SOLVING OF THE FIELD PROBLEM IN CASE OF CHARGED PARTICLE DYNAMICS OPTIMIZATION

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Abstract

Optimization problem of accelerated particle's dynamics is widely investigated if it reduces to minimization of some functional on the ordinary differential equations trajectories. Usually field problem is not solved and control functions are estimated if certain physical limitations are taken into account. In this report more general problem of trajectory optimization is considered when optimal controls are expressed in physical parameters of system realizing these controls. That is field problem is solved simultaneously with trajectory optimization. Examples are given for electrostatic beam forming injection systems of ion linacs.

INTRODUCTION

At the present time methods of non-linear programming theory are widely applied under optimization of charged particle dynamics in accelerated structures. As controls such physical functions are used as electrostatic field strength, accelerating wave phase velocity, accelerating gap lengths and so on [1]. The optimisation problem of particle dynamics is well investigated when it reduces to minimization of some functional on the ordinary differential equation trajectories. In this case the field problem is not solved and control functions are estimated according to certain algorithm, which is determined by used method in the presence of physical limitations. For example, in [1] one considered task of the particle dynamics optimization in waveguide buncher with motion equations

$$\frac{du_m}{dz} = -\alpha(z)\sin\varphi_m$$

$$\frac{d\varphi_m}{dz} = 2\pi\left(\frac{1}{\beta(z)} - \frac{u_m}{\sqrt{u_m^2 - 1}}\right) \qquad (1)$$

and functional

$$P = a\sum_{m=1}^{N}\left(\frac{u_m(L)}{\overline{u}(L)} - 1\right)^2 + b\sum_{m=1}^{N}(\varphi_m(L) - \overline{\varphi})^2 \qquad (2)$$

where $m = 1,2,...,N$; N – number of particles; z – longitudinal coordinate; L – buncher length in unit of working wave length λ; u_m, φ_m - particle's energy in unit of the rest energy and particle's phase accordingly; $\alpha = eE\lambda/mc^2$ is control function. It is in proportion

amplitude of the electrical field strength E. Both α and its derivative have physical limitations

$$\alpha_{min} \le \alpha \le \alpha_{max}$$

$$\left|\frac{d\alpha}{dz}\right| \le \alpha'_{max} \qquad (3)$$

Physical realization of the control $\alpha(z)$ is not considered and is separate technical problem. In reality distribution of the $E(z)$ may be determined by solving of Maxwell equations under suitable boundary conditions in waveguide. Geometrical sizes of the waveguide buncher are determined by requirements of $E(z)$ realization. In much the same way (as task (1), (2), (3)) optimization problems of radial three-dimensional particle dynamics may be solved. However, the problem becomes more complicated if simultaneously under dynamics optimization one needs to determine constructive parameters of the physical system, realizing optimal control functions. In this case it is necessary to solve not only the ordinary differential equations system, describing the charged particle beam dynamics, but also partial differential equations, describing the electrostatic fields whereupon those and others to solve many times.

ELECTROSTATIC FIELD PROBLEM UNDER BEAM DYNAMICS OPTIMIZATION

An attempt of optimization of the particle trajectories in electrostatic fields, taking into account the electrode system, creating these fields at the same time, may be considered as the first step in solving the complicated problem in point. Example of such electrostatic system may be accelerating-focusing system of the *rf* ion linac injector [2]. Usually it is named LEBT. Constructive parameters of LEBT system may be controls. Their alterations change LEBT construction and therefore electrostatic field, which determines beam dynamics. In this way modelling complex, optimizing of the beam forming and beam transport, must include a few modules of programs. There are modules of *2D* and *3D* – simulation of electrostatic fields, module of particle dynamics simulation, functional calculation and choice (or changing) control parameter module. (optimization module) It is useful to have a few different algorithms of the control changing's determination. They may conform to local or non-local search. Scheme of the program complex may look like on Fig. 1.
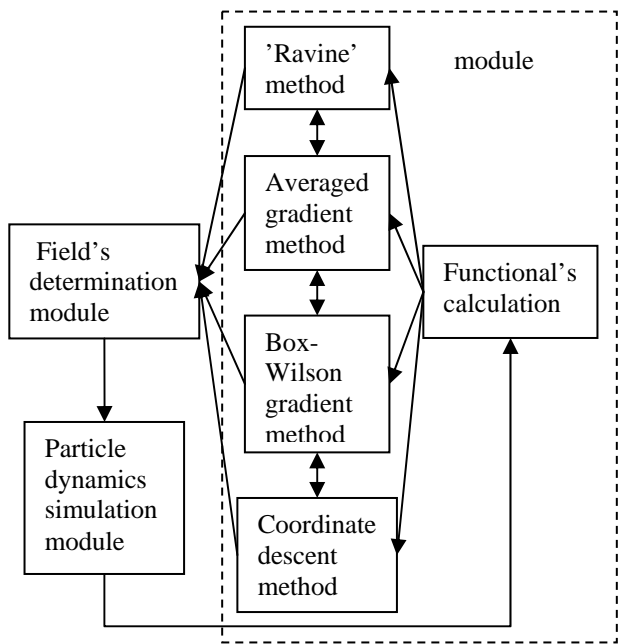
Figure 1: The scheme of the program complex

