LOW-FREQUENCY WIGGLER RADIATION

F. Méot, CEA, DSM/DAPNIA/SEA, 91191 Saclay, France*

Abstract

Classical formalism for synchrotron radiation interference between short sources is applied to analytical formulation of wiggler radiation in the low frequency domain.

1 INTRODUCTION

Considering the lasting interest in synchrotron radiation (SR) for beam diagnostics [1], due also to raising specific demand such as infrared SR [2], a detailed insight in the interference phenomenon between separate radiating sources and its understanding remains of concern. The case of wiggler SR has already been addressed in terms of interference [3] and more recently implications of the low frequency hypothesis have been discussed [4]. We re-visit the subject with recently developed material [5] and derive by this means a detailed analytical formulation of wiggler SR in the low frequency domain.

2 ANALYTICAL MATERIAL

2.1 Low frequency model

In regular conditions of SR production the spectral angular energy density observed at large distance r (assumed constant) is given by $\frac{\partial^3 W}{\partial\omega\partial\phi\partial\psi} = 2\epsilon_0 c \mid {}^r\tilde{\vec{E}}(\phi,\psi,\omega)|^2$ (the r-independent quantity ${}^r\tilde{\vec{E}} = r\,\tilde{\vec{E}}$ is introduced for simplicity) where $\tilde{\vec{E}}(\phi,\psi,\omega)$ is the Fourier transform of the radiated electric field $\vec{E}(\phi,\psi,t)$ (see Fig. 1) and ω is the observed frequency. In the low frequency domain one has

$$\begin{aligned} {}^r\tilde{E}_\sigma(\phi,\psi,\omega) &= \frac{q\gamma}{(2\pi)^{3/2}\epsilon_0 c}\Big(\frac{K-\gamma\phi}{1+(K-\gamma\phi)^2+\gamma^2\psi^2} \\ &+ \frac{K+\gamma\phi}{1+(K+\gamma\phi)^2+\gamma^2\psi^2}\Big) \qquad (1) \\ {}^r\tilde{E}_\pi(\phi,\psi,\omega) &= \frac{q\gamma}{(2\pi)^{3/2}\epsilon_0 c}\,\gamma\psi\Big(\frac{1}{1+(K-\gamma\phi)^2+\gamma^2\psi^2} \\ &- \frac{1}{1+(K+\gamma\phi)^2+\gamma^2\psi^2}\Big) \end{aligned}$$

where indices σ and π designate polarisation components respectively parallel to the bend plane and normal to $\vec{E_\sigma}$; angles ϕ in the bend plane and ψ normal to it define the observation direction. Eqs. 1 holds over a few rms aperture (with $\gamma\phi_{rms} = \gamma\psi_{rms} = \sqrt{(1+K^2)}$) and up to a fraction of ω_{limit} as defined by

$$\omega_{limit} = \omega_c/(3K(1+K^2)) = \gamma^2 c/(L(1+K^2)) \qquad (2)$$

in which $\omega_c = 3\gamma^3 c/2\rho$ is the critical frequency of the standard formalism, γ is the Lorentz relativistic factor, ρ is the curvature radius, α is the particle total deviation, $L = \rho\alpha$ is the trajectory length in the magnetic field, and $K = \alpha\gamma/2$ is the deflection parameter.

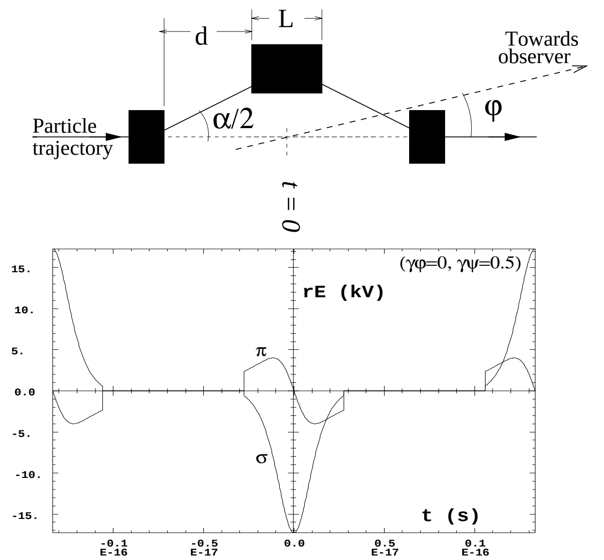

Figure 1: *Up* : A three-dipole wiggler. Definition of observation angle φ in the bend plane. *Down* : Typical shape of the electric field impulse ${}^{3r}E_{\sigma,\pi}(\varphi,\psi,t)$ at the observer.

