POWER LOSS IN THE COAXIAL REGION OF A LHC-LIKE RING

S. Petracca, Th. Demma, University of Sannio, Benevento, Italy, and INFN
R.P. Croce, University of Salerno, Salerno, Italy

Abstract

We outline a general framework for computing the fields in a closed ring machine due to the coupling between the liner and the toroidal coaxial region through the pumping holes.The power dissipated per unit length into the walls of the coaxial region is accordingly readily computed.

INTRODUCTION

In closed ring machines both the beam liner and the region between the external wall of the liner and the internal wall of the surrounding vacuum chamber are toroidal electromagnetic resonators. These two regions are coupled through the pumping holes. Accordingly, in the steady regime, particle bunches circulating in the liner set up stationary fields in the coaxial region. These fields couple-back to produce stationary fields in the liner. In this paper we sketch the derivation of a first order analytic solution for these fields for the simplest case of a multibunch beam and circular pipes.

BUNCHED BEAM FIELD IN LINER

The charge density of a N_b equally spaced point-like bunches of equal charge Q circulating on axis along a ring-liner with circular cross section can be written:

$$\rho(\vec{r},t) = \delta(r)\delta(z - \beta ct \; mod \; (L/N_b)). \tag{1}$$

The corresponding field can be written:

$$\vec{e}_i(\vec{r},t) = \frac{N_b Q}{\pi\epsilon_0 L}\frac{\vec{u}_r}{r}\left\{\frac{1}{2} + \sum_{m=1}^{\infty} Re\left[e^{jmN_b(\omega_b t - k_b z)}\right]\right\}, \tag{2}$$

where

$$T_b = \frac{L}{\beta c}, \quad \omega_b = \frac{2\pi}{T_b}, \quad k_b = \frac{\omega_b}{\beta c}, \tag{3}$$

are the bunch circulation time, (angular) frequency and wavenumber, and we used the Fourier representation of the periodic δ-function.

Real world bunches can be better described by a gaussian charge distribution, viz.:

$$\rho(\vec{r},t) = \delta(r)\delta(z - \beta ct \; mod \; (L/N_b)) * f(z),$$

$$f(z) = (2\pi)^{-1/2}\sigma^{-1}\, e^{-z^2/2\sigma^2}, \tag{4}$$

where $*$ denotes convolution, σ is the r.m.s. bunch-length, and we assume that $L/N_b \gg \sigma$. Hence[1]

$$\vec{e}_i(\vec{r},t) = \frac{N_b Q}{\pi\epsilon_0 L}\frac{\vec{u}_r}{r}\cdot$$

$$\cdot\left\{\frac{1}{2} + \sum_{m=1}^{\infty} Re\left[F(mN_b k_b)e^{jmN_b(\omega_b t - k_b z)}\right]\right\}. \tag{5}$$

where:

$$F(k) = \pi^{1/2} e^{-\sigma^2 k^2/2} \tag{6}$$

is the Fourier transform of the gaussian distribution.

This field (5) is a superposition of time-harmonic forward (counterclockwise) propagating waves, at $\omega = mN_b\omega_b$, with complex (phasor) representation:

$$\vec{E}_{i,m} = \frac{F(mN_b k_b)}{1+\delta_{m0}}\frac{N_b Q}{\pi\epsilon_0 L}\frac{\vec{u}_r}{r}e^{-jmN_b k_b z}, \tag{7}$$

δ_{hk} being the Kronecker function. The corresponding magnetic field is:

$$\vec{H}_{i,m} = Y_0\frac{F(mN_b k_b)}{1+\delta_{m0}}\frac{N_b Q}{\pi\epsilon_0 L}\frac{\vec{u}_r}{r}e^{-jmN_b k_b z}. \tag{8}$$

