Application Limit of SR Interferometer for Emittance Measurement

Y. Takayama*,T. Okugi, T. Miyahara, Tokyo Metropolitan University, Tokyo, Japan
S. Kamada, J. Urakawa , T. Naito, KEK, Tsukuba, Japan

Abstract

We investigate the application limit of the SR interferometer for emittance measurement at the KEK-ATF. We need to consider two important problems, which are the limitation of the availability of the van Cittert-Zernike theorem and the diffraction effect due to a narrow vertical aperture of the SR extraction line. The former problem is analyzed with the theory introduced in another paper [1]. The latter one is studied with the numerical calculation, where the narrow aperture is assumed to be an optical slit with an adequate vertical width. We show that these problems must be solved for the accurate measurement of the electron emittance, especially in the vertical direction.

1 INTRODUCTION

A measurement of the electron beam emittance is one of the most important theme for the accelerator physics. At the KEK-ATF damping ring, several attempts are performed to estimate the emittance. Especially, the SR interferometer which measures the spatial coherence (visibility) has some advantages compared with other methods [2]. The electron beam size can be obtained by performing the Fourier transformation of the spatial coherence, which is called as the van Cittert-Zernike theorem. However, it is not trivial whether this theorem is available for the bending magnet radiation. Recently, some conditions to judge whether this theorem is available or not for the bending magnet radiation were derived by the authors [1]. We investigate whether these conditions are satisfied for the SR interferometer at the ATF damping ring.

We need to consider another important problem, the effect of the narrow vacuum chamber at the SR extraction line. If the light is cut by this vacuum chamber, the spatial coherence at the downstream changes. For an extreme example, if the width of the vacuum chamber is much smaller than the wavelength of light, the spatial coherence at the downstream is perfect for any electron beam parameters. Therefore, in order to measure the electron beam size with the SR interferometer, we must investigate how the vacuum chamber affects the spatial coherence.

In this paper, we investigate above two points in detail and judge whether the SR monitor is available to measure the electron beam size at the ATF damping ring.

* Email: takayama@phys.metro-u.ac.jp

2 AVAILABILITY OF THE VAN CITTERT-ZERNIKE THEOREM

The Twiss parameters and dispersion at the position where the electron beam size is evaluated, and emittance in the horizontal and vertical directions are shown in the Table 1 [3]. Here, we assumed that there is no vertical dispersion and the coupling is 1 %. We investigate whether the van Cittert-Zernike theorem is available in the horizontal and vertical directions individually. The details of derivation of the conditions are shown in another paper [1].

Table 1: Design value of the ATF damping ring at the point where the SR interferometer is installed.

	horizontal (x)	vertical (y)
α	0.3590	-1.1690
β (m)	0.3796	2.8435
γ (1/m)	2.9739	0.8323
η	0.0491	0
η'	-0.1458	0
ε (nm·rad)	1.08	0.0108

2.1 Horizontal direction

In the horizontal direction, three conditions must be satisfied for the electron beam size σ_x in order to use the van Cittert-Zernike theorem, which are

$$\sigma_x \gg \frac{\rho\bar{\beta}_x\varepsilon_x}{L^2}, \quad (1)$$

$$\sigma_x \gg \frac{\rho\bar{\beta}_y\varepsilon_y}{L^2\left(1+\frac{\bar{\beta}_y\varepsilon_y}{\bar{\sigma}_p^2}\right)}, \quad (2)$$

$$\sigma_x \gg \sqrt{\frac{\bar{\beta}_x}{\beta_x}}\frac{\rho\varepsilon_x}{L}, \quad (3)$$

where $L = 7.04$ m is the distance between the light source and double slit, $\rho = 5.73$ m is the bending radius. α, β, γ and ε are the Twiss parameters and emittance after integrating energy spread, respectively. $\bar{\beta}$ is defined as $\bar{\beta} = \beta - 2\alpha L + L^2\gamma$. σ_p and σ'_p are the beam size and beam divergence of the light at the waist in the vertical direction which is emitted by a single electron. $\bar{\sigma}_p$ is defined as $\sqrt{\sigma_p^2 + L^2{\sigma'_p}^2}$. For the wavelength $\lambda = 500$ nm, σ_p and σ'_p are calculated as 19.4 μm and 2.06 mrad, respectively.

Using the parameters in Table 1, we have $\sigma_x = 34.0\ \mu\mathrm{m}$.

The conditions in (1), (2) and (3) are written as

$$\begin{aligned}\sigma_x &\gg 5.20\times10^{-2}\,\mu\mathrm{m},\\ \sigma_x &\gg 7.56\times10^{-5}\,\mu\mathrm{m},\\ \sigma_x &\gg 3.42\times10^{-2}\,\mu\mathrm{m},\end{aligned}$$

respectively. These condition are well satisfied for the design value and the van Cittert-Zernike theorem is safely used to estimate the beam size in the horizontal direction.

