

# A NEW SCHEME FOR DIRECT ESTIMATION OF PID CONTROLLER PARAMETERS

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## Abstract

This paper presents a novel scheme for the direct estimation of a PID (Proportional Integral Derivative) controller parameters ( $K_p$ ,  $T_i$ ,  $T_d$ ). The proposal discussed here is only applicable to first and second order stable systems. The formulation begins with system parameter identification (Transfer function of the process), which has been obtained using system identification toolbox of MATLAB. The pole zero cancellation technique is applied to estimate PID controller parameters which in turn results into the matched coefficients of the system parameters to the Controller parameters. An additional tuning parameter  $\alpha$  is proposed in our method, which provides an additional flexibility of tuning the response time of the controller without disturbing the controller parameters. The proposed scheme is bench marked using real time case of dc motor speed control. The effectiveness and robustness of the proposed auto tuning algorithm are verified by the simulation results.

## INTRODUCTION

A Proportional-Integral-Derivative (PID) controller is the most widely used controller in the industry today. The popularity of PID controller is due to its simplicity which uses only three, parameters to tune. Proportional ( $K_p$ ) term which controls the plant (system) proportional to the input error. Integral ( $T_i$ ) term which provides the change in the control input proportional to the integral of the error signal and the last one is the Derivative term ( $T_d$ ) that control the system by providing control signal proportional to the derivative of the error signal. Derivative action is used in some cases to speed up the response time and to stabilize the system behavior [1]. Error value is the difference between the set Reference value and the output value.

Although the PID controller is known to be the simplest and efficient controller, but it requires effective and optimized tuning of the control parameters ( $K_p$ ,  $T_i$ ,  $T_d$ ). Many PID controller tuning methods have been proposed in the literature, some of the popular methods are Ziegler-Nichols tuning, Cohen-Coon tuning, internal model control, direct synthesis method, neural networks based methodologies, relay based auto tuning method and may be much more [2]. In this paper a simple auto-tuning method is proposed which can be applied to most of the stable first and second order systems without having time delay. The proposed tuning method comprises of two parts. First part is used for finding the system transfer function identification. Second part is used for obtaining PID parameter by arranging the parameters in such a way

that the pole of the second order transfer function can be cancelled by the zeros of the PID parameters. In this way by introducing an extra parameter that is gain ' $\alpha$ ' we can optimize the rise time and settling time of the system. This technique basically tunes the PID parameter in a single iteration and thus it is fast and easy to implement. The paper is organized as follows. In section 2 we have discussed how to obtain the transfer function of an unknown system whereas in section 3 the PID tuning methodology is discussed. In section 4 we have presented the real time simulated case study of DC motor speed control that is done in MATLAB Simulink environment. A brief conclusion is presented in section 5.

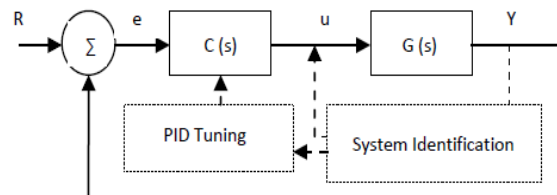


Figure 1: Auto Tuning Scheme of PID Controller.

## SYSTEM IDENTIFICATION

The system identification is the method of finding the transfer function of an unknown system by observing the input and output sequences of the system as shown in the figure 1. The transfer function (TF) identification was carried out by using system identification toolbox of the MATLAB. Let the measured input and output sequences are given by matrix:

$$\Psi(k) = \begin{Bmatrix} -y(k-1), -y(k-2) \cdots -y(k-n), \\ u(k-1), u(k-2) \cdots u(k-n) \end{Bmatrix}^T \quad (1)$$

The dynamics of the system is given by:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}; \boldsymbol{\theta}) \quad (2)$$

where  $\mathbf{x}$  is the state variable,  $\mathbf{u}$  is the input vector and  $\boldsymbol{\theta}$  is the parameter vector.

$$\boldsymbol{\theta} = [a_1 \ a_2 \ \cdots \ a_n \ b_1 \ b_2 \ \cdots \ b_n]^T \quad (3)$$

We have to choose the value of parameter vector  $\boldsymbol{\theta}$  in such a way so that the difference between  $\dot{\mathbf{x}}$  and  $\mathbf{f}(\mathbf{x}, \mathbf{u}; \boldsymbol{\theta})$  will impose minimum error. There are several techniques available in literature. MATLAB has in build procedure to find out the TF of the system by measuring input and

output sequences using system identification toolbox. The input and output sequences are observed in a discrete domain with a sample time  $T_s$  after that the system identification toolbox uses parametric system identification technique and gives the TF in discrete domain (z domain) that is given by:

$$H(z) = \frac{b_n z^n + b_{n-1} z^{n-1} \dots b_1 z^1 + b_0}{z^{n+1} + a_n z^n \dots a_1 z + a_0} \quad (4)$$

After getting the plant transfer function in z domain it is finally converted into continuous domain by using the analogy  $z = e^{sT_s}$ . In the MATLAB we use the function d2c (discrete to continuous) which converts the z domain TF to s domain using Tustin approximation [3].

