LASER COOLING IN ELECTRON STORAGE RING AND ITS LIMITS

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Abstract

The evolution of synchrotron and betatron emittances of an electron beam under action of laser irradiation and consequent emission of hard quanta is analyzed. Dependencies of the cooling rate on structure functions at the irradiation point as well as on the parameters of the electron and laser beams are calculated and optimized. The invariance of the sum of the decrements is proved.

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INTRODUCTION

The possibility of generation of monochromatic X-ray radiation by backward scattering of laser light at a relativistic electron beam attracts now special attention [1, 2]. The frequency transformation easily follows from simple kinematic relations and has an order of magnitude of $4\gamma^2$, where γ is Lorentz factor. Besides, the scattered X-rays are well directed (angle $\sim 1/\gamma$)what is typical for radiation from high energy electrons. A weak point of the method is a rather small cross-section which imposes serious requirements upon the laser power and the beam density.

It is rather obvious that electron beams circulating in a storage ring are preferable from this point of view if their life-time is large enough. The latter depends on many factors including transverse spreading of the beam inherent in the method and caused by the recoil of emitted hard quanta. This effect is well known for cyclic accelerators in connection with quantum fluctuations of synchrotron radiation. However, in our case a quantum is essentially harder and requirements to the beam transverse size are more stressed.

Similarly, one could count on radiation cooling also inherent in the scattering process. Really, a radiation quantum should be re-emitted practically along the instantaneous velocity of a relativistic electron which gets both longitudinal and transverse recoil momentum. The first is restored by the RF compensating system while the second produces radiation "friction" exactly in the same way as in synchrotrons. Although one can not expect really strong damping for existing parameters the effect has to be considered because the spectral and angular distribution of the scattered light differs from that in synchrotrons and depends on parameters of the laser beam.

LASER COOLING

We neglect below intrinsic damping due to synchrotron radiation and consider electrons performing independent synchrotron and betatron oscillations:

$$y = x + R\psi u\,; \tag{1}$$

$$x = \sqrt{\frac{\varepsilon}{\pi}}\beta^{1/2}\cos\int\frac{\mathrm{d}s}{\beta}\,; \tag{2}$$

$$x' = \sqrt{\frac{\varepsilon}{\pi}}\beta^{-1/2}\left(\frac{\beta'}{2}\cos\int\frac{\mathrm{d}s}{\beta} - \sin\int\frac{\mathrm{d}s}{\beta}\right)\,;$$

$$u = \sqrt{\frac{\epsilon}{\pi}}\sqrt{\frac{\Omega}{q\alpha}}\cos\int\Omega\mathrm{d}s\,; \tag{3}$$

$$u' = -\sqrt{\frac{\epsilon}{\pi}}\sqrt{\frac{q\alpha}{\Omega}}\sin\int\Omega\mathrm{d}s. \tag{4}$$

Here u is a relative energy deviation from the equilibrium value, β and ψ are periodic structure functions of the magnetic system, R is the mean radius of the equilibrium orbit, primes denote derivatives with respect to the equilibrium orbit arc s, q is an RF field harmonic number, $\alpha = \overline{\psi}$, Ω is a synchrotron oscillations frequency in rotational frequency units. The values ε and ϵ have the meaning of area enclosed by phase trajectory ellipsis in phase planes (x, x') and (u, u') correspondingly being integrals of motion. For bounding phase trajectories they are identified as transverse and longitudinal emittances. Being expressed via phase plane coordinates they are equal to

$$\frac{\varepsilon}{\pi} = \frac{x^2}{\beta} + \frac{1}{\beta}\left[\frac{x\beta'}{2} - \beta x'\right]^2\,; \tag{5}$$

$$\frac{\epsilon}{\pi} = \frac{q\alpha}{\Omega}u^2 + \frac{\Omega}{q\alpha}{u'}^2\,. \tag{6}$$

