MEASURING INDUCTIVE COMPONENT OF LONGITUDINAL COUPLING IMPEDANCE IN IHEP PS USING GAMMA-TRANSITION JUMP

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Abstract

Matching process of intense proton bunches using the so-called gamma-transition jump method is considered. Longitudinal self-field of bunches is generalized to a case of the non-uniform vacuum chamber. Formulae allowing to calculate sign and value of reactive part of the longitudinal coupling impedance proceeding from the optimal position of the current pulse forming the transition jump with respect to a magnetic cycle of an accelerator are obtained. Based on the analysis of experimental data, sign and value of the reactive part of the longitudinal coupling impedance of the IHEP U-70 PS are estimated.

INTRODUCTION

The most effective way to avoid longitudinal matching of intense proton bunches at transition is to increase speed of crossing the dangerous area by beam. This way of longitudinal matching of proton bunches referred to as gamma-transition jump method, has been put forward in [1]; it allows, at use thin quadrupole lenses to perturb dispersion function, to shift transition gamma significantly while keeping betatron frequencies constant. First, this method was realized in CERN PS [2]; later the similar systems were constructed in U-70 PS (IHEP, Russia) [3] and also in AGS (BNL, USA) [4].

It is hardly possible to obtain with calculations a full picture of coupling impedance of a real vacuum chamber. In the present paper, the technique to measure an inductive component of the longitudinal coupling impedance of the accelerator with use of the γ-transiton jump system is described. The given technique is based on the results of phase equation integration from which it follows that the optimal position of γ-transition jump with respect to a magnetic cycle of the accelerator, at the given set papameters of a proton beam, is completely defined by the longitudinal coupling impedance in a long-wave approximation. With the help of the experimental data, the value and the sign of the longitudinal coupling impedance at transition in the U-70 synchrotron are found.

LONGITUDINAL ELECTRIC FIELD

It is convenient to express longitudinal electric field driven by beam in a surrounding equipment of an accelerator in terms of the longitudinal coupling impedance. In the simplest case of a smooth vacuum chamber homogeneous in longitudinal direction, longitudinal coupling impedance $Z_n(\omega)$ is defined by the equation identity:

$$-2\pi R_0 \mathcal{E}_n(\omega) = Z_n(\omega) J_n(\omega), \tag{1}$$

linear due to linearity of the Maxwell's equations. In (1), the following notations are used: R_0 is average radius of accelerator; $J_n(\omega)$ is amplitude n-th harmonic of the beam current at frequency ω; $\mathcal{E}_n(\omega)$ is the longitudinal electric field harmonic corresponding to the given beam current harmonic and averaged over distribution of particles in the beam cross-section. We imply that the dependence of harmonics upon time t and longitudinal coordinate s is given by factor $\exp(-i\omega t + ins/R_0)$.

Generally speaking, in case of a non-uniform vacuum chamber, the n-th harmonic of the longitudinal electric field is given by all the beam current harmonics. However, the systematic effect on the n-th beam current harmonic is imposed by the resonant wave of longitudinal electric field at frequency $\omega \simeq n\omega_0$ (ω_0 is revolution frequency of beam) whose phase velocity is approximately equal to the beam velocity. For this reason, in calculations of longitudinal electric self-field of bunches it is still possible to use a more simple definition of the coupling impedance (1).

By now, a longitudinal dynamics of particles near transition when the beam is shielded by a smooth ideally conducting vacuum chamber (longitudinal coupling impedance of which is actually a negative inductance) is well understood. It can attain a big value at lower energies. Still, near transition energy, its value usually does not exceed 10 Ohm. Nevertheless, its maximum effect on the beam occurs at transition energy due to strong dependence of longitudinal electric self-field on length of bunches which is minimal in this region.

It is, however, necessary to note that a real vacuum chamber is not-uniform — it contains numerous equipment longitudinal coupling impedance of which at lower frequencies is a positive inductance and, hence, has the opposite sign in comparison with a smooth chamber. So, for example, the longitudinal coupling impedance of a smooth vacuum chamber in U-70 is almost completely compensated due to that of pickup electrodes [5].

