CHARGING OF A SPHERICAL HIGH-VOLTAGE TERMINAL BY PARTICLE BEAM

B.Yu. Bogdanovich, V.V. Kapin# and A.V. Nesterovich,
Moscow Engineering Physics Institute (State University)*, Russia

Abstract

Recently, the accelerator complex for custom examinations had been discussed. The complex consists of RF electron linac and the direct-voltage proton accelerator. The 2-MeV electron beam used for the X-ray generation is also utilized for electrostatic charging of the high-voltage terminal of a proton accelerator. High-voltage terminal can be constructed as a large sphere to minimize peak electric field stress. In this report, the charging process of the spherical target by electron beam is studied. The particle motion is described by the classical Coulomb scattering in the field of the charged sphere. The effects of secondary emission and resistive divider are included as leakage currents. The differential equation for the time-derivative of the sphere charge is derived in a non-relativistic approach. The time-dependences for the charge accumulated on sphere and the energy of hitting electrons are obtained analytically. The final values of the sphere potential and the energy of hitting electrons depend on total value of leakage current. The numerical model for studying relativistic effects is outlined. The simulations are performed under conditions of negligible leakage currents.

INTRODUCTION

Nowadays, X -ray radiation is widely used for the custom examinations of sizes and forms of baggage elements. The γ -rays generated on the target by proton beam detect composition of smuggling, like explosives drugs etc. Recently, the accelerator complex for custom examinations had been discussed [1]. The Figure 1 shows layout of the complex. It consists of RF electron linac and the direct-voltage proton accelerator. The 2-MeV electron beam used for the X-ray generation is also utilized for electrostatic charging of the high-voltage terminal of a proton accelerator. High-voltage terminal can be constructed as a large sphere to minimize peak electric field stress. In this report, the charging process of the spherical target by electron beam is studied.

ELEMENTARY MODEL

Let's an electron beam with current $I_{\rm e}$ charges a spherical body with radius $R_{\rm sph}$ to a charge $Q_{\rm sph} < 0$. The ion beam with the current $I_{\rm i}$ is discharging sphere. To ensure charging process, the electron current should be larger of ion current. The total charging current is equal to $I_{\rm sph} = I_{\rm e} - I_{\rm i}$. The sphere charge is time-dependent $|Q_{\rm sph}| = I_{\rm sph} t$. The sphere has a negative potential $\Phi_{\rm sph} = Q_{\rm sph}/C_{\rm sph}$, where $C_{\rm sph} = 4\pi\varepsilon_0 R_{\rm sph}$ is electrical capacity. The charging process will stop when the energy of electron beam becomes to be equal to the sphere potential, $W_{\rm e} \le e|\Phi_{\rm sph}|$. Total charging time is estimated by the formulae:

$$\tau_{\rm sph} = 4\pi\varepsilon_0 W_{\rm e}^{[\rm eV]} R_{\rm sph} / (I_{\rm e} - I_{\rm i}) \qquad (1)$$

When $I_{\rm e} \approx I_{\rm i}$, the charging time is infinite. The elementary model gives only approximate description of the charging process, because it does not take into consideration details of electron motion.

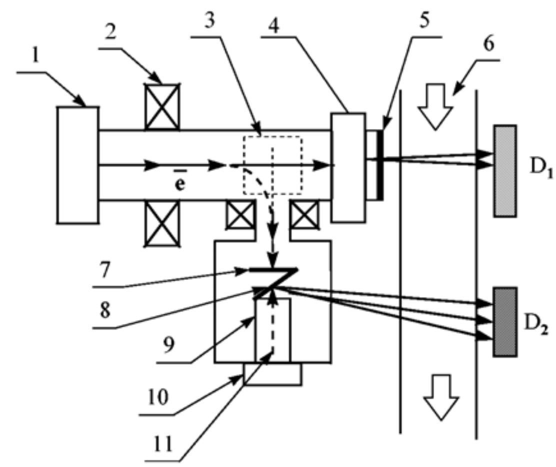

Figure 1: Layout of the accelerator complex: 1 - linear accelerator of electrons; 2 - focusing magnet; 3-deflecting magnet; 4 - scanning system; 5, 7, 8 - targets, 6 -conveyer belt; 9 - optical system of the ion accelerator; 10 - ion source; 11 - ion beam, Dl- X -ray detector, D2 - γ -ray detector.