FIELDS IN TOROIDAL RESONATORS

The time-harmonic fields in a toroidal cavity can be written [2] (phasor notation, $\exp(j\omega t)$ time factor dropped):

$$\begin{cases} \vec{E} = \sum_{q=1}^{\infty}\left[V_q\vec{\mathcal{E}}_q^{(e)} + \mathcal{V}_q\vec{\mathcal{E}}_q^{(h)}\right], \\ \vec{H} = \sum_{q=1}^{\infty}\left[I_q\vec{\mathcal{H}}_q^{(e)} - \mathcal{I}_q\vec{\mathcal{H}}_q^{(h)}\right], \end{cases} \tag{9}$$

where the (complex) constants V_q, $\mathcal{V}_q$, I_q and $\mathcal{I}_q$ having the dimensions of voltages and currents, respectively, are to be determined. The basis "fields" in (9)are solutions of

$$\begin{cases} \nabla\times\vec{\mathcal{E}}_n^{(e,h)} = k_n\vec{\mathcal{H}}_n^{(e,h)}, \\ \nabla\times\vec{\mathcal{H}}_n^{(e,h)} = k_n\vec{\mathcal{E}}_n^{(e,h)}, \end{cases} \tag{10}$$

where $k_n = \omega_n/c$, describe free-field oscillations with angular frequency ω_n.They satisfy the following conditions

$$\begin{cases} \vec{\mathcal{E}}_n^{(e)} = 0, \\ \vec{\mathcal{H}}_n^{(h)} = 0, \end{cases} \quad at\; z = 0, L, \tag{11}$$

[1] Equation follows taking the $z \rightarrow k$ Fourier transform of eq. (4), using Borel theorem, and then switching back to the $z-$domain

[2] For notational ease we use a single suffix to represent the modal indexes.

and earn the following orthogonality properties:

$$\left\{\begin{array}{l}\int_V \vec{\mathcal{E}}_n^{(i)}\cdot\vec{\mathcal{E}}_p^{(i)}\,dV=\int_V \vec{\mathcal{H}}_n^{(i)}\cdot\vec{\mathcal{H}}_p^{(i)}\,dV=\\ =\iint_{\partial V}\vec{\mathcal{H}}_n^{(i)}\cdot\vec{\mathcal{H}}_p^{(i)}\,dS=0,\ p\neq n,\ i=e,h\\ \\ \int_V \vec{\mathcal{E}}_n^{(e)}\cdot\vec{\mathcal{E}}_p^{(h)}\,dV=\int_V \vec{\mathcal{H}}_n^{(e)}\cdot\vec{\mathcal{H}}_p^{(h)}\,dV=\\ =\iint_{\partial V}\vec{\mathcal{H}}_n^{(e)}\cdot\vec{\mathcal{H}}_p^{(h)}\,dS=0,\quad\forall\,p,n.\end{array}\right.\tag{12}$$

Moreover, $\forall n$

$$\left\{\begin{array}{l}\int_V\vec{\mathcal{E}}_n^{(i)}\cdot\vec{\mathcal{E}}_n^{(i)}\,dV=\int_V\vec{\mathcal{H}}_n^{(i)}\cdot\vec{\mathcal{H}}_n^{(i)}\,dV=A,\\ \\ \iint_{\partial V}\vec{\mathcal{H}}_n^{(i)}\cdot\vec{\mathcal{H}}_n^{(i)}\,dS=B,\end{array}\right.\tag{13}$$

where V and ∂V are the ring volume and its (complete) boundary.

The unknown constants $V_q,\ \mathcal{V}_q,\ I_q,\ \mathcal{I}_q$ can be determined in terms of the source terms, represented here by Bethe equivalent electric and magnetic dipoles sitting at the holes connecting the beam pipe to the vacuum chamber, discussed in the next section. The fields (9) obey Maxwell equations:

$$\left\{\begin{array}{l}\nabla\times\vec{E}=-j\omega\mu_0\vec{H}-j\omega\mu_0\vec{M},\\ \nabla\times\vec{H}=j\omega\epsilon_0\vec{E}+j\omega\vec{P},\end{array}\right.\tag{14}$$

We dot-multiply the first equation in (14) by $\vec{\mathcal{H}}_n^{(i)}$, $(i=e,h)$, use the obvious vector identity, $\nabla\cdot(\vec{a}\times\vec{b})=\nabla\times\vec{a}\cdot\vec{b}-\vec{a}\cdot\nabla\times\vec{b}$, together with equations (10), and the Leontóvich boundary conditions:

$$\hat{n}\times\left[\hat{n}\times\vec{E}-Z_{wall}\vec{H}\right]_{\partial V}=0,\tag{15}$$

