MATHEMATICAL MODELING OF THE ACCELERATION PROCESS IN RACE-TRACK MICROTRON

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Abstract

The precise calculations of beam dynamics are needed to make choice of optimal design parameters of race-track microtron. As a result, the necessary physical requirements to the accelerator systems become found. For calculation of the magnetic field, POISSON LANL code is used. Acceleration of the beam is investigated with the help of the program of MathCad.

Nonlinear distribution of the field in magnets of microtron with adjustable reverse field was simulated. The equation of motion of a beam in bending magnets of recirculation system are found and solved by a numerical method. Trajectories of the beam for all orbits in a microtron are received.

The recursive equation for calculation of the largest area of injected beam phase and power spreads providing steady acceleration process is written. The acceleration of the beam with maximal phase-energy area through all orbits of microtron was simulated.

The velocity of accelerated particles on first orbits differs from velocity of light. The minimal energy of injection provided their successful acceleration under this condition is determined.

INTRODUCTION

At an input and an output from each magnet of a race-track microtron the beam crosses strongly non-uniform fringing field which will defocus it in a vertical direction. To decrease this influence, additional poles [1] are provided in a microtron bending magnets. These poles are located along input gap, with the reverse to main magnetic field direction. In this case space distribution of fringing field also changes. Additional poles influence on focusing may be controlled by adjusting the reverse field.. The presence of the reverse field effects the distance between orbits and increases the nonlinearity in dependence of length of orbit on energy of the beam. This of nonlinearity results in reduction of a range of allowed input injection phases. That influences negatively on conditions of acceleration stability and restrict the lower limit of injection energy. Also, the difference between beam velocity and velocity of the light influences minimal allowed injection energy.

To illustrate this mathematical modeling, the 175 MeV CW race-track microtron project developed at the Institute of Nuclear Physics of Moscow State University [2] is taken as an example.

MATHEMATICAL MODELING

Study of process of acceleration and passage of beam through the orbits was carried out by modeling of trajectories of the beam by using MathCAD7 program [3]. The field in magnets was modeled with the help of the program POISSON, kindly granted by Los Alamos Accelerator Code Group LANL.

The motion of the electrons was considered in the system of coordinates shown in Fig. 1.

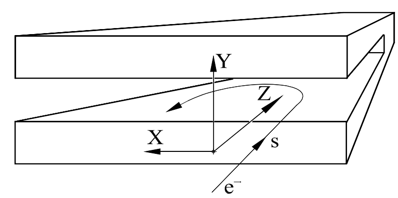

Fig. 1. System of coordinates

The two-dimensional distribution of the magnetic field and its first derivative was received by Poisson program taking into account the gap geometry (Fig. 2) Space distribution of the fringing field in the gap controlled by non-dimensional factor **k**.

It is assumed in all calculations: the coordinates dimension [x, y, z, s] = cm, magnetic induction dimension – –[B] = Gs, energy – [E] = MeV.

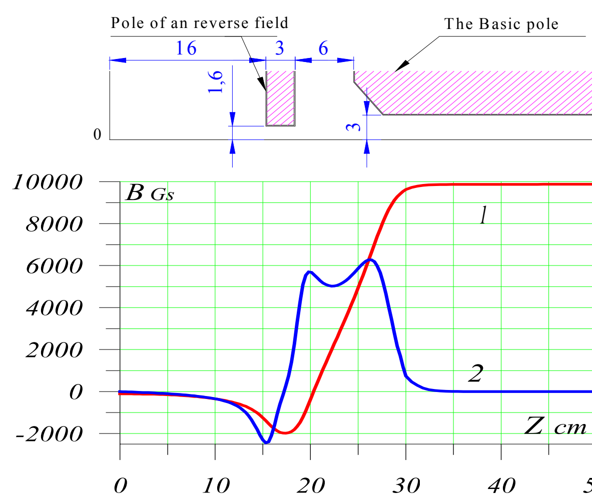

Fig.2. Magnetic field and its first derivative in fringing field of the gap area. On the top the gap geometry in the same scale is shown.

