THEORETICAL AND EXPERIMENTAL STUDY OF BEAM ENERGY SPREAD DIAGNOSTICS WITH VAVILOV – CHERENKOV RADIATION AT OPTICAL AND RF WAVELENGTH

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Abstract

Use of Cherenkov radiation (CR) in the optical and in the microwave ranges for the accelerator beam energy and the energy spectrum measurements is described. The methods are based on the radiation intensity dependence on a phase velocity of an electromagnetic wave in Cherenkov radiator medium. The details of a mathematical procedure to extract the energy spectrum from the measured intensity are given.

INTRODUCTION

The development of the non-intercepting methods for the electron beam energy and the energy spectrum measurements with high accuracy in the energy range 1-20 MeV and above is of great importance for irradiation process control in industry and medicine.

We consider methods of the electron beam energy and the energy spectrum measurement based on the dependence of the CR intensity (CRI) and its spectral distribution on the phase velocity of the electromagnetic waves in the optical and microwaves ranges.

The methods are practically non-intercepting and can be used for the energy and energy spectrum measurements of high intensity electron beams.

OPTICAL RANGE

CRI Dependence on Refraction Index

For the first time the possibility of average electron velocity determination in the electron accelerator beams with the measurements of the CRI dependence on a radiator refraction index n in an optical range was demonstrated in [1-2].

The gases, which refraction index varies with pressure p as $n(p) = 1 + kp$, if kp<<1, are natural choice of the CR medium in this method. The CRI grows nonlinearly with kp between the thresholds of Cherenkov radiation for a maximal and minimum electron velocity in beam in the detector ($1/\beta_{\max} \le n(kp) \le 1/\beta_{\min}$), but for $n(kp) > 1/\beta_{\min}$ the dependence is linear.

The intersection of the extrapolated CR (CRI) linear part with a background level corresponds to the average electron velocity in the beam [1]. The same method was proposed 15 years later [3] without reference to [1-2]. No nonlinear part in I(n) dependence was noted.

In [4-6] it was shown that the nonlinear part of the intensity curve can be used for obtaining the velocity distribution and the energy distribution of the particles in the beam.

If the same fraction of the CR light is registered with the photodetector independently of the position and the direction of a particle passing through the radiator (as for example in the spherical Cherenkov detector) then the velocity distribution and the energy spectrum can be obtained for the arbitrary angular distribution of the particles. In the case under consideration (the accelerator monitoring) this condition can be easily fulfilled.

Following [6] a number of the CR photons $N_{ph}(n(\nu))$ in a unit frequency band reaching the photodetector can be written as:

$$g \int_{\max\left\{\frac{1}{n},\beta_{\min}\right\}}^{\beta_{\max}} \left(1 - \frac{1}{n(\nu)^2 \beta^2}\right) f(\beta)\,d\beta = N_{ph}(n(\nu)) \tag{1}$$

where $f(\beta)$ is a particle velocity distribution; $g = 2\pi\alpha N_e kL$, where α is the fine structure constant, N_e – the number of the electrons, L- the length of particle path in detector, k – the photon collection factor and ν is a photon frequency.

The equation (1) is Volterra integral equation of the first kind with the right part having experimental errors. Although it is ill - posed task the several methods for solution of such equations have been developed during the past decades. These methods can be used to extract the particle velocity distribution from the measured dependence of N_{ph} on the refraction index n

The equation (1) can be reduced to equation of convolution type [6]. Let us change variables:

$$\beta = y^{1/2};\quad \Psi(y) = \frac{f(y^{1/2})}{2y^{3/2}};\quad \frac{1}{n(\nu)} = z^{1/2} \tag{2}$$

Then

$$g \int_{z}^{z_{\max}} \left(y - z(\nu)\right)\Psi(y)\,dy = N_{ph}(z(\nu)) \tag{3}$$

By successively differentiating (3) we obtain the solution as:

$$\Psi\left(z^{1/2}\right) = \frac{1}{g}\frac{d^2 N_{ph}(z)}{dz^2} \tag{4}$$

The first derivative of the N_{ph} in (3) is an integral spectrum, i.e. the number of particles above some energy. In this case the CR monitor can be used at the accelerators as the threshold sensor producing a signal when the beam energy exceeds some prescribed value.