Meeting a laser photon at the light and electron beams crossing point the electron energy is instantaneously changed by the value Δu keeping the coordinates y and u' and the instantaneous velocity y' constant (see Fig.1).The latter means that the scattered photon is emitted perfectly along the electron trajectory. With the same precision one can neglect the energy change when a relatively soft laser photon is absorbed. As a result, the integrals ε and ϵ experience instantaneous changes:

$$\frac{\Delta\varepsilon}{2\pi R\Delta u} = x\left[-\frac{\psi}{\beta} + \beta'\beta^{1/2}\left(\frac{\psi}{\beta^{1/2}}\right)'\right] - \tag{7}$$

$$- \beta^{3/2}x'\left(\frac{\psi}{\beta^{1/2}}\right)' + \ldots\,;$$

$$\frac{\Delta\epsilon}{2\pi R\Delta u} = \frac{q\alpha}{\Omega}u + \ldots\,. \tag{8}$$

Here and below the structure functions and their derivatives are taken at the crossing point. A destination of the second order changes will be considered later.

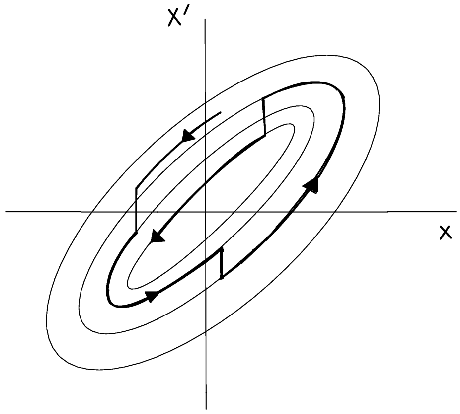

Figure 1:

To find the average variation rate $\mathrm{d}\varepsilon/\mathrm{d}s$ and $\mathrm{d}\epsilon/\mathrm{d}s$ the expressions above are to be multiplied by the scattering probability $P(\Delta u, y, y', u, u')$ and averaged over all betatron and synchrotron phases. To do this we present the probability as an expansion

$$P(\Delta u, y, y', u, u') = \\ = P_0 + \frac{\partial P}{\partial y} y + \frac{\partial P}{\partial y'} y' + \frac{\partial P}{\partial u} u + \frac{\partial P}{\partial u'} u' \,. \tag{9}$$

The first term here is the probability of "ideal" collision with the equilibrium electron, the second and the third ones describe coordinate and angular discrepancies between the electron and light beam axis. The last term correspond to a possible influence of lack of synchronization between electron and light pulses.

Note that after averaging over phases denoted below by the angular brackets all terms of the first order with respect to y, y', u, u' vanish. Besides, it follows from (1) that:

$$\begin{aligned} &\langle xy \rangle = \frac{\varepsilon}{2\pi}\beta; \quad \langle xy' \rangle = \frac{\varepsilon}{4\pi}\beta'; \\ &\langle xu \rangle = \langle xu' \rangle = 0\,; \quad \langle x'y \rangle = \frac{\varepsilon}{4\pi}\beta' \\ &\langle x'y' \rangle = \frac{\varepsilon}{2\pi\beta}\left(\beta'^2/4 + 1\right)\,; \\ &\langle x'u \rangle = \langle x'u' \rangle = \langle uu' \rangle = 0\,; \\ &\langle u^2 \rangle = \frac{\epsilon}{2\pi}\frac{\Omega}{q\alpha}\,; \quad \langle u'^2 \rangle = \frac{\epsilon}{2\pi}\frac{q\alpha}{\Omega}\,. \end{aligned} \tag{10}$$

So, we get:

$$\begin{aligned} &\left\langle \frac{\Delta\varepsilon}{\varepsilon} \right\rangle = \Delta u \left[-\psi\frac{\partial P}{\partial y} - \psi'\frac{\partial P}{\partial y'} \right] R\,; \\ &\left\langle \frac{\Delta\epsilon}{\epsilon} \right\rangle = \Delta u \left[R\psi\frac{\partial P}{\partial y} + R\psi'\frac{\partial P}{\partial y'} + \frac{\partial P}{\partial u} \right]. \end{aligned} \tag{11}$$