On taking longitudinal coupling impedance as $Z_n/n = \pm i \cdot const$, we shall take into account both variants — with positive and negative inductances. Multiplying both parts of the impedance definition (1) at $\exp(ins/R_0)$ and, then, summarizing over n, we shall get the following expression for the longitudinal electric field of bunches

$$\mathcal{E}(s,t) = -\mathrm{sgn}\left(\frac{Z_n}{n}\right) \frac{v}{2\pi} \left|\frac{Z_n}{n}\right| \frac{\partial \rho(s,t)}{\partial s}, \tag{2}$$

where $\rho = J/v$ is linear density of charge , v is beam velocity.

PHASE EQUATION

Phase equation accounting for a longitudinal electric field of bunches $\mathcal{E}$, is:

$$\frac{d\,\Delta p}{dt} = \frac{eV}{2\pi R_0}(\cos\varphi - \cos\varphi_s) + e\mathcal{E}(\varphi, t);$$
$$\frac{d\varphi}{dt} = \frac{\eta\omega_{RF}}{p_s}\Delta p, \quad (3)$$

where $\Delta p = p - p_s$ is momentum difference between the particle in question and a synchronous particle, e is charge of a particle, V is rf voltage amplitude, φ is phase of a particle about crest of the rf wave, φ_s is synchronous phase, ω_{RF} is rf frequency, phase slippage $\eta = \alpha - 1/\gamma^2$ (α is momentum compaction factor, γ is relativistic factor).

Phase oscillations of particles close to transition considered as small. Therefore, it is convenient to proceed in (3) to phase $\chi = \varphi - \varphi_s$, $|\chi| \ll 1$. We shall also assume that the transition value itself depends on time t, so the parameter η is described by:

$$\eta \simeq 2\frac{\dot{\gamma}_{tr}t - \Delta\gamma_{tr}(t)}{\gamma^3_{tr0}}, \quad (4)$$

where γ_{tr0} is the unperturbed transition value, $\dot{\gamma}_{tr}$ is the transition crossing speed in the absence of $\Delta\gamma_{tr}$ jump, time $t = 0$ corresponds to $\gamma = \gamma_{tr0}$.

As the major contribution to longitudinal mismatching of bunches at transition is given by the main (linear) part of electric field $\mathcal{E}$, it is convenient to proceed from a linear charge density depending on phase χ under the parabolic law. In such case we shall receive from (2) the final expression for the field $\mathcal{E}$:

$$\mathcal{E} = \mathrm{sgn}\left(\frac{Z_n}{n}\right)\frac{3qI_0}{2R_0}\left|\frac{Z_n}{n}\right|\frac{\chi}{\hat{\chi}^3}, \quad (5)$$

where q is the harmonic number, I_0 is dc current of beam containing q identical bunches, $\hat{\chi}$ is phase half-length of bunches.

On linearizing external accelerating field in the first equation of system (3), and also on taking into account the expressions (4), (5) and replacing variables $(\Delta p, t)$ by $(y, \tau = t/t_0)$,

$$t_0 = \left(\frac{\pi m_0\gamma^4_{tr0}R_0^2}{qeV|\sin\varphi_s|\dot{\gamma}_{tr}}\right)^{1/3}; \quad \Delta p = \frac{eV|\sin\varphi_s|t_0}{2\pi R_0}y, \quad (6)$$

where m_0 is the proton rest energy, we shall transform phase equation to the following form:

$$\frac{d}{d\tau}\frac{1}{\tau - f(\tau)}\frac{d\chi}{d\tau} = \left(\pm 1 + \frac{\kappa}{\hat{\chi}^3}\right)\chi. \quad (7)$$

Here the signs "plus" and "minus" accordingly stand for beam energies below and above transition, the $\Delta\gamma_{tr}$-jump is described by the function $f(\tau) = \Delta\gamma_{tr}(\tau)/(d\gamma_{tr}/d\tau)$, the parameter κ is proportional to beam dc current I_0 and longitudinal coupling impedance:

$$\kappa = -\mathrm{sgn}\left(\frac{Z_n}{n}\right)\frac{3\pi qI_0}{V|\sin\varphi_s|}\left|\frac{Z_n}{n}\right|. \quad (8)$$

Thus, $\kappa > 0$ in case of positive inductance.