2.2 Vertical direction

As same with the case of the horizontal direction, three conditions are necessary in order to use the van Cittert-Zernike theorem, which are

$$\sigma_y \gg \frac{\rho}{L^2}\sqrt{\frac{\varepsilon_x\varepsilon_y\bar\beta_x\bar\beta_y}{\left(1+\frac{\bar\beta_y\varepsilon_y}{\bar\sigma_p^2}\right)}}, \quad (4)$$

$$\sigma_y \gg \frac{L\varepsilon_y}{\bar\sigma_p}, \quad (5)$$

$$\sigma_y \gg \frac{L}{2k\bar\sigma_p}\sqrt{\frac{\varepsilon_y\bar\beta_y}{\bar\sigma_p^2+\varepsilon_y\bar\beta_y}}, \quad (6)$$

where σ_y = 5.54 μm is the electron beam size in the vertical direction. There is an extra condition on the divergence of the light beam under which the light beam reaches at the observer points strongly enough, which is

$$\sigma_y > \frac{\sigma_p}{\sqrt{1+\left(\frac{\sigma'_y}{\sigma'_p}\right)^2}}, \quad (7)$$

where we put the electron beam divergence as $\sigma'_y = \sqrt{\varepsilon_y\gamma_y}$.

As for the conditions in (4), (5) and (6), we have

$$\begin{aligned}\sigma_y &\gg 1.98\times10^{-3}\,\mu\mathrm{m},\\ \sigma_y &\gg 5.25\times10^{-3}\,\mu\mathrm{m},\\ \sigma_y &\gg 2.41\times10^{-1}\,\mu\mathrm{m},\end{aligned}$$

respectively. Therefore, these conditions are well satisfied. However, the condition (7) is written as

$$\sigma_y > 19.9\,\mu\mathrm{m},$$

which is not satisfied in this case. This means that the whole curve of the visibility can not be obtained due to the weak intensity of the light for large slit separation and the accurate measurement of the electron beam size is very difficult in the vertical direction.

3 EFFECT OF VERTICAL APERTURE

Between the light source and the double slit, there is a narrow space of the vacuum chamber in the vertical direction whose width is only 4 $\sim$ 5 mm. It is important to investigate whether this narrow space affects the coherence in the vertical direction. For this purpose, we suppose that the narrow space is equivalent with the vertical entrance slit with the width d, as shown in Figure 1. We put the distance between the light source and the entrance slit to be L_1 = 0.56 m and the distance between the entrance slit and the double slit to be L_2 = 6.48 m, respectively.

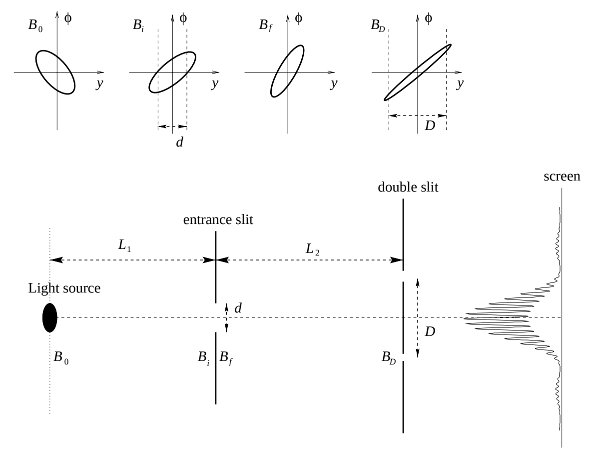

Figure 1: Transformation of the phase space of light.

If we denote the electric field on the entrance slit and on the double slit as $E_i(y)$ and $E_D(y)$, respectively, the correlation of the fields are related with the equation

$$\Gamma_D(y_1,y_2) = \int dy_a dy_b T^*(y_a)T(y_b)\,\Gamma_i(y_a,y_b)\frac{\mathrm{e}^{-ik(r_{1a}-r_{2b})}}{r_{1a}r_{2b}}, \quad (8)$$

where the integration is performed on the entrance slit. k is the wave number of light and

$$\begin{aligned}\Gamma_i(y_1,y_2) &= \langle E_i^*(y_1)E_i(y_2)\rangle,\\ \Gamma_D(y_1,y_2) &= \langle E_D^*(y_1)E_D(y_2)\rangle,\\ r_{1a} &= \sqrt{L_2^2+(y_1-y_a)^2},\\ r_{2b} &= \sqrt{L_2^2+(y_2-y_b)^2}.\end{aligned}$$

$T(y)$ is the transmittance function of the entrance slit. (8) is valid if the wavelength of the light is much smaller than the entrance slit. Intuitively, (8) means that the electric field on the double slit can be obtained by summing up the spherical wave emitted by the point sources on the entrance slit. The spatial coherence on the double slit is written as

$$\gamma(D) = \frac{\Gamma_D(D/2,-D/2)}{\sqrt{\Gamma_D(D/2,D/2)\,\Gamma_D(-D/2,-D/2)}},$$

where D is the separation of the double slit.