ELECTRON BEAM DYNAMICS

The motion of electrons in the field of the charged sphere is described by the classical Coulomb scattering (Fig. 2). In a polar coordinates the trajectory equation is

$$\tilde{r}(\varphi) = \tilde{b}^2 / \left[-\Xi(1+\cos\varphi) + \tilde{b}\sin\varphi \right] \qquad (2)$$

where $\Xi = q\Phi_{\rm sph}/pv$, $\tilde{r} = r/R_{\rm sph}$ and $\tilde{b} = b/R_{\rm sph}$ are dimensionless parameters. With the analysis of the electron trajectories, the portion of the electron beam current hitting the sphere can be calculated. Figure 3 shows 3 trajectories for $\Xi = 0.5$ with different parameter b : $b = 0.1R_{\rm sph}$, $b = 0.5R_{\rm sph}$, $b = 1.0R_{\rm sph}$. The

kapin@mail.ru
* MEPhI, mailbox-600, Kashirskoe sh.31, Moscow 115409, Russia

initial beam cross-section and its image on target are shown. It is seen that only a central part of the beam reaches the sphere and charges it.

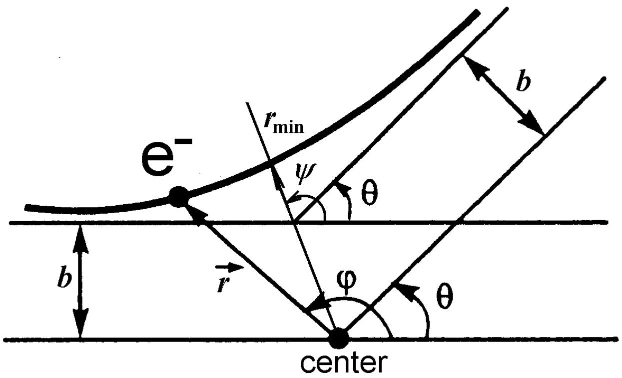

Figure 2: Motion in the central field.

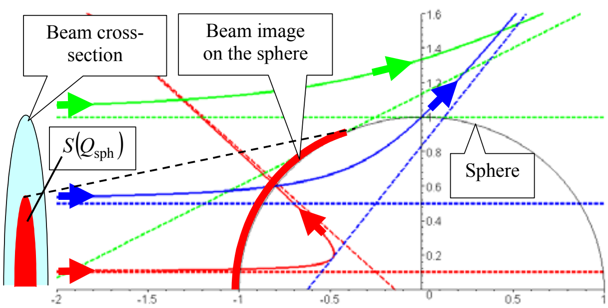

Figure 3: The trajectories at $\Xi = 0.5$.

With increasing parameter Ξ from 0 to 0.5 due to the growth of the sphere potential $\Phi_{\rm sph}$, the beam portion reaching sphere drop down from 100 % to 0, while Ξ=0.5 when kinetic energy of electrons is equal to the sphere potential.

EQUATION FOR CHARGE EVOLUTION

The sphere charge $Q_{\rm sph}$ is related to the beam current on the sphere $I_{\rm sph}$ by equation $dQ_{\rm sph}/dt = I_{\rm sph}$. Only paraxial part of beam reaches the sphere surface. The ratio of currents is the function of the sphere charge, $I_{\rm sph}/I_{\rm b} = f(Q_{\rm sph})$, where $I_{\rm b}$ is the beam current. The resulting equation for the sphere charge is

$$dQ_{\rm sph}/dt = I_{\rm b} f(Q_{\rm sph}) \qquad (3)$$

where $f(Q_{\rm sph})$ is a function to be defined. For a uniform beam cross-section, $f(Q_{\rm sph}) = S(Q_{\rm sph})/S_{\rm b}$, where $S(Q_{\rm sph})$ is the cross-section of paraxial beam reaching the target, and $S_{\rm b}$ is the total beam cross-section. For the round beam with radius R_b, the function is

$$f(Q_{\rm sph}) = \left[b_{\max}(Q_{\rm sph})/R_{\rm b}\right]^2 . \qquad (4)$$

The analytical solution for dimensionless parameter $\tilde{b}^2_{\max}$ the has been derived in the following form:

$$\tilde{b}^2_{\max}(Q_{\rm sph}) = 1 - 2\,qQ_{\rm sph}/pvC_{\rm sph} . \qquad (5)$$

