ON VIOLATION OF THE ROBINSON'S DAMPING CRITERION AND ENHANCED COOLING OF PARTICLE BEAMS IN STORAGE RINGS

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Abstract

Limits of applicability of the Robinson's damping criterion and the problem of enhanced cooling of particle beams in storage rings beyond the criterion are discussed.

INTRODUCTION

Losses of energy by particles in external fields caused by friction forces lead to a decrease of the six-dimensional phase space (emittance) occupied by particle beams in storage rings and damping of a particle transverse and longitudinal oscillations. There is a correlation of damping decrements determined by Robinson's damping criterion [1]-[5]. This criterion limits the rate of particle cooling in storage rings. Below, we would like to pay attention on violation of this criterion in schemes of selective interaction of particles with targets. Schemes of enhanced cooling of ion, electron and muon beams are discussed. We start by reviewing Robinson's damping criterion.

LIMITS OF APPLICABILITY OF THE ROBINSON'S DAMPING CRITERION

The motion of a particle in storage rings is described by its deviations from the ideal orbit in transverse radial x, vertical y directions and a by deviation $\varphi = \psi - \psi_s$ of the particle phase from the synchronous one in a curvilinear coordinate system (x, y, φ). In a linear approximation deviations (x,y,φ) are described by linear, differential equations. We can introduce a six-dimensional coordinate vector $\vec{u}$ with components $(x, x', y, y', \varphi, \Delta\varepsilon)$ and write the equations in the form of a system of six linear, differential equations, where $x' = \partial x/\partial s$; $y' = \partial y/\partial s$; s, the longitudinal coordinate of a particle along the ideal (reference) orbit; $\Delta\varepsilon = \varepsilon - \varepsilon_s$, the deviation of the particle energy from synchronous one. In the matrix presentation:

$$\frac{d\vec{u}(s)}{ds} = Q(s)\vec{u}(s), \qquad (1)$$

where $Q(s) = ||q_{ji}(s)||$ is a six-order matrix with components $q_{ji}(s)$ (j,i = 1...6). The Eq. (1) has six linear independent solutions $\vec{u_j}(s)$ with components $u_{ji}(s)$ ($u_{j1} = x_j$, $u_{j2} = x'_j$, $u_{j3} = y_j$,... $u_{j6} = \Delta\varepsilon_j$,). The solution of the Eq. (1) has the form $\vec{u}(s) = M(s) \cdot \vec{u}(0)$, where $M(s) = ||m_{ij}(s)||$ is a transfer matrix, $\vec{u}(0)$ the initial vector. Six linear independent solutions $\vec{u_j}(s)$ compose the matrix $U(s) = ||u_{ji}(s)||$. The Wronskian of this matrix, $W(s) = |U(s)| = |Q(s)| \cdot |U(0)|$, represents the six-dimensional volume of the polyhedron in the phase space occupied by the beam. The values $dW(s)/ds = SpQ \cdot W(s)$, $W(s) = W(0) \cdot \exp(\int SpQ \cdot ds) \simeq W(0) \cdot \exp(< SpQ > \cdot s)$, where $SpQ = \sum_{j=1}^{6} q_{jj}$, $W(0)$ is the initial Wronskian, sign $<>$ denotes averaging. This is the Jacobean formula [2], [6]. On the other hand, $u_{ji}(s) \sim \exp(\alpha_i \cdot s)$ and the rate of change of the 6-dimensional volume of the polyhedron $\sim \exp[2\sum \alpha_i s]$, where $\alpha_i = \alpha_x, \alpha_y, \alpha_\varepsilon$ are fractional averaged damping decrements. Therefore