LONGITUDINAL MATCHING OF BEAM AT TRANSITION

Transition crossing by beam in U-70 is extremely quick in comparison to period of phase oscillations. So, as a first approximation, it is possible to assume that transition crossing by beam at presence of the γ_{tr}-jump occurs infinitely quick. In that case, as following from fig. 1 at the moment $\tau_3 = \tau_2$, changing of parameter $\tau - f(\tau)$ in equation (7) is adiabatic everywhere except for a point $\tau = \tau_2$ where a discontinuous jump of function f takes place. We get from (7) the following equations for the boundary phase trajectories:

$$y_{1,2}(\chi) = \pm\left[\frac{1}{\tau_2 - f(\tau_2)}\left(1 \pm \frac{\kappa}{\hat{\chi}^3_{tr}}\right)(\hat{\chi}^2_{tr} - \chi^2)\right]^{1/2}, \quad (9)$$

where $\hat{\chi}_{tr}$ is phase half-length of bunches at $\gamma = \gamma_{tr}$, indexes 1, 2 under y are accordingly related to beam directly prior to transition energy ($\tau_2 - f(\tau_2) = \Delta f - f_{max}$) and right after transition ($\tau_2 - f(\tau_2) = \Delta f$).

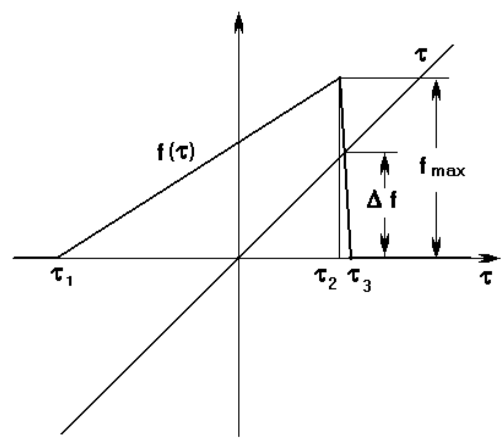

Figure 1: The scheme of the γ-transition jump.

To get longitudinal matching of bunches after transition, the following condition for peak values of $y_{1,2}$ should be fulfilled: $\hat{y}_1 = \hat{y}_2$. Therefrom, the necessary locking of current pulses forming the γ_{tr}-jump to the time moment corresponding to unperturbed γ-transition value can be found:

$$\tau_2 = \frac{f_{max}}{2}\left(1 - \frac{\kappa}{\hat{\chi}^3_{tr}}\right). \quad (10)$$

In such case parameter $\tau_2 = \Delta f$ can be measured in accelerator. Thus, it is possible to find the ratio $\Delta f/f_{max}$

and, accordingly, the value and sign of κ,

$$\kappa = \hat{\chi}_{tr}^3 \left(1 - 2\frac{\Delta f}{f_{max}}\right). \qquad (11)$$

Taking into account the definition (8) of κ one can see from (11) that the sign (Z_n/n) depends on the value of $\Delta f/f_{max}$: if $0 < \Delta f/f_{max} < 0.5$ then longitudinal coupling impedance is a positive inductance and it is negative one in the case of $0.5 < \Delta f/f_{max} < 1$.

Equation (11) could be transcendental if the phase half-length of bunches $\hat{\chi}_{tr}$ depended on κ. Fortunately, it is not the case. Using formulae (9) and (11) we get the following expression for $\hat{\chi}$ at transition:

$$\hat{\chi}_{tr}^2 = \frac{S}{\pi}\sqrt{\frac{f_{max}}{2}}, \qquad (12)$$

where $S = \oint y\, d\chi$ is phase volume of a bunch.

The process of transition crossing by the intense beam was also studied with the help of numerical integration of equation for phase oscillations amplitude

$$\frac{d}{d\tau}\frac{1}{\tau - f(\tau)}\frac{d\hat{\chi}}{d\tau} = \pm\hat{\chi} + \left(\frac{S}{\pi}\right)^2 \frac{\tau - f(\tau)}{\hat{\chi}^3} + \frac{\kappa}{\hat{\chi}^2} \qquad (13)$$

with function $f(\tau)$ approximately describing $\Delta\gamma$-transition jump in U-70. Results represented above, as one would expect because of small duration of the $\Delta\gamma$-transition roll-off part, are completely confirmed [5].