$\hat{n}$ being the outward unit vector normal to ∂V, and $Z_{wall}=(\omega\mu_0/2\sigma)^{1/2}$ the appropriate wall impedance. Hence $(i=e,h)$:

$$\begin{array}{l}\int_V(\nabla\times\vec{E})\cdot\vec{\mathcal{H}}_n^{(i)}\,dV=\\ =\int_{\partial V}Z_{wall}\,\vec{H}\cdot\vec{\mathcal{H}}_n^{(i)}dS+nk_0\int_V\vec{\mathcal{E}}_n^{(i)}\cdot\vec{E}\,dV=\\ =-j\omega\mu_0\int_V\vec{H}\cdot\vec{\mathcal{H}}_n^{(i)}dV-j\omega\mu_0\int_V\vec{M}\cdot\vec{\mathcal{H}}_n^{(i)}dV,\end{array}\tag{16}$$

whence, using eq.s (9), and (12), (13),

$$\left\{\begin{array}{l}(\kappa-j\alpha)I_n-jk_nY_0V_n=-\kappa i_n^{(e)},\\ (\kappa-j\alpha)\mathcal{I}_n+jk_nY_0\mathcal{V}_n=\kappa i_n^{(h)},\end{array}\right.\tag{17}$$

where:

$$i_n^{(e,h)}=\frac{1}{\pi L}\int_V\vec{M}\cdot\vec{\mathcal{H}}_n^{(e,h)}dV,\tag{18}$$

and:

$$\kappa=\omega/c,\quad\alpha=Z_{wall}Y_0\,\frac{B}{A}.\tag{19}$$

Similarly, we dot-multiply the second equation in (14) by $\vec{\mathcal{E}}_n^{(i)}$, and proceed as before to get $(i=e,h)$:

$$\begin{array}{l}\int_V(\nabla\times\vec{H})\cdot\vec{\mathcal{E}}_n^{(i)}\,dV=k_n\int_V\vec{\mathcal{H}}_n^{(i)}\cdot\vec{H}\,dV=\\ =j\omega\epsilon_0\int_V\vec{E}\cdot\vec{\mathcal{E}}_n^{(i)}\,dV+j\omega\int_V\vec{P}\cdot\vec{\mathcal{E}}_n^{(i)}\,dV,\end{array}\tag{20}$$

whence,

$$\left\{\begin{array}{l}-jk_nZ_0I_n=\kappa V_n+\kappa v_n^{(e)},\\ jk_nZ_0\mathcal{I}_n=\kappa\mathcal{V}_n+\kappa v_n^{(h)},\end{array}\right.\tag{21}$$

where:

$$v_n^{(e,h)}=\frac{Z_0c}{\pi L}\int_V\vec{P}\cdot\vec{\mathcal{E}}_n^{(e,h)}dV.\tag{22}$$

From (17) and (21) we finally get:

$$\left\{\begin{array}{l}I_n=-\dfrac{\kappa^2 i_n^{(e)}+jk_nY_0\kappa v_n^{(e)}}{\kappa^2-j\alpha\kappa-k_n^2},\\ \\ V_n=\dfrac{jk_n\kappa Z_0 i_n^{(e)}-(\kappa^2-j\alpha\kappa)v_n^{(e)}}{\kappa^2-j\alpha\kappa-k_n^2},\\ \\ \mathcal{I}_n=\dfrac{\kappa^2 i_n^{(h)}+jk_nY_0\kappa v_n^{(h)}}{\kappa^2-j\alpha\kappa-k_n^2},\\ \\ \mathcal{V}_n=\dfrac{jk_n\kappa Z_0 i_n^{(h)}-(\kappa^2-j\alpha\kappa)v_n^{(h)}}{\kappa^2-j\alpha\kappa-k_n^2}.\end{array}\right.\tag{23}$$

