A DISCRETE HYSTERESIS MODEL FOR PIEZOELECTRIC ACTUATOR AND ITS PARAMETER IDENTIFICATION

Y. Cao and X. B. Chen[#]
Department of Mechanical Engineering
University of Saskatchewan, Canada

Abstract

Hysteresis is an important nonlinear effect exhibited by piezoelectric actuators (PEA) and its modelling has been drawing considerable attention. This paper presents the development of a novel discrete model based on the concept of auto-regressive moving average (ARMA) for the piezoelectric-actuator hysteresis, and its parameter identification method as well. Experiments were carried out to verify the effectiveness of the developed model. The result obtained shows that the developed model can well represent the hysteresis of the PEA.

INTRODUCTION

Piezoelectric actuators (PEA) have been widely used in nanopositioning applications, such as AFM , STM , DVD disc reading and writing [1], diamond lathe machine [2], lithography, X-ray imaging [3]. However, the performance of a PEA can be significantly degraded by its hysteresis. Hysteresis is a memory effect of piezoelectric actuators and, as a result, the hysteresis exhibited at an given time instant depends on not only the input at the present time but also the operational history of the system considered. In order to develop control schemes on PEA, modelling of PEA has been drawing considerable attention and several models have been resulted to describe the hysteresis effect, such as Preisach model [4], the ferromagnetic material model [5] and the nonlinear auto-regressive moving average model with exogenous input (NARMAX model) [6]. However, most of the models developed in literatures are continuous and the model-based controller design is proceeded in the continuous time domain. With the advance of computer technology nowadays, controllers are mostly implemented digitally. Note that not all the continuous controllers can work on the sampled digital system as desired since the discrete sampling can sometimes make the continuous system unstable. Therefore, it is advantage to develop a discrete hysteresis model of PEA for its digital controller design. Unfortunately, little work about the discrete hysteresis model or the digital controller design for PEA has been found yet. In this paper, the ferromagnetic material hysteresis model is discreted and, by combining it with the concept of auto-regressive moving average (ARMA), a novel model is developed to represent the hysteresis of PEA. Specifically, the next section of this paper is the introduction to the discrete ARMA-based hysteresis model, which is followed by the experimental identification and verification results by using the discrete ARMA-based hysteresis model as compared to the general discrete form of hysteresis model [7]. The last section gives the conclusions of the paper and future work.

[#] xbc719@mail.usask.ca

DISCRETE ARMA-BASED HYSTERESIS MODEL

The ferromagnetic material hysteresis model introduced by Adriaens and Koning [5] is illustrated in the following:

$$\dot{y} = \alpha|\dot{x}|[f(x) - y] + \dot{x}g(x) \tag{1}$$

where x is the input of the hysteresis and y is the output, $f(x)$ and $g(x)$ are functions of x with which you can "shape" the hysteresis loop. It has been experimentally verified that this differential equation is also suitable for describing electric hysteresis such as PEA. In theory, PEA shows the length saturation. In practise, however, the displacement of the PEA stays far away from saturation. Therefore, chose $f(x) = ax/\alpha$ and $g(x) = b$ as the shape function, Equation (1) can be rewritten as:

$$\dot{y} = |\dot{x}|(ax + cy) + b\dot{x} \tag{2}$$

where $c = -1/\alpha$. [7] applied the difference equation to discrete Equation (1) as follows:

$$\begin{aligned} y(k+1) - y(k) = {} & |x(k+1) - x(k)|[ax(k) + cy(k)] \\ & + b[x(k+1) - x(k)] \end{aligned} \tag{3}$$

This paper discrete Equation (1) by integral.

Discrete form of the ferromagnetic material hysteresis model

When the input signal is monotonically increasing, $\dot{x} > 0$, take integral on both side of equation (2) in one sampling interval, one can derive:

$$\int_{kT}^{(k+1)T} \dot{y}dt = a\int_{kT}^{(k+1)T} \dot{x}xdt + c\int_{kT}^{(k+1)T} \dot{x}ydt + b\int_{kT}^{(k+1)T} \dot{x}dt \tag{4}$$

where T is the sampling interval.

Equation (4) leads to:

$$\begin{aligned} y(k+1) - y(k) = {} & \frac{1}{2}a[x^2(k+1) - x^2(k)] + c\int_{x(k)}^{x(k+1)} ydx \\ & + b[x(k+1) - x(k)] \end{aligned} \tag{5}$$

which is the discrete form of the first order hysteresis differential equation (2).

Using trapezoid equation to estimate the integral term, Equation (5) yields:

$$y(k+1) = a\frac{\alpha(k+1)}{2 - c\beta(k+1)} + b\frac{2\beta(k+1)}{2 - c\beta(k+1)}$$

$$+\frac{2+c\beta(k+1)}{2-c\beta(k+1)}y(k) \qquad (6)$$

where $\alpha(k+1)=x^2(k+1)-x^2(k)$,

$\beta(k+1)=x(k+1)-x(k)$.