### PID TUNING

PID tuning comprises the selection of best value of  $K_p$ ,  $T_i$  and  $T_d$  of the PID controller so that the system performance can be increased. As we already discussed in introduction that there are various schemes involved in tuning the PID controller parameters. In this section we would form a simple and effective PID tuning method that can be applied to any stable first and second order systems. The tuning basically based on the logic that if we can be able to make the closed loop transfer function of the system in such a way that the poles of the open loop TF of the system exactly cancelled by the zeroes of the PID controller. Then we can make the closed loop transfer function (CLTF) of first order and then there should be no overshoot. Rise time and settling time can be tuned by introducing an extra parameter ' $\alpha$ ' in cascade with the input of controller as shown in figure 2. Let the second order system can be represented by:

$$G(s) = \frac{K}{s^2 + as + b} \quad (5)$$

The transfer function of the PID controller is given by:

$$C(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) e(s) \quad (6)$$

So the open loop TF (OLTF) of the system with PID is  $G(s)C(s)$ . Now if we tune the parameters  $K_p$ ,  $T_i$  and  $T_d$  in such a way so that the poles of the system TF (eq. 5) can be cancelled by the zeroes of the PID parameter than the closed loop TF (CLTF) becomes of first order type. By comparing the coefficients of the numerator of PID controller TF with the denominator of the system TF we can find that the value of PID parameters that make the CLTF of first order type are:

$$\begin{bmatrix} K_p \\ T_d \\ T_i \end{bmatrix} = \begin{bmatrix} a/K \\ 1/a \\ a/b \end{bmatrix} \quad (7)$$

When these values are put in the PID parameters then the CLTF becomes first order type. In order to have some control over rise time and settling time we introduced another parameter ' $\alpha$ ' at the input of controller as shown in the figure 2. This way the CLTF become in the form of:

$$\frac{Y_{o/p}}{R_{ef}} = \frac{\alpha}{s + \alpha} \quad (8)$$

Finally, we have a control over the rise time and settling time with the parameter alpha  $\alpha$ . The Rise time is given by  $T_r = 2.2/\alpha$  and the settling time is given by  $T_{ss} = 4/\alpha$  in case of first order system.

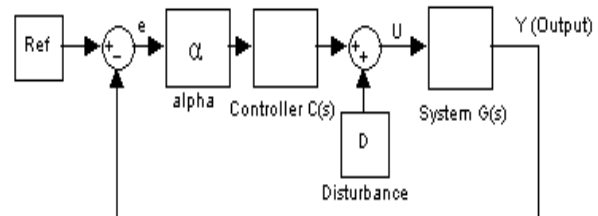


Figure 2: Block Diagram of the Closed Loop system with PID and parameter ' $\alpha$ '.

### CASE STUDIES

The effectiveness of the proposed tuning method is demonstrated on speed control of the dc motor [4]. The block diagram of the DC motor is shown in figure 3:

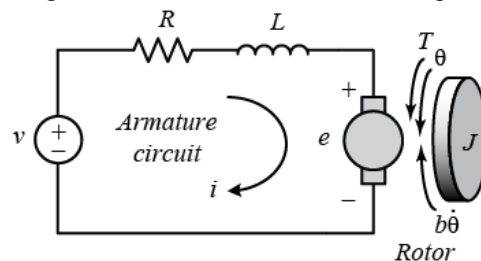


Figure 3: DC Motor

The motor transfer function was calculated as

$$\frac{\text{speed}(\dot{\theta})}{\text{Voltage}(V)} = \frac{K}{JLs^2 + (JR + Lb)s + bR + K^2} \quad (9)$$

where,

- Moment of Inertia of rotor (J) = 0.01 Kg m<sup>2</sup>/s<sup>2</sup>
- Damping ratio of mechanical system (b) = 0.1 Nm/s
- Electromotive force constant (K) = 0.01 Nm/A
- Electric resistance (R) = 1  $\Omega$