THE HOLE COUPLING

Each of the time harmonic terms in (5) with phasor representation (7) will generate a TEM field in the coaxial region. In the frame of Bethe's approximation [1], the electromagnetic coupling between the beam field in the liner and the coaxial region through the pumping holes (assumed identical) can be described by the (phasor) source terms

$$\begin{array}{l}\vec{P}_m=\epsilon_0\alpha_e\sum_p\delta(\vec{r}-\vec{r}_p)\hat{u}_r\hat{u}_r\cdot\vec{E}_{i,m},\\ \\ \vec{M}_m=\alpha_m\sum_p\delta(\vec{r}-\vec{r}_p)(\bar{\bar{I}}-\hat{u}_r\hat{u}_r)\cdot\vec{H}_{i,m},\end{array}\tag{24}$$

where α_e and α_m are the external hole electric and magnetic polarizabilities [1],$\vec{E}_{i,m},\ \vec{H}_{i,m}$ are given by (7), (8),and $\{\vec{r}_p\}$ are the holes positions, viz.:

$$\vec{r}_p\equiv\left\{a,s\frac{L}{N_s},l\frac{2\pi}{N_l}\right\}\tag{25}$$

$\frac{L}{N_s}$ and $\frac{2\pi}{N_l}$ being the longitudinal and azimuthal inter hole spacings, N_l, N_s the circumferential number of holes (number of holes per section) and the longitudinal number of holes, respectively.

STATIONARY FIELDS IN THE COAXIAL REGION

Under usual conditions, the spectral content of the circulating bunch current is well below the lowest, higher order TE and TM cutoff frequency of the coaxial region. Therefore the basis fields in (10) assume the form:

$$\begin{cases} \vec{\mathcal{E}}_n^{(h)} = \frac{\vec{u}_r}{r}\,cos(nk_0 z), \quad \vec{\mathcal{H}}_n^{(h)} = -\frac{\vec{u}_\phi}{r}\,sin(nk_0 z), \\ \vec{\mathcal{E}}_n^{(e)} = \frac{\vec{u}_r}{r}\,sin(nk_0 z), \quad \vec{\mathcal{H}}_n^{(e)} = \frac{\vec{u}_\phi}{r}\,cos(nk_0 z), \end{cases} \tag{26}$$

where $k_0 = \omega_0/c$, $\omega_0 = 2\pi/T$ and $T = L/c$ is the light round-trip time. The constants A and B in (13) are $A = \pi L\;log(b/a)$ and $B = \pi L\;\frac{a+b}{ab}$, where a and b are respectively the (external) liner and the (internal) vacuum chamber radii. Substituting (24), (26), (7) and (8) in (18) and (22), we obtain:

$$v_{m,n}^{(e)} = \frac{Z_0 c}{\pi L}\int_V \vec{P}_m \cdot \vec{\mathcal{E}}_n^{(e)} dV =$$

$$= -j\,\frac{N_l N_s N_b Q \alpha_e}{4\pi^2 L^2 a^2 \epsilon_0}\,\frac{F(mN_b k_b)}{1+\delta_{m0}}\;\delta_{n,mNb}, \tag{27}$$

$$v_{m,n}^{(h)} = \frac{Z_0 c}{\pi L}\int_V \vec{P}_m \cdot \vec{\mathcal{E}}_n^{(h)} dV =$$

$$= \frac{N_l N_s N_b Q \alpha_e}{4\pi^2 L^2 a^2 \epsilon_0}\,\frac{F(mN_b k_b)}{1+\delta_{m0}}\;\delta_{n,mNb}, \tag{28}$$

$$i_{m,n}^{(e)} = \frac{1}{\pi L}\int_V \vec{M}_m \cdot \vec{\mathcal{H}}_n^{(e)} dV =$$

$$= \frac{Y_0 N_l N_s N_b Q \alpha_m}{2\pi^2 L^2 a^2 \epsilon_0}\,\frac{F(mN_b k_b)}{1+\delta_{m0}}\;\delta_{n,mNb}. \tag{29}$$

$$i_{m,n}^{(h)} = \frac{1}{\pi L}\int_V \vec{M}_m \cdot \vec{\mathcal{H}}_n^{(h)} dV =$$

$$= j\,\frac{Y_0 N_l N_s N_b Q \alpha_m}{2\pi^2 L^2 a^2 \epsilon_0}\,\frac{F(mN_b k_b)}{1+\delta_{m0}}\;\delta_{n,mNb}. \tag{30}$$