2.2 Interference

As one knows the underlying physics in SR interference is in time coherence resulting from the geometrical arrangement of sources, which entails space and frequency modulation of the radiated signal. A series of N sources radiate electric field of the form

$${}^N E_{\sigma,\pi}(\phi,\psi,t) = \sum_{i=1}^{N} \delta(t+T_i) * E_{i\sigma,\pi}(\phi,\psi,t) \qquad (3)$$

where $E_{i\sigma,\pi}(\phi,\psi,t)$ describes the impulse from magnet i, T_i is the emission time of signal i, δ is the Dirac distribution and $*$ denotes the convolution product. The Fourier transform gives the interferential amplitude density

$${}^N\tilde{E}_{\sigma,\pi}(\phi,\psi,\omega) = \sum_{i=1}^{N} e^{i\omega T_i}\,\tilde{E}_{i\sigma,\pi}(\phi,\psi,\omega) \qquad (4)$$

whose modulus square provides the energy density

$$\begin{aligned} \frac{\partial^{3\,N} W_{\sigma,\pi}}{\partial\omega\partial\phi\partial\psi} = &\quad 2\epsilon_0 c\,\Big(\big(\textstyle\sum_{i=1}^{N} {}^r\tilde{E}_{i\sigma,\pi}\cos(\omega T_i)\big)^2 \\ + &\quad \big(\textstyle\sum_{i=1}^{N} {}^r\tilde{E}_{i\sigma,\pi}\sin(\omega T_i)\big)^2\Big) \qquad (5) \end{aligned}$$

* Email : fmeot@cea.fr

Times T_i are obtained from the geometry of the magnet assembly by combining the duration $\Delta T = \frac{L}{2\gamma^2 c}(1 + K^2/3 + \gamma^2(\phi^2 + \psi^2))$ of the impulse issued from a magnet, with the time of flight (in observer time) $\Delta T_d = \frac{d}{2\gamma^2 c}(1 + \gamma^2(\phi^2 + \psi^2))$ between magnets distant d.

Note that Eqs. 4, 5 involve the exact Fourier transform of $\delta(t + T_i)$ hence possible low frequency approximation validity domain depends only on the characteristics (L and K) of SR sources, not on their distance d.

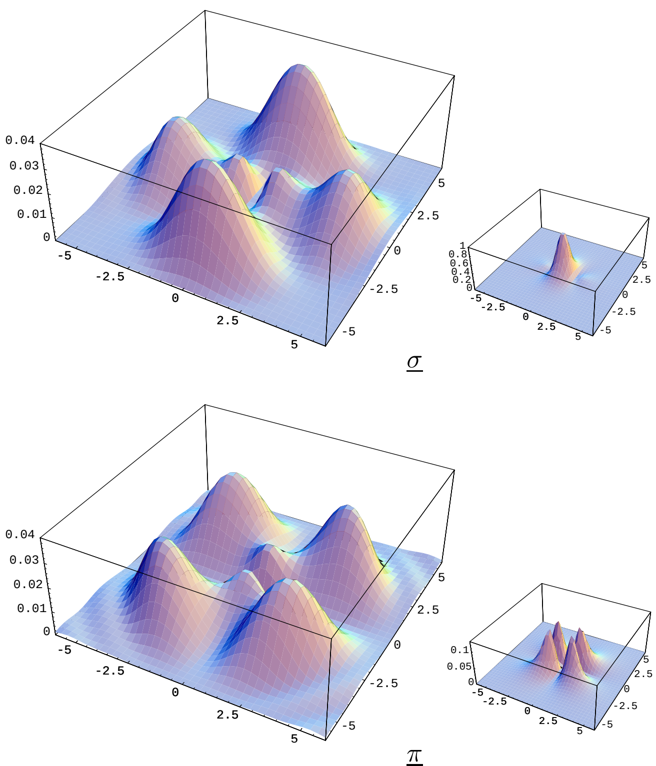

Figure 2: Low frequency energy density from a three-dipole wiggler (Eq. 6 for 2.5 GeV electron, at $\omega = 1.9\,10^{15}$rad/s - $\lambda = 10^{-6}$m). The small boxes show the energy density from the central $(2/\gamma)$ dipole alone ; comparison reveals a damping of about $0.04/1$ due to the wiggler interference. (g.phi and g.psi stand for coordinates $\gamma\phi$ and $\gamma\psi$.)

3 SR FROM A 3-DIPOLE WIGGLER

For simplicity a single-bump wiggler based on $(1/\gamma)$- and $(2/\gamma)$-deviation magnets[1] is considered (Fig. 1). However what follows can be extended to arbitrary N, and as well to low frequency undulator radiation (N large and $K < 1$).