Substituting Eq. (5) into Eq. (1) provides a differential equation for sphere charge:

$$dQ_{\rm sph}/dt = I_{\rm b}\left(1 - 2\,qQ_{\rm sph}/pvC_{\rm sph}\right). \qquad (6)$$

The solution of the equation is:

$$Q(t) = Q^{\max}_{\rm sph}\left[1 - \left(1 - Q_0/Q^{\max}_{\rm sph}\right)\exp\left(-t/\tau^{\min}_{\rm sph}\right)\right], \qquad (7)$$

where $\tau^{\min}_{\rm sph} = Q^{\max}_{\rm sph}/I_{\rm b}$ $\tau^{\min}_{\rm sph} = Q^{\max}_{\rm sph}/I_{\rm b}$ is the characteristic time of charging, Q_0 is a charge at initial time $t = 0$. The time $\tau^{\min}_{\rm sph}$ coincides with charging time obtained with elementary model without beam dynamics effects. The energy of the electrons on the sphere surface is obtained from the energy conservation law:

$$W^{\rm sph}_{\rm kin} = W^0_{\rm kin}\left(1 - Q_0/Q^{\max}_{\rm sph}\right)\exp(-t/\tau^{\min}_{\rm sph}) \qquad (8)$$

Figure 4,a shows the time dependencies of the normalized sphere charge and the electron energy on sphere at $Q_0 = 0$.

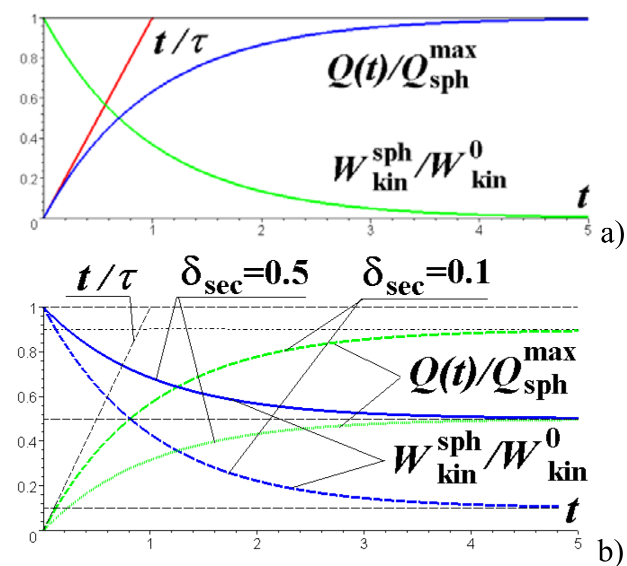

Figure 4: The $Q(t)/Q^{\max}_{\rm sph}$ and $W^{\rm sph}_{\rm kin}/W^0_{\rm kin}$ versus time at zero (a) and non-zero (b) current of secondary particles.

CHARGING AT LEAKAGE CURRENTS

The effects of secondary emission and resistive divider can be included as leakage currents $I_{\rm sec}$, which can be expressed via the current of charging beam as $\delta_{\rm sec} = I_{\rm sec}/I_{\rm b}$. Eq. (6) becomes as follows:

$$dQ_{\rm sph}/dt = I_{\rm b}\left(1 - 2\,qQ_{\rm sph}/pvC_{\rm sph}\right) - I_{\rm sec} . \qquad (9)$$

The solution for charge and energy is written as

$$Q(t) = Q^{\max}_{\rm sph}\left[(1-\delta_{\rm sec}) + \left(1 - \delta_{\rm sec} - \frac{Q_0}{Q^{\max}_{\rm sph}}\right)\exp\left(-t/\tau^{\max}_{\rm sph}\right)\right],$$

$$W^{\rm sph}_{\rm kin}(t) = W^0_{\rm kin}\left[\delta_{\rm sec} + \left(1 - \delta_{\rm sec} - \frac{Q_0}{Q^{\max}_{\rm sph}}\right)\exp\left(-t/\tau^{\max}_{\rm sph}\right)\right]. \qquad (10)$$

Figure 4,b shows the time dependencies of the normalized sphere charge and the electron energy on sphere at two values of the relative leakage $\delta_{sec} = 0.1$ and $\delta_{sec} = 0.5$, while the initial charge is zero ($Q_0 = 0$).