To avoid the gas dissociation by the electron beam usage of the single atomic gases is preferable. For the narrow energy spectra typical for the microtron beams or the beams from the high brightness accelerators, the required change of the pressure will not exceed several percents. The narrow band filter installed in front of the photodetector and measurement of gas refraction index with a build-in interferometer can decrease sufficiently the experimental errors. The instant gas expansion by high power beam pulse can be neglected if a pulse is very short ($\leq 10^{-8}$ s.)

The yield of the CR near the threshold for the most electron accelerators is more than sufficient for the reliable intensity registration with the photodetectors (photomultipliers). The energy loss fluctuations in the CR detector should be taken into account at the determination of a device resolution power.

Experimental Results

Experiments were conducted with the beam of 35 MeV race track microtron [7]. The beam with an energy in the range 4.8 – 34.2 MeV, an energy spread ~200 keV can be extracted from the different orbits with a step ~2.4 MeV. The beam pulses following with the repetition frequency 50 Hz have a duration ~5 ns with a charge ~ 1 nC/pulse. The experimental set-up is shown in Fig. 1. The electron beam from the accelerator produces CR on its path "AB" in the chamber (1) filled by a gas which pressure is regulated by a membrane (5) and controlled by the manometer (6). A light pulse reflected by a mirror (3) is registered by a photomultiplier (4). The beam pulse charge is controlled by Faraday cup (2).

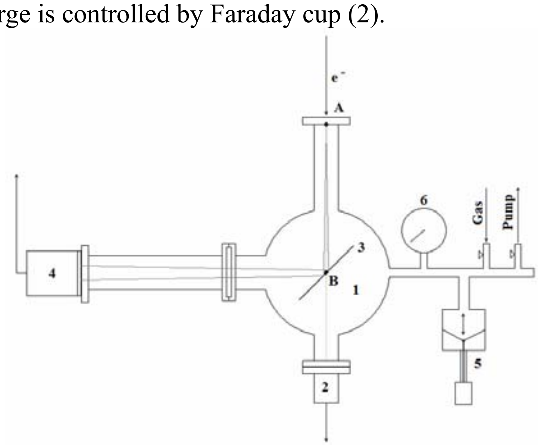
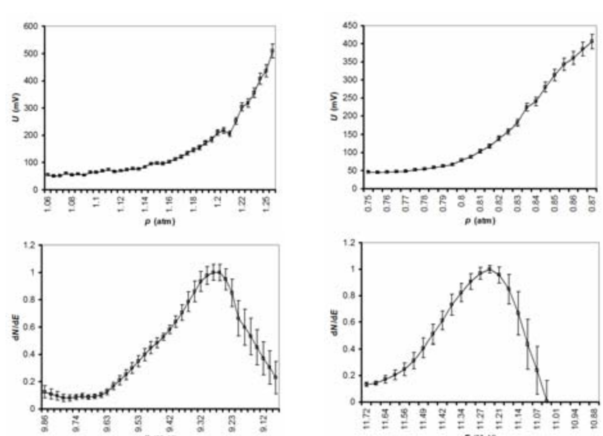


Figure 1: Experimental set-up.



(a) (b)

Figure 2: CRI measured for (a) 2[d] and (b) 3[d] RTM orbits and corresponding energy spectra.

In Fig. 2 we show CRI measured depending on Freon R-12 gas pressure and corresponding the energy spectra extracted using formula (4) with regularization algorithm.