Given the zero initial condition that $y(1)=0$,one can derive:

$$y(2)=a\frac{\alpha(2)}{2-c\beta(2)}+b\frac{2\beta(2)}{2-c\beta(2)}$$

$$y(3)=a[\frac{\alpha(3)}{2-c\beta(3)}+\frac{2+c\beta(3)}{2-c\beta(3)}\cdot\frac{\alpha(2)}{2-c\beta(2)}]$$

$$+b[\frac{2\beta(3)}{2-c\beta(3)}+\frac{2+c\beta(3)}{2-c\beta(3)}\cdot\frac{2\beta(2)}{2-c\beta(2)}]\cdots$$

Therefore, the output y can always be represented as a function of input x by recursion:

$$y(k+1)=ay_1(k+1)+by_2(k+1) \qquad (7)$$

where

$$y_1(k+1)=\frac{\alpha(k+1)}{2-c\beta(k+1)}+\frac{2+c\beta(k+1)}{2-c\beta(k+1)}y_1(k) \qquad (8)$$

$$y_2(k+1)=\frac{2\beta(k+1)}{2-c\beta(k+1)}+\frac{2+c\beta(k+1)}{2-c\beta(k+1)}y_2(k) \qquad (9)$$

When the input signal is monotonically decreasing, $\dot{x}<0$, repeating the above process, one can derive:

$$y(k+1)=ay_1(k+1)+by_2(k+1) \qquad (10)$$

where

$$y_1(k+1)=\frac{-\alpha(k+1)}{2+c\beta(k+1)}+\frac{2-c\beta(k+1)}{2+c\beta(k+1)}y_1(k) \qquad (11)$$

$$y_2(k+1)=\frac{2\beta(k+1)}{2+c\beta(k+1)}+\frac{2-c\beta(k+1)}{2+c\beta(k+1)}y_2(k) \qquad (12)$$

In order to verify the effectiveness of the discrete form of the first order hysteresis differential equation, a SIMULINK model was built to generate the simulation data. Parameters a, b, c are chosen to be 0.0064, -0.0378, 0.1144 such that the output displacement of the SIMULINK model can fit the measured displacement. Meanwhile, another group of output data was generated by Equations (3) and (7)-(12). Table 1 shows the comparison of the discrete error using Equation (3) and Equation (7)-(12). The input is a sinusoidal signal whose frequency is set to be 1Hz ~300Hz with 70V magnitude. From the result, it can be concluded that the discrete hysteresis Equation (7)-(12) is more accurate in describing hysteresis than the general Equation (3).

Table 1: Discrete error by using different methods

Input Frequency (Hz)	**50**	**100**	**200**	**300**
By using Equation (7)-(12)	0.0167	0.0335	0.0671	0.1010
By using Equation (3)	0.0171	0.0341	0.0681	0.1022

Discrete ARMA-based hysteresis model

The general ARMA model has the form as follows:

$$z(t)=\sum_{i=1}^{N_z}a_i z(t-i)+\sum_{i=0}^{N_y}b_i y(t-i) \qquad (13)$$

X.B. Chen, Q.S. Zhang et. al. [8] have made a conclusion that the second order system can be used to approximately represent the dynamics of the piezoelectric stage if the mass ratio between the stage and the actuator increases. Thus, using a second order ARMA model, one can derive:

$$z(t)=a_1z(t-1)+a_2z(t-2)+b_0y(t)+b_1y(t-1)+b_2y(t-2) \qquad (14)$$

Substitute the discrete hysteresis Equation (7) and (10) into Equation (14), the discrete ARMA based hysteresis model will be derived as:

$$z(t)=a_1z(t-1)+a_2z(t-2)+b_0'y_1(t)+b_0''y_2(t)$$

$$+b_1'y_1(t-1)+b_1''y_2(t-1)+b_2'y_1(t-2)+b_2''y_2(t-2) \qquad (15)$$

It will be used to describe the rate-dependent performance of a piezoelectric actuator later.

PARAMETER IDENTIFICATION AND EXPERIMENTS

Experiments are implemented on a PEA (P-753, Physik Instrumente). The actuator can generate displacement in a range of 15 µm with a resolution of 0.5 nm. For displacement measurements, a capacitive displacement sensor of the P-753 PEA is used. It is a built in sensor with a resolution of 1nm. Both the actuator and the sensor are connected to a host computer via an I/O board (PCI-DAS1602/16, Measurement Computing Corporation) and controlled by SIMULINK programs. All measured displacements used in this study were measured with a sampling interval of 0.05 ms. The unit of the measured displacements is µm. The mass ration of the stage and the PEA is 49.8 which indicates that the dynamics of the piezoelectric driven stage can be regarded as a second order system approximately according to our previous study [8].