For example, LEBT system consists of electrodes, which look like washers of different thickness (see Fig. 2). Physical and geometrical parameters of LEBT may be considered as controls under optimization. They are electrode potentials, inner radii, thicknesses and cross-sections of electrodes, electrode-to-electrode distances.
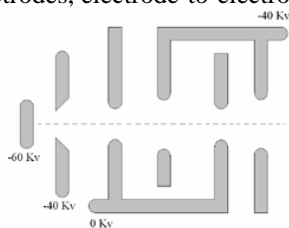

Figure 2: Scheme of the four-electrode LEBT system

have many control parameters. However, these parameters will have different influences on the particle dynamics. As example, one may consider particle dynamics in LEBT with four electrodes of thick washer's type, where potentials and distances between electrodes are controls (see Fig. 3). This is the most economical scheme of LEBT system. Proton source produces beam with $20\,keV$ energy and normalized emittance $10^{-7}\,rad \cdot m$. Output energy of the beam is $W_{out} = 60\,keV$. Pre-calculations establish that changes of both u_2 and u_3 potentials have maximal influences on the beam dynamics. Radii of electrodes, thicknesses of electrodes and distances between electrodes are determined by pre-calculations and later are fixed.
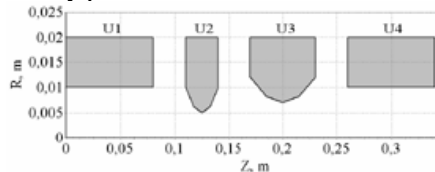

Figure 3: The geometry of four-electrode LEBT system

Quality of LEBT system is determined by K_{er} function

$$K_{er} = E_{out} / E_{in}$$

where E_{out} and E_{in} – values of output and input emittances of the beam accordingly. K_{er} was examined as function of two given parameters u_2 and u_3, and the optimization problem was the minimization of the function K_{er} under limitations $|K_{er}| \le 2$, $W_{out} = 60\,keV$ and $N_{bl} = 0.1 \cdot N_{b0}$. N_{bl}, N_{b0} - beam losses during transportation process and number of beam particles before transportation accordingly.
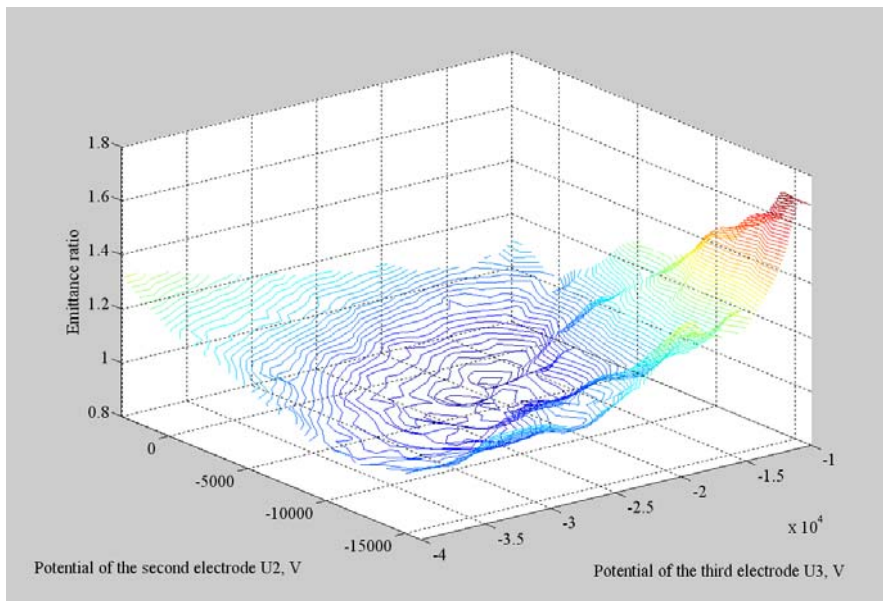