LONGITUDINAL COUPLING IMPEDANCE OF U-7O

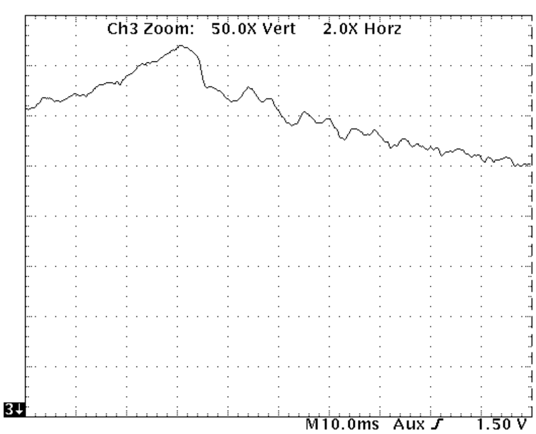

Figure 2: Oscillogram from U-70 peak detector signal ($\Delta\gamma_{tr}$-jump is on; sweep — 10 ms/dev).

Fig. 2 shows oscillogram of the peak detector signal; it corresponds to the case when longitudinal matching with the help of $\Delta\gamma_{tr}$-jump for five bunches with total intensity of $1.6 \cdot 10^{12}$ ppp has taken place. The U-70 magnetic field was $H_1 = 1404$ Oe at the moment when $\Delta\gamma_{tr}$-jump was switched on and the jump was switched off at the field $H_2 = 1576$ Oe. Magnetic field rate at transition is 4.44 Oe/ms ($\dot{\gamma}_{tr} = 26.7$ s^{-1}) so the time interval Δt during which the magnetic field of the accelerator is changed from H_0 up to the value H_2, where $H_0 = 1520$ Oe corresponds to unperturbed value of the transition energy ($\gamma = \gamma_{tr0}$), is equal $\Delta t = (H_2 - H_0)/\dot{H} = 12.6$ ms. On taking into account that during time Δt relativistic factor γ increases by $\Delta\gamma = 0.35$ and also the jump amplitude $(\Delta\gamma_{tr})_{max} = 0.9$, we have $\Delta f/f_{max} = \Delta\gamma/(\Delta\gamma_{tr})_{max} = 0.39$.

Since we have got $\Delta f/f_{max} < 0.5$, it is at once possible to make a conclusion on a sign of the inductive impedance of the U-70 vacuum chamber — it is positive, as one would expect, because $\kappa > 0$ according to (11).

For estimating the value of longitudinal coupling impedance, it is necessary first to calculate from formula (11) parameter κ, and, then, to calculate $|Z_n/n|$ from definition (8) of κ. Taking into account ratio (11) for the observed value $\Delta f/f_{max} = 0.39$ we have $\kappa/\hat{\chi}_{tr}^3 = 0.22$. Substituting $f_{max} = 8.7$ in formula (12) and average value of the phase volume which equals $S = 0.11$ according to measurements in the accelerator (in $(\Delta p, s)$ coordinates it equals 0.8 eV·s) we shall get $\hat{\chi}_{tr} = 0.27$ and $\kappa = 4.3 \cdot 10^{-3}$ accordingly.

Substituting then into (8) the obtained value of κ and also the U-70 parameters $q = 30$, $V|\sin\varphi_s| = 390$ kV, $I_0 = 0.3$ A corresponding to the data shown in fig. 2 we have the following result for the longitudinal coupling impedance — $|Z_n/n| \simeq 18$ Ohm. As carried out analysis has shown [5], the accuracy of the average S value measuring during the experiments was not worse than 10%. Therefore, the measurement error of $|Z_n/n|$ should be within the limits of $\pm$15%, as it is clear from the formulae (11) and (8). Thus the value of the U-70 longitudinal coupling impedance in a lower frequency range is estimated as 15 Ohm $< |Z_n/n| <$ 20 Ohm.

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