In order to calculate the spatial coherence, the correlation of the fields on the entrance slit is needed. This can be obtained by approximating the radiation field with the Gaussian beam. With this approximation, the brightness function is represented analytically. Since the brightness function $B(y, \phi)$ and the correlation of the fields $\Gamma(y_a, y_b)$ are related with the Fourier transformation [4]. $\Gamma(y_a, y_b)$ is also obtained analytically. Using this property, the visibility is numerically calculated if we set the electron beam parameter at the emitting point [5].

3.1 Numerical calculation

We calculate the visibility with the formula discussed in the previous section. We put ε_y = 0.01 nm·rad. Figures 2, 3 and 4 are plots of the numerical calculations for σ'_p = 1.0 mrad, 2.0 mrad and 3.0 mrad, respectively. For each light divergence, four types of the entrance slits are used. In each figure, three curves are drawn. The curve with "relative intensity" is the plot of the intensity on the double slit normalized by that for $D = 0$. The curve with "without slit" is the curve of the spatial coherence without the entrance slit. The curve with "d mm slit", where d = 2, 4, 5 or 6, is the curve of the spatial coherence with the d mm entrance slit. The horizontal axis represents the separation of the double slit D, and the vertical axis represents the relative intensity or the spatial coherence.

Although the intensity distribution on the double slit is Gaussian, the behaviors of the relative intensity on the double slit are very complicated, especially for small entrance slit and large light divergence. It is due to the diffraction effect. The vibrations of the spatial coherence with the entrance slit are caused by the Fraunhofer diffraction. For the region where this vibration remarkably appears, the spatial coherence with the entrance slit has great discrepancy with that without the entrance slit. Fortunately, this discrepancy is hard to observe since the light intensity is extremely weak for such region.

As we pointed out in the previous section, it is difficult to obtain the exact curve of the coherence in the vertical direction, since the intensity decreases rapidly before the coherence decreases.

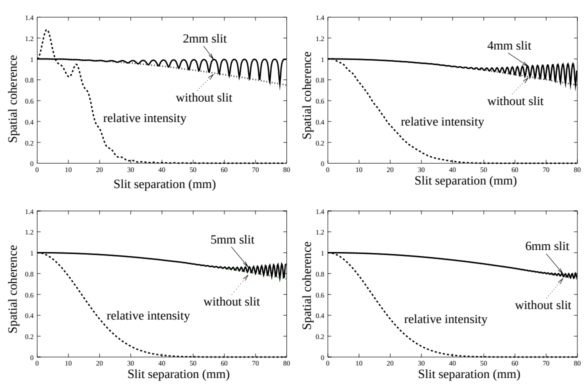

Figure 2: Calculation for σ'_p = 1.0 mrad.

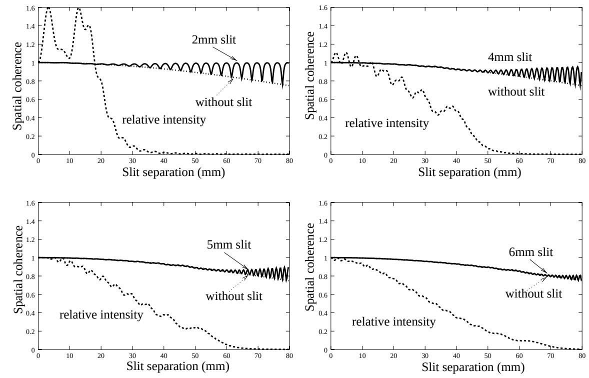

Figure 3: Calculation for σ'_p = 2.0 mrad.

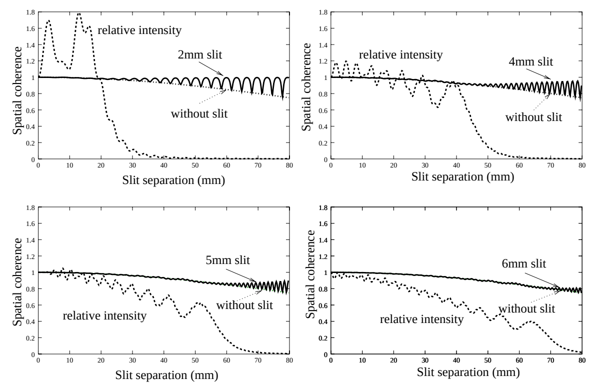

Figure 4: Calculation for σ'_p = 3.0 mrad.

4 CONCLUSION

The calculation in this paper shows that the electron beam size in the vertical and horizontal directions can be estimated by using the van Cittert-Zernike theorem at the ATF damping ring. There exists an experimental difficulty in the vertical direction because of the weakness of light intensity. This difficulty will be overcome by using the vertical bend as mentioned in another paper [1]. However, the vertical intensity distribution on the double slit might have a very complex form due to the vacuum chamber, which has a chance to improve the spatial coherence. Therefore, the measurement of the intensity distribution on the double slit is significant.

5 ACKNOWLEDGEMENT

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