The low frequency limit simplifies into $\omega_{limit} \approx \omega_c/6 = \gamma^2 c\,/\,2\,L$ (Eq. 2 with $K = 1$) leading for instance to validity range $\omega < 4\,10^{16}$ rad/s ($\lambda > 40\,10^{-9}$ m) for a 2.5 GeV electron traversing a, *e.g.*, 670 kG, $L = 5\,10^{-2}$ m long dipole. Fig. 1 shows the typical shape of electric field impulse series so generated (Eq. 3 with $N = 3$), the total duration of which is $\Delta T \mid_{\frac{L}{2},\frac{K}{2},\phi=\varphi-\frac{K}{2\gamma}} + \Delta T_d \mid_{\phi=\varphi-\frac{K}{\gamma}} + \Delta T \mid_{\phi=\varphi} + \Delta T_d \mid_{\phi=\varphi+\frac{K}{\gamma}} + \Delta T \mid_{\frac{L}{2},\frac{K}{2},\phi=\varphi+\frac{K}{2\gamma}}$. Taking the origin at the centre of the central dipole, times in Eq. 3 write, $T_2 = 0$ and

$$-T_1 = \Delta T \mid_{\frac{L}{2},\frac{K}{2},\phi=\varphi-\frac{K}{2\gamma}} + \Delta T_d \mid_{\phi=\varphi-\frac{K}{\gamma}} + \frac{1}{2}\Delta T \mid_{\phi=\varphi}$$

$$+T_3 = \frac{1}{2}\Delta T \mid_{\phi=\varphi} + \Delta T_d \mid_{\phi=\varphi+\frac{K}{\gamma}} + \Delta T \mid_{\frac{L}{2},\frac{K}{2},\phi=\varphi+\frac{K}{2\gamma}}$$

Fig. 2 shows the resulting interferencial patterns which express as

$$\frac{\partial^{3\,3}W_{\sigma,\pi}}{\partial\omega\partial\varphi\partial\psi} = 2\,\epsilon_0\,c$$
$$\Bigg(\Big(\,{}^r\tilde{E}_{\sigma,\pi}|_{\frac{K}{2},\phi=\varphi-\frac{K}{2\gamma}}\cos(\omega T_1) + {}^r\tilde{E}_{\sigma,\pi}|_{-K,\phi=\varphi}$$
$$\pm\ {}^r\tilde{E}_{\sigma,\pi}|_{\frac{K}{2},\phi=\varphi+\frac{K}{2\gamma}}\cos(\omega T_3)\Big)^2 + \Big(ST\Big)^2\ \Bigg) \quad (6)$$

where (ST) designates the complementary sin term. Fig. 3 compares the radiation spectrum from the wiggler to that of a single $K = 1$ dipole and to classical body SR from a $K \gg 1$ dipole.

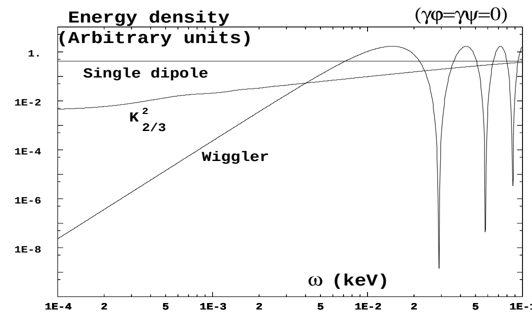

Figure 3: Forward spectrum of wiggler σ component (Eq. 6), and for comparison, spectra due to the central $(2/\gamma)$-deviation dipole alone ($K = 1$ in Eq. 1), and due to body SR from a strong dipole (regular $K^2_{2/3}(\omega)$ shape).

[1] The latter has the merit of producing highest brightness low frequency SR.

4 REFERENCES

[1] A. Hofmann, Diagnostics with SR, Proc. Cern Acc. School “Synchrotron Radiation and Free-Electron Lasers”, Grenoble (F), 22-28 April 1996.

[2] Workshop on infrared SR, 1^{st} meeting : the source, J.-L. Laclare, M.-P. Level and P. Roy organisers, Gif-sur-Yvette, France, 8-9 Dec. 1997.

[3] K.-J. Kim, Characteristics of SR, AIP Conf. Proc. 184, US Part. Acc. School, Fermilab, 1987 ; R. P. Walker, Interference effects in undulator and wiggler radiation sources, Internal Report Sincrotrone Trieste, 1993.

[4] R. A. Bosch, Long-wavelength radiation along the straight-section axis in anelectron storage ring, NIM A, vol. 386 (1997) 525-530.

[5] F. Méot, A theory of low frequency far-field synchrotron radiation, report DSM/DAPNIA/SEA-98-05, CEA/Saclay (1998) ; F. Méot, N. Ponthieu, On low frequency far-field interference between short SR sources, report DSM/DAPNIA/SEA-98-53, CEA/Saclay (1998).