Besides re-emission process the transverse emittance is influenced by the RF field necessary for radiation losses compensation. We shall suppose it being concentrated within a narrow accelerating gap normal to the equilibrium orbit. A particle gets there an instantaneous increase in energy, keeping y and u' constant. There is a simultaneous change of y' because the accelerating field does not change the transverse momentum $p_{mathrmy}$. For this reason the change of the emittance has to be calculated under condition $p_{\mathrm{y}} =$ const, or[1].

$$\Delta y' = \Delta\frac{p_{\mathrm{y}}}{p} = -\frac{p_{\mathrm{y}}}{p}u = -y'u, \tag{12}$$

as far as in the ultrarelativistic case the relative changes of energy and of total momentum are equal to each other. Moreover, the probability of the gain is now identically equal to unity because the energy income does not depend on the possible scattering at previous turns.

It easy to see that after averaging over phases the condition (12) gives an additional change of the emittance

$$\Delta\varepsilon = -\varepsilon u\,; \quad \Delta\epsilon = 0 \tag{13}$$

which does not depend on structure functions at the point of compensation.

Noting that $-P\Delta u$ and Δu are equal to the relative energy W emitted per one turn we get the increments of betatron and synchrotron oscillations damping :

$$\Gamma_{\mathrm{b}} = -R\psi\frac{\partial W}{\partial y} - R\psi'\frac{\partial W}{\partial y'} + W\,; \tag{14}$$

$$\Gamma_{\mathrm{s}} = R\psi\frac{\partial W}{\partial y} + R\psi'\frac{\partial W}{\partial y'} + \frac{\partial W}{\partial u}\,. \tag{15}$$

They have much common with the usual radiation damping decrements but contain the local values of the structure functions. In particular the theorem about the decrements sum [3] looks as

$$\Gamma_{\mathrm{b}} + \Gamma_{\mathrm{s}} = \frac{\partial W}{\partial u} + W\,. \tag{16}$$

It says that the coordinate and angular discrepancies of laser and electron beams do non influence the total phase volume and yield a decrement re-distribution only. By the way, the term proportional to the coordinate shift between the beams vanishes unless the beams have a zero crossing angle.

Bearing in mind that the intensity of radiation of a relativistic particle is proportional to the square of its energy the relation (16) can be rewritten as

$$\Gamma_{\mathrm{b}} + \Gamma_{\mathrm{s}} = 3W\,. \tag{17}$$

The laser cooling as opposed to synchrotron radiation one depends on the laser power and thus on the final output of hard quanta. Even in certain ambitious projects [2] the damping time can not be less than a second. This hardly might provide a serious limitation of the emittance growth due to quantum fluctuations discussed below.

[1] We do not consider here the influence of the magnetic component of the RF field which creates no additional damping [3]

EXCITATION BY RECOIL MOMENTUM FLUCTUATIONS

A quantum nature of radiation is described by the next terms of expansion of ε and ϵ over powers of Δu. A single emission act gives

$$\Delta_2\varepsilon = \pi R^2 \left(\Delta u\right)^2 \left[\frac{\psi^2}{\beta} + \beta^2 \left(\frac{\psi}{\beta^{1/2}}\right)^{\prime 2}\right]; \quad (18)$$

$$\Delta_2\epsilon = \pi R \frac{q\alpha}{\Omega} \left(\Delta u\right)^2 . \quad (19)$$

In the limit of $\hbar \to 0$ the value

$$\langle \left(\Delta u\right)^2 P_0 \rangle = E_{\rm q} W$$

where $E_{\rm q}$ is a relative energy of the emitted quantum defined as

$$E_{\rm q} = \langle \left(\Delta u\right)^2 P_0 \rangle / \langle \Delta u P_0 \rangle .$$