$$\alpha_{6D} = 2\sum_{i=1}^{3} \alpha_i = < SpQ > . \qquad (2)$$

SpQ is determined by the diagonal elements of the matrix $||q_{ji}(s)||$. In the transverse plane, the particle momentum loss does not lead to a change of the direction of the momentum and position of the particle ($q_{11} = q_{33} = q_{55} = 0$). Acceleration of the particle changes the direction of the momentum on the value $|\Delta\vec{p}|/|\vec{p}| = \overline{P} \cdot s/c \cdot \varepsilon_s$. It leads to matrix elements $q_{22} = q_{44} = -\overline{P}/c \cdot \varepsilon_s$, where $\overline{P}$ is the average rate of particle energy loss; ε, the particle energy; subscript s refers to the reference orbit; c, the velocity of light. The rate of change of the particle energy is $\partial\varepsilon/\partial t = -(\partial\overline{P}/\partial\varepsilon)|_s \cdot \Delta\varepsilon + (\partial P_{rf}/\partial\psi)|_s \cdot \varphi$ and matrix element $q_{66} = -(\partial\overline{P}/c \cdot \partial\varepsilon)|_s$. Substitution of diagonal matrix elements to (2) leads to generalized Robinson damping criterion [3], [4]:

$$\sum \alpha_i = \frac{1}{2}\alpha_{6D} = -\frac{1}{c}\frac{\overline{P_s}}{\varepsilon_s} - \frac{1}{2c}\frac{\partial\overline{P}}{\partial\varepsilon}|_s. \qquad (3)$$

The proof of the Robinson's damping criterion was reduced to application mathematical Jacobean formula. Nondiagonal matrix components responsible for the beam dynamics of particles in a lattice was not used. Two diagonal components responsible for damping in the transverse plane are determined by the average power of the particle energy loss and one diagonal component responsible for damping in the longitudinal plane is determined by the partial derivative of the power energy loss. The value α_{6D} in (2), (3) determine the rate of damping of the 6-dimensional phase space volume (emittance) occupied by the beam (cooling). Coefficients α_i and α_{6D} can be both positive and negative [2], [3].

If there is no coupling between radial x and vertical y planes in a storage ring, the same calculations can be performed separately for vertical and longitudinal damping coefficients:

$$\alpha_y = -\frac{1}{2c}\frac{\overline{P_s}}{\varepsilon_s}, \qquad \alpha_\varepsilon = -\frac{1}{2c}\frac{d\overline{P}}{d\varepsilon}|_s. \qquad (4)$$

The radial decrement follows from Eq. (3):

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$$\alpha_x = -\frac{1}{2c}\left[\frac{\overline{P_s}}{\varepsilon_s} + \frac{\partial \overline{P}}{\partial \varepsilon}|_s - \frac{d\overline{P}}{d\varepsilon}|_s\right]. \quad (5)$$

Damping times $\tau_{i,\,min} = -1/c\alpha_{i,\,max}$ are limited by Robinson's damping criterion (3) by the value

$$\tau_{i,min} = \frac{\varepsilon_s}{J_{i,max}\overline{P_s}}. \quad (6)$$

where $J_{i,max}$ is determined by the dependence $\overline{P}(\varepsilon)$ if all decrements are greater or equal zero. The value $J_{i,max} = 2$ if synchrotron radiation (SR) or backward Compton scattering are used ($\overline{P} \sim \varepsilon^2$) [7]. In the case of radiative ion cooling (backward Rayleigh scattering, $\overline{P} \sim D/(1+D)$, $D \sim \varepsilon$): $J_{i,max} = (3+2D)/2(1+D)$, where D is the saturation parameter [8]. For ionization muon cooling $(\partial\overline{P}/\partial\varepsilon)|_s$ is rapidly decreasing with the energy for $\varepsilon_\mu < 0.3$ GeV, but is slightly increasing for $\varepsilon_\mu < 0.3$ GeV ($J_{i,max} \sim 1$) [9]. The partial derivative $(\partial\overline{P}/\partial\varepsilon)|_s$ can be very high only in the case of laser cooling of ion beams by homogeneous broadband laser beam with rapidly increasing linear dependant spectral intensity in the frequency range corresponding to the ion energy spread $\sigma_{\varepsilon,0}$. In this case $\overline{P} \simeq \overline{P}_s(\varepsilon - \varepsilon_s + \sigma_{\varepsilon,0})/\sigma_{\varepsilon,0}$ $(0 < \varepsilon - \varepsilon_s < 2\sigma_{\varepsilon,0})$, $(\partial\overline{P}/\partial\varepsilon)|_s = \overline{P}_s/\sigma_{\varepsilon,0} \gg \overline{P}_s/\varepsilon_s$, $J_{i,max} \simeq \varepsilon_s/2\sigma_{\varepsilon,0}$,

$$\tau_{i,min} = \frac{2\sigma_{\varepsilon,0}}{\overline{P}_s}. \quad (7)$$

Next conditions were used to prove the Robinson's damping criterion: 1) cooling in the radio frequency (RF) bucket, 2) linear dependence $\overline{P}(\Delta\varepsilon)$, 3) stationary process. Violation of these conditions can lead to the violation of the criterion, the concept of decrement, non-exponential damping and fast cooling.

