OPTIMISATION OF THE LHC DYNAMIC APERTURE VIA THE PHASE ADVANCE OF THE ARC CELLS

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Abstract

The phase advances of the arc cells of storage rings are traditionally chosen to be simple fractions of π in order to take advantage of second order achromats they constitute.

For the LHC, such a choice is not relevant because of the existence of high order systematic multipole components in the main dipoles. In this case it is better to choose the phase advances to cancel the driving term for the largest possible number of non–linear resonances, which is straightforward for an ensemble of identical cells. This can also be achieved for an actual LHC arc featuring dispersion suppressors. The associated improvement of the dynamic aperture is shown in this paper.

1 INTRODUCTION

The working point in the tune diagram is usually determined by a systematic scanning. The SPEAR upgrade projects is a good example [1].

A more subtle approach consists of building a machine from blocs which do not contribute to the excitation of non–linear resonances. The details are explained in [2] and the application to the LHC is shown in section 2 below.

In order to have a basis for comparison, several LHC optics have been constructed. Their characteristics are given in section 3.

In order to help the understanding the resonance strengths have been computed by Normal Form [4]. Lastly, tracking results, performed with SIXTRACK [3], are presented for these optics.

2 A “RESONANCE FREE” LHC ARC

The main problem associated with multipole errors in the LHC arises from the arc dipoles. These dipoles are constructed by several different firms. Each fabrication line may produce dipoles with different systematic multipole errors. As the number of fabrication lines is comparable with the number of LHC arcs, which is eight, the concept of “systematic multipole per arc” comes naturally into the game. Such a component takes a constant value over a given arc and this value varies randomly from arc to arc.

In this context it is attractive to design the arc optics such that it does not contribute to the excitation of low order resonances [2]. To this end both cell phase advances have to be set to the values $k_1 \cdot 2\pi/N_c$ in the horizontal plane and $k_2 \cdot 2\pi/N_c$ in the vertical plane. Under these conditions only those resonances are excited which satisfy,

$$n_x k_1 + n_y k_2 = k_3 N_c, \tag{1}$$

where k_3 is any integer.

N_c is determined for the LHC as follows. Each arc is composed of 23 FODO cells plus one dispersion suppressor at each end. Each cell is composed of two quadrupoles and six dipoles. The dispersion suppressors consist of four quadrupoles and eight dipoles. Thus, a dispersion suppressor is a little longer than one cell, it has a larger phase advance and it contains one third more dipoles. Even though, each dispersion suppressor is taken as one cell. The number of cells is therefore 25 (23 cells plus 2 dispersion suppressors).

The phase advance of the arc cells must be close to 90° because the quadrupole gradient and the aperture of the vacuum chamber have been designed in view of these phase advances. This leaves two possibilities for (k_1, k_2): (7,6) or (6,5). Nevertheless there remain systematically excited single resonances satisfying equation (1). Below order 10 they are: 4,8,9 and 5,6,9 for these two cases respectively. The pair (7,6) has been kept for tracking studies due to its lower β functions.

3 LHC OPTICS STUDIED

In order to have a relevant basis of comparison, several optics have been constructed:

1. A simple model with 25 FODO cells and at each end a transfer matrix to simulate a LHC arc
2. An optics with a tune–split of 5 (Qx=64.28, Qy=59.31). A tune–split of 5 has been chosen as the nominal LHC lattice version 6.
3. An optics which minimises some adverse effects of non–linearities for multipole components which have the same value in all dipoles Qx=65.28, Qy=58.31 [5]. Such an optics is likely not to bring any improvement in the case where the systematic by arcs dominate.
4. A “resonance free” lattice with the phase advances per cell: $mu_{x,c} = \frac{7}{25} \times 2\pi$ and $mu_{y,c} = \frac{6}{25} \times 2\pi$. Qx=68.28, Qy=59.31. The phase advance in the cells has been set and the insertions have been rematched using the minimum number of variable parameters. Then the tunes have been adjusted by means of IR4 and IR6 to the closest values with the same fractional

parts as above. For this lattice, the resonances are not cancelled as they should be for the case of 25 FODO cells, nevertheless their driving term is substantially reduced compared with other lattices.

4 RESONANCE COMPUTATION

In order to test the efficiency of the resonance cancellation procedure, the resonance driving terms have been computed by means of the formalism in ref. [4].