So, in average, the scattering process results in the emittances increase rate:

$$\frac{{\rm d}\varepsilon}{{\rm d}s} = \pi R^2 E_{\rm q} W \left[\frac{\psi^2}{\beta} + \beta^2 \left(\frac{\psi}{\beta^{1/2}}\right)^{\prime 2}\right]; \quad (20)$$

$$\frac{{\rm d}\epsilon}{{\rm d}s} = \pi R \frac{q\alpha}{\Omega} E_{\rm q} W .$$

This rate has to be compared with damping due to laser cooling. Note that both are proportional to the laser power so that the final steady-state emittance has an universal value of order of

$$\varepsilon_{\rm st} \approx \frac{1}{\Gamma_{\rm b}} \frac{{\rm d}\varepsilon}{{\rm d}s} \approx \pi R^2 E_{\rm q} \left[\frac{\psi^2}{\beta} + \beta^2 \left(\frac{\psi}{\beta^{1/2}}\right)^{\prime 2}\right].$$

This value determines, of course, whether the stored electrons can be exploited for a long time or they would be burning down and require continual reinforcement.

SCATTERING CROSS SECTION AND PHOTON ENERGY

To use the relations obtained above one has to know the the cross section of the process as a function of the incident photon angle. The differential cross section for a solid angle do is [4]:

$$\frac{{\rm d}\sigma}{{\rm d}o} = 2 r_0^2 \frac{\left(\Delta u\right)^2}{\kappa_1^2} \times \quad (21)$$
$$\times \left[4\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right)^2 - 4\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right) - \left(\frac{\kappa_1}{\kappa_2} + \frac{\kappa_2}{\kappa_1}\right)\right]$$

where

$$\kappa_1 = -2\left(\Delta u\right)_{\rm i} \gamma \left(1 - \beta\cos\theta_{\rm i}\right); \quad (22)$$
$$\kappa_2 = 2\left(\Delta u\right) \gamma \left(1 - \beta\cos\theta\right). \quad (23)$$

Index i here and below marks values related to an incident photon, θ is an angle between the direction of the photon and the particle velocity. The energies of an incident and scattered photons are related by:–

$$\left(\Delta u\right) = \left(\Delta u\right)_{\rm i} \frac{1 - \beta\cos\theta_{\rm i}}{1 - \beta\cos\theta + \left(\Delta u\right)_{\rm i}\left(1 - \cos\mu\right)} \quad (24)$$

where μ is an angle between their directions:

$$\cos\mu = \sin\theta \sin\theta_{\rm i} \cos\nu - \cos\theta\cos\theta_{\rm i} \quad (25)$$

and ν is a polar angle in do.

For small angles θ the value $1 - \beta\cos\theta$ reaches its minimum of the order of $1/2\gamma^2$ then increases sharply starting from angles $\approx \gamma^{-1}$. So, for all angles of interest

$$1 - \beta\cos\theta \gg \left(\Delta u\right)_{\rm i}$$

if the particle energy $\gamma \ll mc^2/\hbar\omega_{\rm i}$. This condition means neglect of quantum Compton effect and is well fulfilled for all parameters of interest.

Following the standard procedure we get after some arithmetic in the relativistic limit, i.e. for $\cos\theta_{\rm i}$ differing markedly ($> \gamma^{-2}$) from unity:

$$\frac{W}{W_{\rm i}} = \frac{8\pi}{3} \frac{r_0^2}{BS} \gamma^2 \left(1 - \cos\theta_{\rm i}\right) \quad (26)$$

where S is the average cross section of the interaction region, $B > 1$ is a bunching factor and r_0 is the electron classical radius.

To complete the picture note that in the same limit the averaged frequency multiplication factor is:

$$\left\langle \frac{\omega}{\omega_{\rm i}} \right\rangle = \frac{7}{5} \gamma^2 \left(1 - \cos\theta_{\rm i}\right) . \quad (27)$$

It is lesser than the ideal value $4\gamma^2$ even for head-to-head collisions because of averaging over emission angles. Note that the relations (26) and (27) are not valid for collinear beams.