$$1 - B_y(z)_{y=0};\ 2 - \frac{\partial B_y(z)_{y=0}}{\partial z}\cdot 5;\ k = 1$$

The equations of motion of electrons in the gap of magnets restricted by the first derivatives of a magnetic field [4, 5], look like:

$$\begin{aligned}
&\frac{d^2x}{ds^2} = -D(E)\left[\frac{\partial B_y(z)_{y=0}}{\partial z}\,y\,\frac{dy}{ds} - B_y(z)_{y=0}\cdot\frac{dz}{ds}\right]\\
&\frac{d^2y}{ds^2} = D(E)\frac{\partial B_y(z)_{y=0}}{\partial z}\,y\,\frac{dx}{ds}\\
&\frac{d^2z}{ds^2} = -D(E)\cdot B_y(z)_{y=0}\cdot\frac{dx}{ds}\;;\quad D=\frac{e}{mv}\;;\quad v=\frac{ds}{dt}
\end{aligned} \qquad (1)$$

$$D(E) = \frac{2{,}9979\cdot 10^{-4}}{\sqrt{E^2 - E_0^2}}$$

In specified units: E [MeV] - electron full energy, E_0[MeV]-electron rest energy.

The orbits distribution in bending magnet was calculated by motion equations integration in median plane along a trajectory from an index point **s** = 0 at **z** = 0 up to final – **Smax**, which corresponds to magnet output, also at **z** = 0. Initial and final points of orbits are located at **z = 0**, where the fringing magnetic field is practically equals to zero. On Fig. 3 the first 5 orbits received by numerical integration are shown.

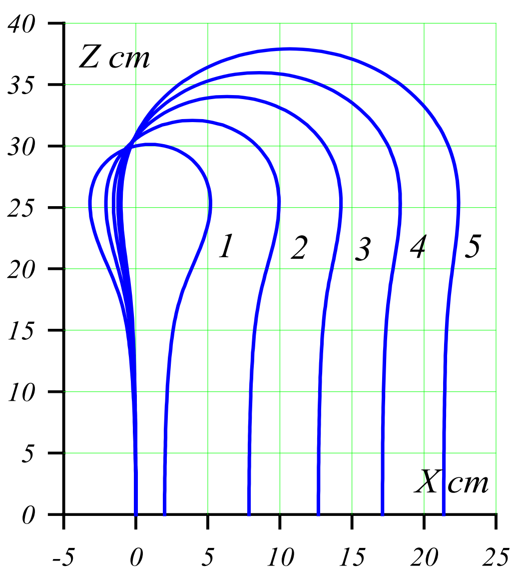

Fig. 3. Five initial equilibrium orbits. B=9808 Gs Es1=11.46 MeV, Es2=17.19 MeV, Es3 = 22.92 MeV, Es4 = 28.65 MeV, Es5 = 34.38 MeV.

During integration, dependence of length of orbit on energy – **Smax(E)** and its deviation from linear law is also determined.

The increasing of the orbit length results in phase shift of accelerated particles. That can cause the disturbance of synchronous acceleration. Infringement of synchronism occurs also because of insufficient electron relativity at initial stage. The biggest phase shift occurs in the first orbits.

By means of special devices, beam was one directed into the common orbit, and it passes linear accelerator, two bending magnets and drift intervals, on one of them there is a linear accelerator. Having turned on 360^0, the beam comes back in magnetic field.

A recursive calculations method is reproduced acceleration process in the microtron.