ENHANCED COOLING OF PARTICLE BEAMS IN STORAGE RINGS BEYOND THE ROBINSON'S DAMPING CRITERION.

Below three examples demonstrate fast laser cooling of ion beams beyond the Robinson's damping criterion. Internal ion selectivity and Rayleigh scattering are used.

1) Monochromatic laser beam target with scanning frequency is used when the RF system of the storage ring is switched off (RF buckets, linear dependence $\overline{P}(\Delta\varepsilon)$ and stationary conditions are absent) [10]. Ions interact with the counter-propagating laser beam at resonance energy, decrease their energy in the process of the laser frequency scanning until all of them reach the minimum energy of ions in the beam. At this frequency the laser beam is switched off. The higher the energy of ions, the earlier they start interacting with the laser beam, the longer the time of interaction. Ions of minimum energy do not interact with the laser beam at all. Cooling is available.

2) Ion and broadband laser beams interact in a straight section of a storage ring. The laser beam is homogeneous in limits of the ion-laser beam interaction region (IR) and has sharp frequency edges. The frequency band of the laser beam is sufficient for all ions to interact with the laser beam. The RF system of the storage ring is switched off (violations of conditions 1, 2). The minimum initial energy of ions corresponds to interaction with laser beam photons of high-frequency edge. Ions decrease their energy until all of them reach the minimum energy of ions in the beam. Cooling is available.

In these examples the energy spread of the ion beam is decreased by a linear (non-exponential) law to a small value limited by quantum processes of Rayleigh scattering. The damping time is determined by (7). Both non-exponential and exponential damping times determined by (7) are $\varepsilon/4\sigma_{\varepsilon,0} \sim 10^2 \div 10^3$ times shorter than damping time for radiative ion cooling if scattered powers are the same[1]. These methods work only longitudinally and can use both a dispersion and dispersion-free laser-ion IR. To cool the beam in the transverse plane the emittance exchange through a synchro-betatron resonance [11] or dispersion coupling by a wedge-shaped or by a moving broadband laser target [12], [13] can be used.

3) Ion and broadband laser beams interact in a straight section of a storage ring. The laser beam is homogeneous in limits of the ion beam and has sharp frequency edges. The RF system of the storage ring is switched on. The synchronous energy of ions corresponds to interaction with photons of high-frequency edge of the laser beam. The spectral intensity of the laser beam is linearly decreased from a maximum at the low-frequency edge to zero at the high-frequency edge. The power of the scattered radiation depends on the ion energy according to the law: $\overline{P} =< P_{max} > [(\varepsilon - \varepsilon_s)/\sigma_{\varepsilon,0}]$ at $\varepsilon_s < \varepsilon < \varepsilon_s + \sigma_{\varepsilon,0}$ and $\overline{P} = 0$ at $\varepsilon < \varepsilon_s$, where $\overline{P}_{max} = \overline{P}(\varepsilon - \varepsilon_s = \sigma_{\varepsilon,0})$. A discontinuity in the rate of energy loss was introduced: ions with an energy more than the synchronous energy interact with the laser beam and ions with less energy do not.

In this case, a synchronous ion does not lose energy. There is no friction and damping of synchrotron oscillations at the energy $\varepsilon < \varepsilon_s$ and there is damping at the energy $\varepsilon > \varepsilon_s$. The minimum damping time will be determined by (7) if we accept $\overline{P}_s = \overline{P}_{max}/2$.

Below a scheme for the emittance exchange and for three-dimensional cooling of muon beams is considered.