ON OPTIMIZATION OF THE STRUCTURE FUNCTIONS

It is easy to see that to make the emittance growth smaller[2] the value of

$$U = \frac{\psi^2}{\beta} + \beta^2 \left(\frac{\psi}{\beta^{1/2}}\right)^{\prime 2} \quad (28)$$

[2] Sometimes this is a square of transverse deviation what should be minimized

should be as small as possible while the mutual geometry of the electron and light beams can influence the decrements redistribution only (here and below β is again a structure function).

Note that the structure functions ψ and β are not independent as far as $\psi(s)$ is a periodic solution of

$$\psi'' + g(s)\psi = \frac{K(s)}{R}\,, \tag{29}$$

while the amplitude function β satisfies the nonlinear equation

$$\left(\beta^{1/2}\right)'' + g(s)\beta^{1/2} = \beta^{-3/2}. \tag{30}$$

with the same focussing function $g(s)$. Here $K(s)$ is the equilibrium orbit curvature.

Multiplying (29) by $\beta^{1/2}$ and (30) by ψ gives the general equation relating the structure functions

$$\beta\frac{\mathrm{d}}{\mathrm{d}s}\beta\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\psi}{\beta^{1/2}}\right) + \left(\frac{\psi}{\beta^{1/2}}\right) = \frac{K}{R}\beta^{3/2}. \tag{31}$$

Using this relation and differentiating U with respect to s to find an extremum we have:

$$U' = \left[\frac{\psi^2}{\beta} + \beta^2\left(\frac{\psi}{\beta^{1/2}}\right)'^2\right]' = 2\frac{K}{R}\beta^{3/2}\left(\frac{\psi}{\beta^{1/2}}\right)'. \tag{32}$$

Thus, U reaches its extremal values at the same points where $\psi/\beta^{1/2}$ does while

$$U_{\mathrm{ext}} = \left(\frac{\psi^2}{\beta}\right)_{\mathrm{ext}}. \tag{33}$$

Within a straight section U is a non-zero constant which can be expressed in terms of the positive β function. Really, considering $\phi = \int \mathrm{d}s/\beta$ as an independent variable in (32) we obtain

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\phi^2} + 1\right]\left(\frac{\psi}{\beta^{1/2}}\right) = \frac{K}{R}\beta^{3/2} \tag{34}$$

with the periodicity conditions in the interval $0 < \phi < 2\pi\nu$ where $\nu = (2\pi)^{-1}\int_0^{2\pi R}\mathrm{d}/\beta$ is the betatron oscillation frequency. The solution is straightforward:

$$\frac{\psi}{\beta^{1/2}} = \int_0^{\phi}\frac{K}{R}\beta^{3/2}\sin\left(\phi-\phi'\right)\mathrm{d}\phi' + \tag{35}$$
$$+\frac{1}{2\sin\pi\nu}\int_0^{2\pi\nu}\frac{K}{R}\beta^{3/2}\sin\left(\pi\nu+\phi-\phi'\right)\mathrm{d}\phi'.$$

At a point of an extremum where $\left(\psi/\beta^{3/2}\right)' = 0$ the function U reaches the value

$$U_{\mathrm{ext}} = \frac{1}{4\sin^2\pi\nu}\left[\left(\int_0^{2\pi\nu}\frac{K}{R}\beta^{3/2}\cos\phi'\mathrm{d}\phi'\right)^2 + \left(\int_0^{2\pi\nu}\frac{K}{R}\beta^{3/2}\sin\phi'\mathrm{d}\phi'\right)^2\right]. \tag{36}$$

So, one can say that the excitation of oscillations always takes place. To minimize it a defocusing (?) lense could be helpful at the crossing point as well as negative curvature portions of the equilibrium orbit with large values of β function.

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