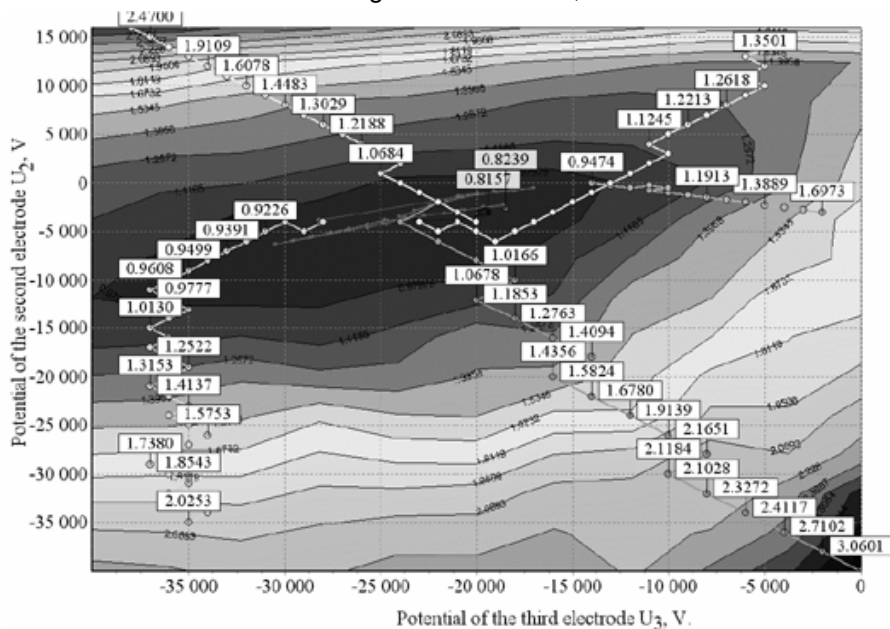
Figure 4: Relief s function K_{er}

Figure 5: Two-parametric optimization of four-electrode LEBT system

Program complex had been worked out by S. A. Kozynchenko according to scheme on Fig. 1 and was used for optimization. First, some preliminarily investigations of the relief surface of the function K_{er} have been made. The results are shown in Fig. 4 and 5. Second, methods of 'averaged' gradient [1] (non-local search), Box-Wilson [4] (local search) and 'ravine' [3] (local search) were applied in succession (see Fig. 4). Final result $K_{er} = 0.8157$ under $u_1 = 0$ kV, $u_2 = -3$ kV, $u_3 = -19$ kV and $u_4 = -40$ kV, perhaps, may be considered as approximation of the global minimum of two-sized problem. Three-, four- and five-parametric optimization problems of the four-electrode LEBT system have been also investigated.

Real LEBT construction for RFQ, which is given in paper [3], also needs four- or more parametrical optimization. Program complex (see Fig. 1) allows the simulation of both axial-symmetrical and 3D electrostatic LEBT on PC. Real time of modelling and 19 - parametrical optimization of five-electrode LEBT with plasma-surface ion source of Dudnikov type is about 18 hours on Pentium 4/1.7 GHz PC. Ion source produces elliptical or axial-symmetrical beam of ions H^- with 17 keV energy and current 15 mA. Output energy of the beam is 100 keV.

Quality of the LEBT system is determined by the function, characterizing the matching of both beam on the LEBT exit and given acceptance of ion linac with RFQ. This optimized five-electrode system allows receiving the final beam with 78% of particles, involved in subsequent acceleration process in the ion linac with RFQ (see Fig. 6).

CONCLUSION

Thus, presented program complex allows directly with modelling to determine optimized construction and physical parameters of injection and transport system. 3D-field codes, which are part of complex, give possibility to optimize injection systems with plasma-surface sources or axial-symmetrical type ones. But multi-parametric optimization of electrostatic systems or resonators and waveguides need adoption of the program complex to power computer.

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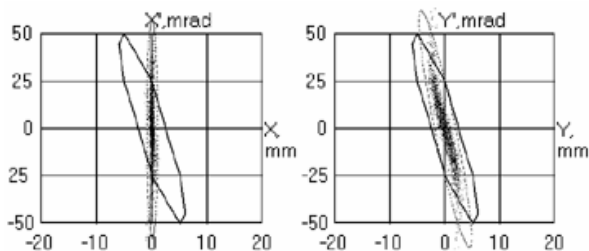
Figure 6: Phase distributions of the H^- – ion beam at the plane of five-electrode LEBT system exit. The solid sloped ellipse is the acceptance of the ion linac with RF