Initial conditions for injected particles located on a boundary ellipse are set in the parametric form. As a parameter, the angular coordinate of j-th particle is used. The inclination angle of a boundary ellipse in respect to the phase coordinates was one found out by multiplication of ellipse vector by a matrix of turn on angle α [see equations (2)]. The ellipse semi-axes are set through its product **SEo** and their ratio – **Eθo**.

$$\begin{bmatrix}\delta Eo_j\\ \delta\theta o_j\end{bmatrix} = \begin{bmatrix}\cos(\alpha) & \sin(\alpha)\\ -\sin(\alpha) & \cos(\alpha)\end{bmatrix}\cdot\begin{bmatrix}\sqrt{SEo\cdot E\theta o}\cos\left(\tau_j\right)\\ \sqrt{\dfrac{SEo}{E\theta o}}\sin\left(\tau_j\right)\end{bmatrix};\ \tau_j = \frac{2\pi}{60}\cdot(j-1); \qquad (2)$$

Input Parameters of Accelerating System and Injected Beam:

Accelerating frequency............. **f** = 2449.76 · 10^6 Hz (*).
Wavelength of an accelerating voltage.... **λg** = 12.242 cm.
Amplitude of an accelerating field.......... **V** = 6.0 MeV/m.
Magnetic field ………………………… **In** = 9874 Гс (*).
Equilibrium energy gain .... **ΔEs** = 5.73 МэВ.
Length of straight
Drift space **Ld** = **(1000 + ΔL)** cm, **ΔL** =-8.5398 cm (*).
Equilibrium phase........... **θs** = **acos (ΔEs/V)** = 0.301 rad.
Injection energy of axial particles ...**Eo** = 19.231 МэВ (*).
The area of a boundary
injected ellipse ………...….…. **πSE0** = 0.072 МэВ · rad.
The ratio of power semi-axis
to phase one …………………… **Eθo** = 0.7 MeV/rad (*).
The angle of turn of
the boundary ellipse ………………….. **α** = **-** 0.8 rad. (*).
Injected particle parameter ………………………….. **τ**.
Injected particle index .... **j** = 1...61.
Deviation of injected boundary particles energy
from axial particles energy..**δEo_j**.
Deviation of the injected boundary particle phase
from a phase of axial particles.................................. **$\delta\theta_j$**.
Phase of **j**-particle injection .... **θo_j**.
Energy of **j**-particle on **n**-orbit................................. **E_{nj}**.
Factor of a reverse field ………………...….**k** = 1.02 (*).
Number of orbits ……………………………… **n** = 26.
(*) - The values provided optimum mode of acceleration for all of orbits.

Passing the drift intervals and bending magnets, particles come back in an initial point. Thus they are displaced on a phase refer to an accelerating voltage not exactly on the angle 2π.

Particles phase shift while passing of the drift interval - **θdr(E)**:

$$\theta dr(E) = 2\pi f\,\frac{Ld}{C}\,\frac{E}{\sqrt{E^2 - Ee^2}} \qquad (3)$$

The length of a drift interval – **Ld = 1000 + ΔL**, **ΔL** – an adjustable part of a drift interval. C – velocity of light.

Gain of a phase of particles at passing of a acceleration interval **θac(E)**:

$$\theta ac(E) = 2\pi f \frac{Ld}{C \cdot \Delta Es} \int_{Es-\Delta Es}^{Es} \frac{E}{\sqrt{E^2 - Ee^2}} dE \quad (4)$$

In calculations, it is one assumed that particles are accelerated in average field and the gain of energy is equal to equilibrium one – **ΔEs**. The length of an accelerating gap is equal to a drift gap – **Ld**.

The particle phase gain at passing of two bending magnets **θ2m(E)**:

$$\theta 2m(E) = 4\pi f \frac{L\max(E)}{C} \frac{E}{\sqrt{E^2 - Ee^2}} \quad (5)$$

Recursive Equations of Changing of Energy and Phase at the Acceleration Process.

Energy **j**-th particle on the first orbit – $\mathbf{E_{1,j}}$ is defined by axial particle energy at injection **Eo**, derivation of **j**-th particle from axial $\mathbf{\delta Eo_j}$ and a gain of energy $\mathbf{\Delta U_{1,j}}$ after first passing of accelerating gap.