CRI Dependence on Wavelength

Dependence of the refraction index on the wavelength requires some precautions when measuring the particles energy spectra by the changing gas pressure as described above. Dispersion of light causes some trouble in Cherenkov counter methods in general. However it becomes "friendly" phenomena in another proposed method, the essence of which consists in following. CR from the radiator output is directed to a spectral device (grating, prism etc.) for spectral decomposition and CR intensity dependence on wavelength is registered with the line of photodetectors. Because of refraction index for any medium depends on a wavelength, the CR threshold depends on a wavelength also. Thus, the measurements at the different wavelengths are analogous to the measurements with variable gas pressure or to measurements with a large number of the threshold counters with different thresholds.

In the visible light range the refraction index decreases with the wavelength increase. Thus, the higher is electron velocity the longer wavelength CR threshold is. At threshold wavelength for some specific energy a number of the emitted CR photons is equal to zero but increases systematically as the wavelength of the registration is deceased.

The number of the CR photons having a wavelength λ in the unit wavelength range may be written as:

$$N_{ph,\lambda} = g\int_{\beta_\lambda}^{\beta_{\max}} f(\beta)(1-\frac{\beta_\lambda^2}{\beta^2})\lambda^{-2}d\beta \tag{5}$$

(β_λ is the threshold velocity at the wavelength λ, g is as in (1).)

Using Cauchy formula $n(\lambda) \approx a + b\lambda^{-2}$, where a and b are the constants individual for different gases, we obtain the threshold wavelength for given β_λ:

$$\lambda^{-2} = (\beta_\lambda^{-1} - a)/b \tag{6}$$

Substituting (6) into (5) after some transformations we obtain:

$$f(\beta_\lambda) = \frac{\beta_\lambda^2}{2g}\frac{d}{d\beta_\lambda}(\frac{1}{\beta_\lambda}\cdot\frac{d}{\beta_\lambda}(\frac{b}{\beta^{-1}-a}\cdot N_{ph,\lambda})) \tag{7}$$

The equation (5) can be reduced similar to equation (3) to convolution equation (the designations are the same):

$$\int_z^{z_{\max}} (y - z(\lambda))\Psi(y)dy = \frac{\lambda^2}{g}N_{ph,\lambda}(z(\nu)) \tag{8}$$

which can be solved by successive differentiation.

Thus, to get the beam velocity distribution it is necessary to measure the number of photons in the different wavelength ranges and to get their first and second derivatives with respect to the wavelength. The number of photons is known with error and this procedure is ill -posed task, so appropriate methods to obtain stable solution must be applied too.

The proposed method will be convenient for the accelerator monitoring especially in the case of the narrow energy spectra. As an example, at the beam energy 30 MeV and the energy spread of 0.5 MeV the spectral interval for Xenon is in the range from 400 nm to 600 nm.

MICROWAVE RANGE

For continuous beam energy control during an accelerator operation it is desirable to have a monitor with a vacuum beam channel excluding the ionization and the radiation electron energy losses. A charge when passing through the vacuum channel in a dielectric radiates in the same manner as in a continuous medium if the next conditions are fulfilled [8]:

$$b << \frac{\lambda_C \beta}{2\pi\sqrt{1-\beta^2}}, \quad b << \frac{\lambda_C \beta}{2\pi\sqrt{\varepsilon\mu\beta^2 - 1}} \tag{9a}$$

where b – is a radius of the vacuum beam channel in a dielectric with the permittivity ε and the magnetic conductivity μ, λ_C –the CR wavelength. It is supposed that conditions for the CR in a dielectric are fulfilled: $\varepsilon\mu\beta^2 > 1$. In practice a beam channel radius should be not less than 5 mm, so a radiation will take place at mm and cm wavelength, i.e. in the RF range.

For the beam channel radius not satisfying the conditions (9a) the analytical solution for a radiation field distribution and a radiated power exists [8]. It should be noted that a radiated power is cut in the short wavelength range at:

$$\lambda << \frac{4\pi b\sqrt{1-\beta^2}}{\beta} \tag{9b}$$

In the RF accelerator particles are grouped in the bunches with the length $l << \lambda_{RF}$, where λ_{RF} – is the free space wavelength of the accelerating field. For $\lambda_C > 2\pi l$ the amplitudes of the field radiated with the particles within the bunch add coherently and the radiated power is enhanced by the factor approaching to N_e, i.e. by many orders of magnitude.