At first this has been done for the simple model composed of 25 cells and two insertions for which the resonance driving term is exactly zero. It is found to have a value smaller by five order of magnitude compared with the other cases. This shows the validity of the computation and the consistency between the two approaches, namely perturbation calculation and map transformation to calculate resonance driving terms.

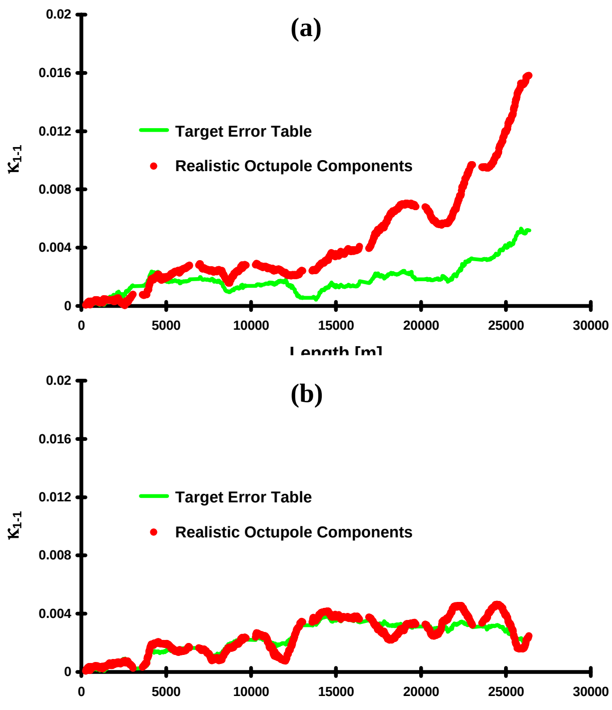

Figure 1: Build–up of the driving term of the (1012) sub–resonance over the length of the LHC. The light gray (green) curve is associated with the “target error table” [6]. The dark gray (red) curve is obtained with larger b_4 and a_4 systematic by arc multipole components, respectively 0.35 and 0.555 in units of 10^{-4} and at a reference radius 17mm.
Part (a) Case 2 in section 2.
Part (b) Case 4 in section 2.

The build–up of the resonance driving term along the LHC lattice is shown in figure 1(a) for the optics with a tune–split of 5 and in figure 1(b) for the “resonance free” lattice respectively. The multipole errors introduced in the lattice are: systematic per arc and random. The error tables can be found in ref. [6]. The resonance cancellation per octant appears quite clearly in figure 1(b): the value of the driving term at the end of the lattice is the same both with and without realistic octupole components.

5 DYNAMIC APERTURE

For the simple model, trajectories have been tracked over 1000 turns without synchrotron oscillations. The maximum initial amplitudes with zero slopes for which the trajectory remains stable are shown on figure 2.

For realistic lattices (case 2, 3 and 4 in section 2), trajectories have been tracked for 10^5 turns with two sets of initial conditions. One set has identical coordinates in both planes, the other set has an amplitude ratio of tan(15°)=0.27, the two trajectory slopes are zero. Both β–functions have the value of 18m at the starting point. The beam emittance is 3.75nm at 1σ. Synchrotron oscillations with an initial relative momentum deviation of 0.00075 are included. 60 different realizations of the multipole errors have been studied in order to achieve a 95% confidence level of the minimum dynamic aperture.

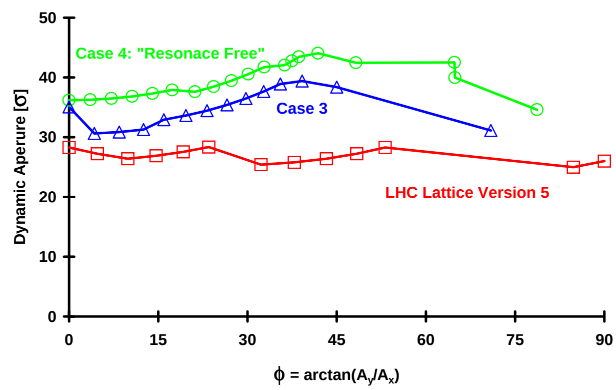

Figure 2: 1’000 turn dynamic aperture of the LHC model for three phase advances per cell: LHC lattice version 5, case 3 and case 4 of section 2. The dynamic aperture is defined as the ensemble of points in the {x,y} plane for which the trajectories starting with the initial coordinates x and y and zero slopes at the beginning of the lattice where both α’s are zero, are stable over 1000 turns. Both horizontal and vertical β functions are equal to 1m at the starting point. The multipole errors are b_4=0.5, a_4=0.5 and b_5=1.4 in units of 10^{-4} and at a reference radius 17mm.