The injected particles phase $\mathbf{\theta o_j}$ is summarized of initial phase deviation, **θo** and a deviation of a phase of **j**-th particle from a phase of an axial particle $\mathbf{\delta\theta o_j}$.

After passing of accelerating section **j-th** particle on the **n**-th orbit increase its energy $\mathbf{\Delta U_{n,j}}$ and owing to passage of an accelerating site, a drift site and two rotary magnets, comes back in an initial point of injection with changed phase $\mathbf{\theta_{n,j}}$. Then the process repeats.

$$\begin{aligned}
&E_{1,j} = Eo + V\cdot\cos(\theta o) + \delta Eo_j; \quad \theta o_j = \theta o + \delta\theta o_j; \\
&\Theta_{1,j} = \theta 2m(E_{1,j}) + \theta d(E_{1,j}) + \theta ac(E_{1,j}) \\
&E_{n+1,j} = E_{n,j} + V\cos\left(\theta o_j + \left[\sum_{n=1}^{n} \theta 2m(E_{n,j}) + \theta dr(E_{n,j}) + \theta ac(E_{n,j})\right]\right) \\
&\theta_{n,j} = \theta o_j + \left[\sum_{n=1}^{n} \theta 2m(E_{n,j}) + \theta dr(E_{n,j}) + \theta ac(E_{n,j})\right] - \Theta_{1,j} \\
&\Delta U_{n,j} = V\cos\left(\theta o_j + \left[\sum_{n=1}^{n} \theta 2m(E_{n,j}) + \theta dr(E_{n,j}) + \theta ac(E_{n,j})\right] - \Theta_{1,j}\right) \\
&\delta E_{n,j} = E_{n,j} - Es0_{n,j} \qquad \delta\theta_{n,j} = \theta_{n,j} - \theta s0_{n,j}
\end{aligned} \quad (6)$$

$\mathbf{Es0_{n,j}}$ – energy of an axial particle in an orbit **n** at **SEo** = 0.

$\mathbf{\theta s0_{nj}}$ – a phase of an axial particle in an orbit **n at** **SEo** = 0.

The equations (6) allow to investigate the behavior of the beam injected in a race-track microtron at all stages of acceleration. By changing entrance boundary ellipse parameters (the semi-axes product – **SEo**, their ratio – **Eθo**, a angle of inclination of ellipse – α), the characteristics of accelerating system (frequency of accelerating field, rate of acceleration, position of an equilibrium phase, magnetic field, energy of injection and an input phase) and observing of change of the shape of an ellipse at passing of the beam on all orbits, it is possible to determine the greatest area of capture and optimum conditions of acceleration and injection.

Varying the parameter **n** one can monitor the acceleration process in each orbit. Changing parameter **j** it is possible to monitor each particle from a boundary ellipse and to determine area from which particles drop out of a mode of acceleration under adverse conditions. At **SEo** = **0** acceleration of axial particles is under investigation.

For a race-track microtron initial speed of particles imposes restriction on the minimal energy of injection.

In a described variant:

The minimal energy of injection providing reasonable capture - **Eo** = 19.23 МэВ.

The greatest area of a boundary ellipse at injection which is accelerated lost-free equals to – **πSEo** = 0.072MeV· rad.

The nearest minimal energy of injection – **Eo** = 13.10MeV.

In this case, the greatest area of a boundary ellipse of injection, in which the particle are accelerated – **πSEo** = $1.89 \cdot 10^{-4}$ MeV· rad.

One can see from the data obtained, that decreasing of injection energy down to the first possible value, leads to disappearing of the capture area practically complete.

Thus, the described calculations show that the race-track microtron with the specified parameters can work successfully. However, it is necessary to provide the fine tuning of a magnetic field, frequency of the accelerating generator and length of straight intervals.

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