To provide vacuum at the beam path and to arrange the conditions for the radiated power collection and registration dielectric with a beam channel should be placed inside a conducting metal tube. The main feature of the radiation generated by the charge in this case is that it takes place at the discrete frequencies which values are determined by the waveguide and the beam channel radii, the dielectric properties and the particle velocity. The charge passing along the beam channel axis will excite with a highest amplitude TM_{01} mode wave with a longitudinal electric field on axis. Thus, if the conditions (9a) are fulfilled, then a frequency of the excited TM_{01} mode wave is connected with a particle velocity by [8]:

$$\nu_{01} \approx \frac{\beta c x_{o1}}{2\pi b\sqrt{\varepsilon\mu\beta^2 - 1}} \tag{10}$$

where $x_{01} \approx 2.405$ – is the first root of zero order Bessel function J_0. The power radiated in this mode is:

$$p_{o1}(\beta) \approx \frac{2e^2 \beta c}{b^2 \varepsilon\varepsilon_0 J_0'(x_{01})} \tag{11}$$

From (10) it follows that when approaching the Cherenkov effect threshold ($\varepsilon\mu\beta^2 = 1$) a generated wave frequency goes to infinity, but in reality limit for a frequency will be defined by the condition (9b). For a beam channel radius not satisfying to (9a) the analytical expressions for a radiation frequency and a power are available [8].

Strong dependence of the generated radiation frequency on the particle velocity and absence of the sharp boundary for a RF signal appearance make difficult to use the method for the energy and energy spectrum determination developed for an optical wavelength range and described above. So, we propose to use for the beam energy and energy spectrum control in the microwave range strong dependence of the generated wave oscillation frequency on the particle velocity. The generated wave oscillation frequency is uniquely connected with the particle velocity via (10) and for the relativistic particles according to (11) a radiated power is nearly independent of the velocity. Thus measurement of the generated radiation spectrum is a direct method for the beam energy spectrum control not requiring a solution of the inverse task.

Energy resolution of proposed method is connected with frequency resolution by:

$$\frac{\Delta E}{E} \approx \frac{(\varepsilon\mu\beta^2 - 1)\beta}{1-\beta^2} \times \frac{\Delta\nu}{\nu} \tag{12}$$

The quite simple RF measurement methods (e.g. using a high quality factor tunable cavity) provide a frequency resolution $\Delta\nu/\nu \approx 10^{-3}$. For the beam energy ~ 10 MeV (the industrial and medical accelerators) and $\varepsilon\mu \approx 1.1$ (aerogel) energy resolution will be about 4%, and for a circular waveguide radius ~ 10 mm radiation will take place at ~8 mm wavelength range.

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REFERENCES

[1] M.R. Bhiday, R.E.Jennings, P.I.P.Kalmus, Proc. Phys. Soc., **72,** 1958, p.973.

[2] R.E. Jennings, P.I.P. Kalmus, Nucl. Instr. Meth., **6**, 1960, p.209.

[3] C.E. De Almeida, P.R. Almond, Phys. Med. Biol., **19**, N 4, 1974, p.476.

[4] K.A. Trukhanov, V.N. Grebenev, V.M. Shlapak, in Proc. IV Int. Cong. Int. Rad. Protection Assoc. Paris, 24-30 April 1977, p. 119.

[5] E.A. Vainer, K.A. Trukhanov, in Proc.3th meeting on charge particle accelerator use in economics", Leningrad, 26-28.06.1979, V. III, p. 345.

[6] K.A. Trukhanov, in Proc. Seminar "Cherenkov detectors and their applications in science and technics" (1984), M. Nauka, 1990, p. 380.

[7] V.I. Shvedunov, R.A. Barday, D.A. Frolov, et al., Nucl. Instr. Meth. **A531**, 2004, p. 346

[8] B.M. Bolotovsky, Usp. Fiz. Nauk, **125,** 1961, p. 295.