The tracking results are shown in table 1. For the “target error table” (the first three cases) we observe no difference in the values for the dynamic apertures. When realistic octupole components b_4 and a_4 are introduced the dynamic aperture is not changed for quasi horizontal motion ($y_0 = 0.27x_0$). For equal amplitudes ($y_0 = x_0$) there is a 10% reduction of the minimum dynamic aperture for the cases 3 and 4 in section 2. This reduction is fully recovered for the "resonance free" lattice.

Case	Realistic	$y_0 = x_0$		$y_0 = 0.27x_0$	
	$b_4\&a_4$	min.	av.	min.	av.
3 (V6 nom.)	off	12.4	14.6	11.3	12.7
4		12.4	14.5	11.0	12.8
5 (Res. free)		12.5	14.8	11.3	12.6
3 (V6 nom.)	on	10.9	13.4	11.1	12.8
4		11.3	13.8	11.3	12.9
5 (Res. free)		12.6	14.5	10.8	12.7

Table 1: Minimum and average dynamic aperture in units of r.m.s. beam size for a series of 60 distributions of errors, both random and systematic per arc. For the first three cases, the values are taken from the "target error table". For the following three cases the errors are the same except for the systematic by arc octupole components which have been set to the more realistic values of a_4=0.555 and b_4=0.35 (in units of 10^{-4} at a reference radius of 17mm) without compensation. From our current experience, the dynamic apertures can be overestimated by 0.2 for the average and 0.5 for the minimum.

In order to observe the beneficial effect of the resonance suppression the same exercise was repeated with values of the realistic b_4 and a_4 components increased threefold. The efficiency of the "resonance free" lattice appears clearly in table 2.

From these three sets of tracking results we conclude that the dynamic aperture of the "target error table" is far from being dominated by low order resonances driven by systematic multipoles.

Case	$y_0 = x_0$		$y_0 = 0.27x_0$	
	min.	av.	min.	av.
3 (V6 nom.)	7.7	10.3	7.6	10.8
5 (Res. free)	10.2	13.5	10.3	12.5

Table 2: Minimum and average dynamic aperture in units of r.m.s. beam size for a series of 60 distributions of errors, both random and systematic per arc. The values are taken from the "target error table" except the octupole errors which have been set to three times their realistic values.

In order to check that there is no artifact in the "resonance free" lattice, the dynamic aperture has been evaluated for 12 ratios of initial amplitudes. The results are shown on figure 3. The maximum dynamic apertures (the two upper curves) are both at the level we expect from the random components only [7], the resonance suppression has obviously no effect in this case. The average dynamic apertures (medium two curves) show an improvement of some 2σ for all amplitude ratios. The minimum dynamic apertures (lower two curves) show a sizable improvement except for quasi vertical motion. This lack of improvement is probably due to higher order effects.

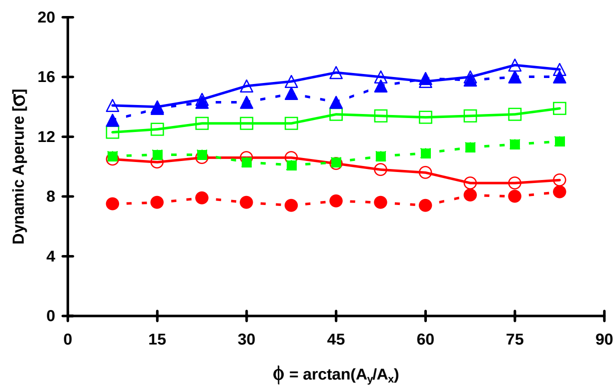

Figure 3: Dynamic aperture in the x,y plane for the two cases of table 2.

6 CONCLUSION

We have demonstrated, for the case of the LHC, that the dynamic aperture can be increased by a suppression of first order resonances with a proper choice of the cell phase advances in the arcs. This is closely associated with the fact that we expect large systematic per arc multipole components to dominate the dynamic aperture.

This optimisation of the cell phase advances provides a large safety margin against unexpected large systematic components of the LHC dipoles.

7 REFERENCES

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