_q - (KL)_q z_i^{CO} \right]$$
$$+ \sum_{sext} c_{is}^z (2SLD)_s \left[\Delta z_s + z_i^{CO} \right] - z_i^{CO}$$

c_{ij} is the vertical response matrix from j to i. We generalize this response matrix C_{ij} including the sum over the correctors : each element d_{jc} represents the value of the corrector c induced by the default j. K, S, L and D are respectively, the strength of the quadrupoles, sextupoles, their length and the horizontal unperturbed dispersion.

To derive a complete analytical formulation of the beam envelop and emittance induced by betatron coupling between the two transverse planes in presence of synchrotron radiation, we study the eigen vectors of the one turn transport coupled matrix. After a tedious calculation, we find that we could simply estimate analytically the coupling K in presence of misalignment by the means of the vertical dispersion and the two complex function A and B with [1] :

$$K = \sqrt{\left[\frac{(\sin\mu_+)^2 |B|^2 + (\sin\mu_-)^2 |A|^2}{(4\sin\mu_+ \sin\mu_-)^2} + \frac{U_3}{\varepsilon_x^0} \right]^2 - \left[\frac{|AB|}{8\sin\mu_+ \sin\mu_-} \right]^2}$$

$$\mu_+ = \frac{\mu_x + \mu_z}{2} \quad \text{and} \quad \mu_- = \frac{\mu_x - \mu_z}{2}$$

μ and ε_x^0 are the tune and the horizontal emittance. Using the same notation than for the spurious dispersion, with β and φ the optical function and phase advance, the two complex functions A and B are :

$$A = \sum_{quad} \sqrt{\beta_{xq}\beta_{zq}} e^{i(\varphi_{xi}+\varphi_{zi})} \left[(2KL)_q \Delta\Phi_q \right]$$
$$+ \sum_{sext} \sqrt{\beta_{xs}\beta_{zs}} e^{i(\varphi_{xi}+\varphi_{zi})} (2SL)_s \left[\Delta z_s + z_s^{CO} \right]$$

$$B = \sum_{quad} \sqrt{\beta_{xq}\beta_{zq}} e^{i(\varphi_{xi}-\varphi_{zi})} \left[(2KL)_q \Delta\Phi_q\right]$$
$$+ \sum_{sext} \sqrt{\beta_{xs}\beta_{zs}} e^{i(\varphi_{xi}-\varphi_{zi})} (2SL)_s \left[\Delta z_s + z_s^{CO}\right]$$

U_3 is the second invariant at four dimension (Courant and Snyder invariant) in average in the bending magnets. This term is function of A, B and the vertical dispersion.

Once we have develop the dispersion and the two function A and B over the errors, it is straight forwards to estimate their rms and mean values. The coupling K being a quadratic function of the errors, its mean value, <K>, can be expressed in function of the square standard deviation of the errors [1].

In summarize, the relevant items concerning the betatron coupling are :

1. All the quantities such as the tree moments and the vertical emittance induced by the betatron coupling are only dependent of the tow functions A and B and their respective complex conjugates A^* and B^*. They are the generating functions of the linear betatron coupling.

2. All the cross terms of the one-turn transverse transport matrix T are only dependent of A and B and their complex conjugates A^* and B^*.

3. This four functions A, B, A^* and B^* are s dependent (along the lattice) and are independent one to each other. We need at least also four skew correctors to cancel them in one location in the lattice. If we cancel this four functions on one point, we have :

    ⇒ an uncoupled one turn transport matrix T
    ⇒ an uncoupled beam matrix (or optical functions)
    ⇒ the two vertical and horizontal emittances are equal to their respective invariant

4. The two quantities $B/\sin\mu_-$, $A/\sin\mu_+$ are the sum of all the coupling resonances.

5. Near the two resonnances, the K coupling tend to :

    ⇒ difference resonance ($2\mu_- = \mu_x - \mu_z \rightarrow 2n\pi$)
    $$K \rightarrow \frac{|B|^2}{(\sin\mu_-)^2}\left(1+\frac{J_x}{J_z}\right)$$
    ⇒ sum resonance ($2\mu_+ = \mu_x + \mu_z \rightarrow 2n\pi$)
    $$K \rightarrow \frac{|A|^2}{(\sin\mu_+)^2}\left(1+\frac{J_x}{J_z}\right)$$

6. The two quantities $|A|^2$ and $|B|^2$ are constant along the lattice except at each skew errors (or correctors), where they have a jump. This jump tends to cancel near the resonance.

3 COUPLING CORRECTION

In order to correct the coupling, we have to reduced the two sources of vertical emittance : the vertical dispersion and the betatron coupling. In both case the correctors are skew quadrupoles. The vertical dispersion is corrected at each Beam Plot Monitor (BPM) using a Singular Value Decomposition (SVD) in a similar manner than the closed orbit. In addition we prospect few techniques to correct simultaneously the betatron coupling. The principal difficulties in correcting the betatron coupling is to be able to estimate and to measure his contribution to the vertical emittance. Actually, among others, two schemes used to correct the betatron coupling are : correction of the two resonances, sum and difference, and correction of Cross Talk Orbit (OCT) at each BPM (closed orbit induced by the kick of a steerer in the other plane via the skew errors [2]).