Hence the field excited in the coaxial region through the pumping holes can be written as:

$$\begin{cases} \vec{E} = \sum_{m=1}^{\infty}\left[V_{mN_b}\vec{\mathcal{E}}_{mN_b}^{(e)} + \mathcal{V}_{mN_b}\vec{\mathcal{E}}_{mN_b}^{(h)}\right], \\ \vec{H} = \sum_{m=1}^{\infty}\left[I_{mN_b}\vec{\mathcal{H}}_{mN_b}^{(e)} - \mathcal{I}_{mN_b}\vec{\mathcal{H}}_{mN_b}^{(h)}\right], \end{cases} \tag{31}$$

where $V_{mN_b}, \mathcal{V}_{mN_b}, I_{mN_b}$ and $\mathcal{I}_{mN_b}$ are obtained using (27)-(30) in (23). Note by comparison (27)-(30) that in view of (6) only a few terms of the series (31) are significant.

PARASITIC LOSS

The power dissipated per unit length into the walls of the coaxial region between the external liner and the internal cold bore surfaces is given by:

$$W = \frac{1}{2L}\int_{\partial V} \vec{E}\times\vec{H}^*\cdot\hat{n}ds = \frac{1}{2L}\int_{\partial V} Z_{wall}\,\vec{H}\cdot\vec{H}^* ds, \tag{32}$$

where ∂V is the complete boundary of the coaxial region, and Z_{wall} is the pipe-wall complex characteristic impedance. Using (31) and (13) it is easily seen that:

$$W = \frac{\pi(a+b)}{2ab\;log(b/a)}\;\sum_{m=1}^{\infty} Z_{wall}\left(|I_{mN_b}|^2 + |\mathcal{I}_{mN_b}|^2\right). \tag{33}$$

In Figure 1 the power dissipated per unit length is showed as a function of σ/a for numerical values of the parameters appropriate for LHC.

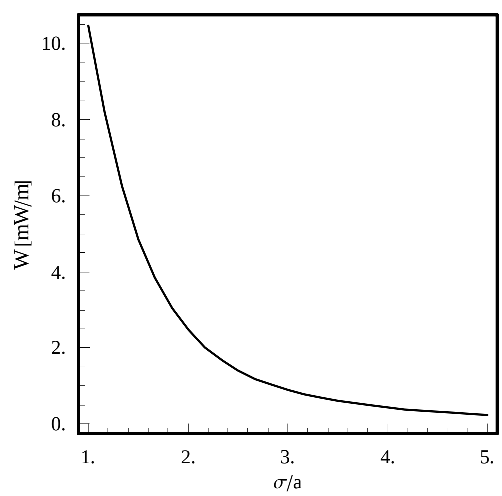

Figure 1: Power dissipated per unit length W versus σ/a. $L = 2.7\cdot 10^4 m$, $N_l = 10^7$, $N_s = 5$, $N_b = 2.8\cdot 10^3$, $a = 36.8mm$, $b = 48.3mm$, $\rho_{tube} = 5\cdot 10^{-10} Ohm\,m^{-1}$, $Q = 2.5664\cdot 10^{-27}C$, $\beta = 1$.

CONCLUSIONS

We outlined a general framework for computing the fields in a closed ring machine due to the coupling between the liner and the toroidal coaxial region through the pumping holes. The main relevant quantities of interest (peak field amplitudes, parasitic losses) can be accordingly readily computed. These are possibly more accurate than those obtained for the case of an infinite straight structure [2], or from impedance boundary conditions at the liner's wall [3].

REFERENCES

[1] R.E. Collin, *Field Theory of Guided Waves*, Wiley, 1983.

[2] S. de Santis et al., Phys. Rev. **E56**, 5990, 1997.

[3] S. Petracca, Phys. Rev. **E60**, 6030, 1999.