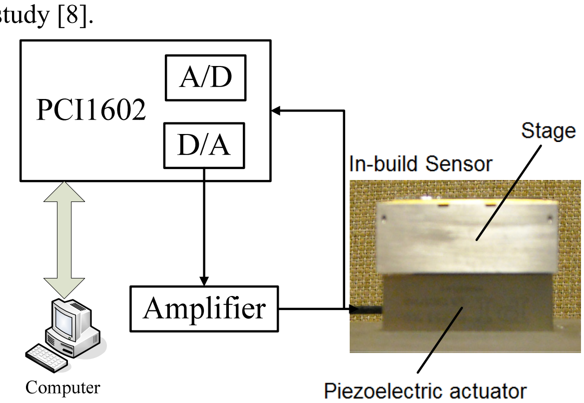

Figure 1: Piezoelectric driven stage

Online estimation method is applied to identify the parameters in the model by giving a bunch of sinusoidal inputs with frequency varying from 10Hz to 200Hz and amplitude being 70V. Table 2 shows the parameter identification results for discrete ARMA-based hysteresis model. The parameter c is identified to be -0.0305 by using Box-Kanemasu method. The initial value of the hysteresis operator parameters a, b, c are still identified from the 1Hz 70V sinusoidal input data using LS method.

Table 2: Discrete error by using different methods

Parameters	a_1	a_2	b_0'	b_0''
Value	1.6531	-0.676	-0.00276	-0.0732
Parameters	b_1'	b_1''	b_2'	b_2''
Value	0.0064	0.174	-0.00353	-0.0976

Another two types of inputs is applied to the piezoelectric actuator and the corresponding output is measured for model verification. One is the piecewise continuous combination of different amplitude sinusoidal inputs with the same frequency. The other one is the superposition of four sinusoidal inputs with different frequency, amplitude and phase delay.

Table 3 shows the estimation error according to discrete ARMA-based hysteresis model when applied a piecewise continuous combination of different amplitude sinusoidal inputs. The frequency varies from 10Hz to 400Hz. In order to show the effectiveness of the discrete method developed in this paper, the estimation error is compared with the general discrete form by using difference equation referred in [7].

Table 3: Estimation error for the Piecewise continuous combination of different amplitude sinusoidal inputs

Frequency (Hz)	10	50	200	400
Discrete ARMA-based hysteresis model	0.0943	0.0989	0.1112	0.1603
General discrete form of hysteresis	0.0946	0.0996	0.1128	0.1627

Compare with the model error corresponding to the same type of input data, it can be concluded that as the input frequency increases, the estimation error increases for both discrete methods. Meanwhile, the discrete ARMA-based hysteresis model has a lower estimation error than the general discrete model shown in [7], especially at high frequencies.

CONCLUSIONS

This paper presents the development of a novel discrete ARMA based hysteresis model to describe the hysteresis of PEA. Online estimation method was applied to identify the model parameters. In order to illustrate the effectiveness of the ARMA-based hysteresis model, experiments are carried out and the results are compared with the general discrete model (3). It shows that the discrete ARMA-based hysteresis model can better predict the hysteresis of PEA. However, the model shows a larger estimation error in high frequency application than in low frequency application due to the estimation of the integral term in the discrete hysteresis equation. Using a higher order polynomial equation to estimate the integral term maybe helpful to improve the discretization. Therefore, a piece of the future work will be on the use of a high order polynomial equation to estimate the integral term to improve the behaviour of the discrete ARMA-based hysteresis model. Moreover, the discrete control scheme will be developed to insure the stability of the digital control system for the PEA.

ACKNOWLEDGEMENT

The support to the present study from the China Scholarship Council (CSC) and the Natural Sciences and Engineering Research Council (NSERC) of Canada is acknowledged.

REFERENCES

[1] Santosh Devasia and S. O. Reza Moheimani, A Survey of Control Issues in Nano-positioning, IEEE Transactions on Control System Technology, Vol.15, No.5, Sep.2007

[2] H. J. Zhang,a_ S. J. Chen, and M. Zhou, Fast tool servo control for diamond-cutting microstructured optical components, J. Vac. Sci. Technol. B 27,,3…, May/Jun 2009

[3] H. Zhang, D. Chapman, Z. Zhong, C. Parham, M. Gupta, Crystal tilt error and its correction in diffraction enhanced imaging system, Nuclear Instrument and Methods in Physics Research A 572 (2007) 961-970

[4] I. D. Mayergoyz, Mathematical Models of Hysteresis, Physical Review Letters, Vol.56, No. 15, April 14th, 1986

[5] Han J.M.T.A. Adriaens, Willem L. de Koning and Reinder Banning, Modeling piezoelectric actuators, IEEE/ASME transactions on mechatronics, VOL.5, No.4, Dec. 2000

[6] Liang Deng, Yonghong Tan, Modeling hysteresis in piezoelectric acuators using NARMAX models, Sensors and actuators, A149, 2009, P106-112

[7] Jozef Vörös, Modeling and Identification of Hysteresis using Special Forms of the Coleman-Hodgdon Model, Journal of Electric Engineering, Vol.60, No.2, 2009, 100-105

[8] X. B. Chen, Q. S. Zhang, D. Kang, and W. J. Zhang, On the dynamics of piezoelectric positioning systems, Review of scientific instrument 79. 116101, 2008