The resonance correction need to shift the tunes to set the machine on the two resonances. It is generally not sufficient to reach the desired coupling. From the analytical point of view, it corresponds to correct the two square modules of the functions A and B (items 4 and 5) once the tunes are shifted. The OCT corrections tend to improve the coupling correction, but the relation between the OCT and the coupling was not clear [5]. It could be shown that there is only a one way relation between a segment of the Cross Orbit induced by a steerer and the two generating functions A and B. The concerned segment is the part of the orbit near the steerer free of skew errors (figure 1). This relation is no longer valid elsewhere.
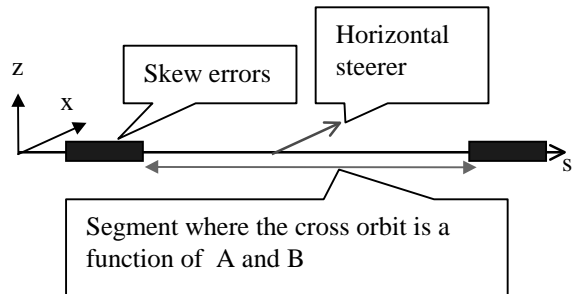

Figure 1

For a better targeting of the betatron coupling, one can only correct the cross orbit at the nearest BPM from the steerer (one steerer one BPM). At best, if there is no skew errors between the BPM and the steerer (both located on the same straight section for instance), one will correct the

betatron coupling via the correction of the cross orbit (item 1) .

To test this new technique of correction, we compared the mean resulting coupling of the four betatron correction methods :

1. Theoretical via direct correction of the square module of A and B in each dipoles, named AB.
2. The tow resonances correction, named RES.
3. All the OCT correction at each BPM, named OCT
4. All the OCT corrected only at the nearest BPM, a Local Cross Talk Orbit (LOCT) correction, named LOCT.

on the SOLEIL (table 1) and ESRF (table 2) machine. In each case, we correct a weighted function of both dispersion and betatron coupling using one of the above technique varying the number of correctors. In order to simplify the computation, we only set tilted quadrupoles (2.10^{-4} rad rms) and sextupoles vertical displacement (1.10^{-4} m rms). Without any correction, the mean coupling of the SOLEIL and ESRF machine are respectively of 1.4 10^{-2} and 17. 10^{-2}.

	4	8	16
AB+Disp	$0.75 \ 10^{-2}$	$0.18 \ 10^{-2}$	$0.63 \ 10^{-3}$
RES+Disp	$0.76 \ 10^{-2}$	$0.20 \ 10^{-2}$	$0.16 \ 10^{-2}$
LOCT+Disp	$0.78 \ 10^{-2}$	$0.22 \ 10^{-2}$	$0.15 \ 10^{-2}$
OCT+Disp	$0.91 \ 10^{-2}$	$0.33 \ 10^{-2}$	$0.22 \ 10^{-2}$

Table 1 : Mean coupling on SOLEIL for the four schemes of correction versus the number of skew correctors.

	16*	16**	32
AB+Disp	$0.62 \ 10^{-2}$	$0.26 \ 10^{-2}$	$0.41 \ 10^{-2}$
RES+Disp	$4.00 \ 10^{-2}$	$0.32 \ 10^{-2}$	$0.10 \ 10^{-2}$
LOCT+Disp	$0.72 \ 10^{-2}$	$0.28 \ 10^{-2}$	$0.64 \ 10^{-3}$
OCT+Disp	$2.50 \ 10^{-2}$	$0.36 \ 10^{-2}$	$0.88 \ 10^{-3}$

Table 2 : Mean coupling on ESRF for the four schemes of correction versus the number of skew correctors.
*Standard corrector setting ** New corrector setting [6]

One observes that the LOCT correction improve by 20 to 30 % the coupling correction compared to the OCT scheme. In the first case on the ESRF machine one gets an improvement of a factor 3.5 (from 2.5 to 0.72 %). An other advantage is to notably reduce the amount of data to handle by only using one BPM per steerer. For instance on the SOLEIL machine, if one wants to correct the vertical closed cross orbits induced by the 48 horizontal steerers at the 112 BPM, the number of constraints is 48×112=5376. By using one BPM per steerer, the number of constraints is reduced to only 48. This improvement of the coupling correction came simply from the fact that, by locally correcting the Cross Orbit we correct the

generating functions, A and B, of the betatron coupling (item 1).

An other application of the one way relation between the Cross talk Orbit and the two functions A and B, is the possibility to fully uncouple the beam (item 4) (and the one turn transport matrix (item 3)). By correcting the Cross Orbit induced by two steerers on two BPM judiciously localized (figure 2), we get four constraints (function of A and B) that can be cancelled with four skew correctors.
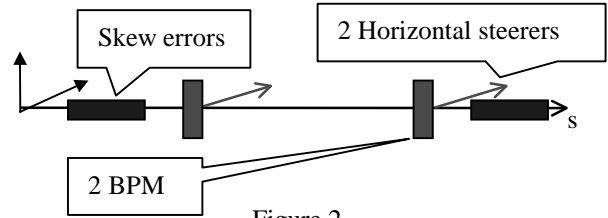

Figure 2

A numerical test of such a Cross Orbit correction on the SOLEIL machine with four correctors effectively cancel the local coupling as well as the one turn transport matrix.

4 CONCLUSION

From a complete analytical treatment of the betatron coupling, we proposed a new technique of the coupling correction based on the local cross talk orbit minimization. By simply correct the cross orbit to the nearest BPM from the active steerer, we improve notably the reduction of the coupling induced by the betatron coupling by a better targeting of the correction. An other advantage of this technique, is to reduce the amount of constraints to handle.

5 REFERENCES

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