Electric Induction (L) = 0.5 H

Output velocity ( $\dot{\theta}$ ) = m/s

So, TF of the DC Motor Speed Control will become

$$G(s) = \frac{\dot{\theta}}{V} = \frac{2}{s^2 + 12s + 20.02} \quad (10)$$

where  $\theta$  is the position and its derivative is the speed of the DC Motor, V is the input voltage given to the motor. Eq. (10) gives the original calculated transfer function of the DC motor. When we apply auto tuned PID controller method to this transfer function than the discrete estimated TF identified by using system identification technique given in MATLAB with sampling time 0.01s is:

$$G(z) = \frac{\dot{\theta}}{V} = \frac{-6.882e^{-9} + 0.0001705z^{-1}}{1 - 1.896z^{-1} + 0.8977z^{-2}} \quad (11)$$

after converting the z domain TF to s domain, the resultant transfer function comes out to be:

$$G(s) = \frac{\dot{\theta}}{V} = \frac{1.8}{s^2 + 10.8s + 18} \quad (12)$$

So we obtain the values of  $K=1.8$ ,  $a=10.8$ ,  $b=18$ . When these values are put in eq. (7) then the PID parameters come out to be:

$$\begin{bmatrix} K_p \\ T_d \\ T_i \end{bmatrix} = \begin{bmatrix} 6 \\ 0.0926 \\ 0.6 \end{bmatrix} \quad (13)$$

Figure 4 shows the comparison of step response between the PID parameters calculated for original and estimated transfer function. Figure 5 shows the step response of the DC motor speed control with the value of PID parameters given by eq. (13). Figure 6 shows the effect of parameter  $\alpha$  over the rise time and settling time.

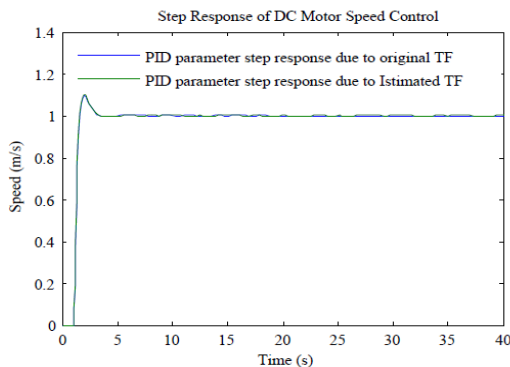


Figure 4: Comparison of step response between the PID parameters calculated for original and Estimated transfer function (TF).

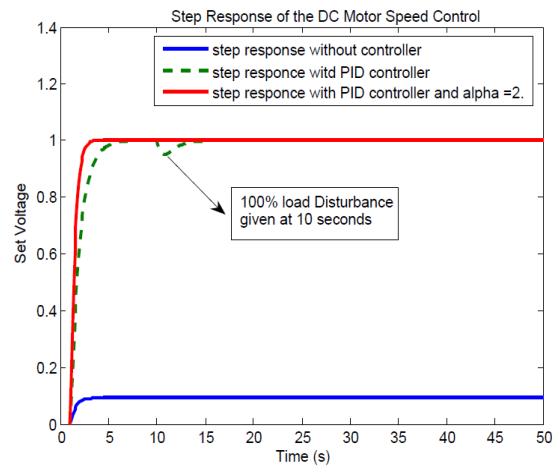


Figure 5: Step Response of the DC motor speed control.

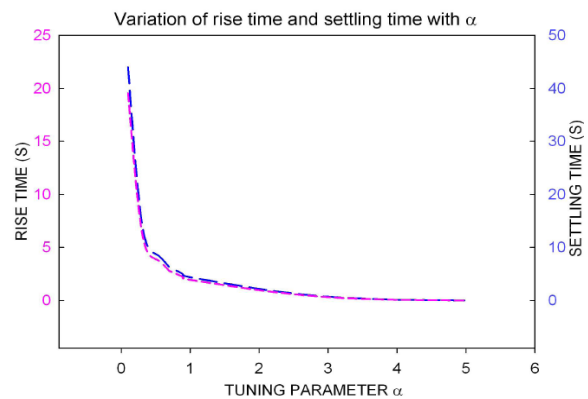


Figure 6: Effect of  $\alpha$  over the rise time and settling time.

## CONCLUSION

A PID auto tuning method has been formulated which directly estimates the PID parameters ( $K_p, T_i, T_d$ ). The proposed tuning method is fast and does not require complex algorithm to tune. An extra parameter  $\alpha$  was used in series with the controller in order to have flexible control over rise time and settling time without affecting the PID controller parameters.

## REFERENCES

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