4). An homogeneous flat material target moves in the proper region of a storage ring to the position of a closed orbit of muons having a minimum energy, stop at this position and is extracted from the region for a short time $\Delta t \ll T_s$, where T_s is the revolution period of the muon. The RF system is switched off. A straight section with low-beta and high-dispersion function is used for efficient selection of closed orbits by the target. The higher the energy of muons, the earlier they start interaction with the target, the longer the time of interaction. Ions of minimum energy and

[1] Exponential damping leads to decrease of the beam dimension $e \simeq 2.7$ times for one damping time while non-exponential damping leads to much greater decrease and much faster cooling.

zero amplitude of betatron oscillation do not interact with the target at all (external selectivity of interaction).

In this method the energy spread is decreased by a non-exponential law to a value limited by a jump processes of the energy loss in the target and the initial spread of betatron oscillations. However friction and external selectivity of interaction of a moving target with a particle beam do not lead to cooling of the beam. In this case particles are deepened in the target to the depth larger then their amplitudes of betatron oscillations for many turns, interact with the target at the deviations from the closed orbit $x_{b\,0}$ of one sign and that is why receive the unwanted increase of amplitudes of betatron oscillations simultaneously with the decrease of the energy spread. This statement follow from the change of the amplitude of betatron oscillations

$$\delta A = -(x_{\beta\,0}/A)\delta x_\eta \tag{8}$$

which is valid in the approximation $|\delta x_\eta| \ll |x_{\beta\,0}| < \sigma_{x,0}$, where δx_η is the jump of the closed orbit; A, $\sigma_{x,0}$, the amplitude and betatron beam size [12], [13][2].

If the tune of muons $\nu_x \simeq m + 1/2$, the relative velocity

$$k_2 = \frac{|v_{T_2}|}{|\dot{x}_{\eta\,in}|} > 2 + \frac{\sigma_{x,0}}{|\delta x_\eta|}, \tag{9}$$

particles are deepened in the target to the depth larger then their amplitudes of betatron oscillations for one turn, interact with the target at deviations from their closed orbits of alternate signs and receive the unwanted increase of amplitudes of betatron oscillations only one time. Here $\dot{x}_{\eta\,in} = \delta x_\eta / T_s$, m is a whole number. Muons having zero initial amplitudes of betatron oscillations obtain amplitudes $A = |\delta x_\eta|$ after the first crossing the target and lose them after the second one (at this moment their position is displaced at a distance $2|\delta x_\eta|$ to a new position of their closed orbit). The initial phase space area occupied by muon beam in the transverse plane will be splitted by 2 composite parts consisting of many regions. Central muons in one part of these regions will be at rest (even crossing the target) and central muons in the regions of the other part will oscillate with the amplitude $|\delta x_\eta|$ (odd crossing the target). This scheme leads only to an emittance exchange.

We can use a sequence of even number of flat moving targets N_T. Targets have to locate in sequence at a distance determined by a 180^o phase advance for the lattice segment. Thicknesses of targets is N_T times less than in case of one target in the ring considered above. In this case the jump of closed orbits at the exit of the target and the corresponding degree of excitation of amplitudes of betatron oscillations will be 2 times less then for one target with N_T times larger thickness [13]. This scheme leads to emittance exchange as well.

[2]This interaction is similar to interaction of ions and monochromatic laser beams with scanning frequency (see above). However, in the last case laser beams overlap being cooled ion beams, interaction occur at the deviations from their closed orbits of different signs and that is why dos not lead to excitation of betatron oscillations.

If one wedge-shaped moving target is used, its thickness is decreased in the direction of the target velocity, the velocity of the target is greater then $\dot{x}_{\eta,\,in}$ at the position of the closed orbit of the particle, the target overlap the muon beam the emittance exchange for motionless target and cooling for moving target will occur. In this case the rate of compression of the ion beam in the longitudinal plane depends on the slope of the target but the rate of antidamping in the transverce plane depends both on the slope and the velocity of the target. Muon can cross the target at positive (in the direction opposite to target velocity) and negative deviations from its closed orbit at different moments of time, thickness of the target at this pair moments and the position of closed orbit can be constant and the amplitude of muon betatron oscillations will not be changed.

CONCLUSION

In this paper we pointed out limits of applicability of the Robinson's damping criterion. New schemes of three-dimensional enhanced cooling of ion beams beyond the criterion where developed. Using external selectivity for emittance exchange and cooling was discussed.

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