The sphere charge is increased according to the exponential law with the characteristic time τ_{sph}^{min}. The limiting value is not equal to Q_{sph}^{max}, the charge is growing to the equilibrium value $Q_{sph}^{lim} = Q_{sph}^{max}(1 - \delta_{sec})$. The energy of electrons on sphere is not tend to zero, its equilibrium value is $W_{kin}^{sph} = \delta_{sec} W_{kin}^{0}$.

Thus, the leakage currents do not allow charging the target to a full potential of electron beam. The final values of the sphere potential and the energy of hitting electrons depend on total value of leakage current.

RELATIVISTIC EFFECTS

The particle motion in the above sections has been considered in a non-relativistic approach. In the relativistic case, the analytical model can provide only approximate estimations. The limiting values of the charge and the energy obtained analytically should be true due to the laws of the energy conservation. However, details of time dependences $Q(t)$ and $W_{kin}^{sph}(t)$ should deviate from non-relativistic curves.

The numerical model for studying relativistic effects has been developed. For the simulation of the particle motion in the electrostatic field of charged sphere and summation of the charge accumulated on the sphere, it is naturally to use the "macro-particles". The essential approach is that the potential (and charge) of sphere does not change during the flight time of particles. Integration of the motion equation has been performed with well-known "leap-frog" algorithm.

Simulation has been performed with 100 MeV zero-emittance electron beam with a uniform transverse distribution. The leakage currents are negligible. The sphere radius is R_{sph} =1 m.

Figure 5,a shows the time dependence for the sphere potential. The ideal curve with characteristic time of 0.11 s calculated with Eq. (1) is also shown. The ideal and numerical curves are very close to each other. According to the numerical simulation the sphere is charged to the full potential during 0.15-0.17 s with a little difference from the characteristic time of 0.11 s.

The time dependencies for the energy and current of electrons reaching the sphere surface are shown in Figure 5,b. During the charging process, the electron energy is decreased from the initial 100 MeV to zero value within 0.17 s. The electron current is also drop from the initial 100 mA to the zero level. Note that the numerical code uses negative values for the electron current.

The general character of curves does not contradict with the theory. However, there is considerable numerical noise related to the restricted number of macroparticles at the end of the charging process.

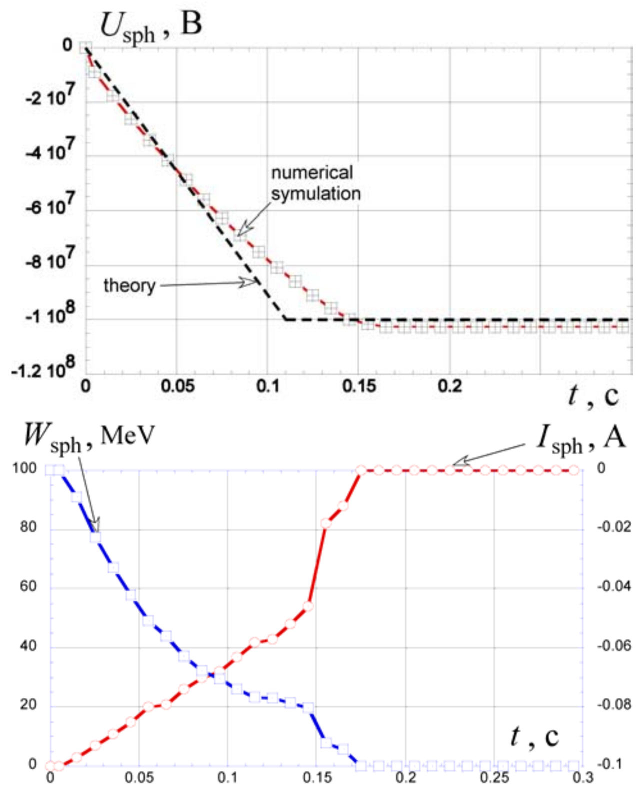

Figure 5: the time dependencies for the sphere potetnial (a) and energy and current of electrons on sphere (b).

REFERENCES

[1] P. Alferov, B. Bogdanovich, A. Nesterovich et al., "Universal Accelerating Complex for the Custom Examination", EPAC'98, Stockholm, p. 812.

[2] R.W. Hockney and J.W.Eastwood, 'Computer Simulations Using Particles', Bristol, 1988 by Adam–Hilger.