# ION BEAM FOCUSING METHODS IN SUPERCONDUCTING LOW ENERGY LINAC

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#### Abstract

Ion superconducting linac is based on independently phased cavities. The low charge state beams require stronger transverse focusing. This focusing can be reached with the help of SC solenoid lenses, electric or magnetic quadruples, and RF focusing fields. In this paper the various focusing methods are compared for low ion velocities. This comparison can be demonstrated with an example a post-accelerator of radioactive ions (RIB linac). This linac must produce high-quality beams over the full mass range, including uranium, with high transmission and efficiency [1].

## **INTRODUCTION**

The initial section of the RIB linac is a low-charge-tomass-ratio superconducting RF (SRF) linac which will accelerate any ion with  $Z/A \ge 1/66$  from 75 keV/u to ~900 keV/u or higher. This section of the RIB linac consists of  $N_{c}\approx 60$  independently phased SC cavities providing a total of  $U \approx 70$  MV accelerating potential. Low-energy RIB linac is based on 4-gaps and 2-gaps quarter wave SC niobium cavities, which can provide typically 1 MV of accelerating potential per cavity in the velocity range 0.011c < v < 0.06c. Beam focusing can be provided with help of SC solenoid lenses, electric or magnetic quadruples following each cavity and with help of special RF fields. The low-charge-state beams and the low velocity require stronger transverse focusing than one is used in existing SC ion linac. The large radial variation of the axial accelerating field induces a beam energy spread, which will accumulate as the beam passes through successive resonators. A key issue with such SC linac is to maintain small longitudinal emittance while maximizing transverse acceptance [2]. The connection between the longitudinal and transverse motion can deteriorate longitudinal beam quality rapidly with increasing beam radius. The numerical investigation of beam dynamics shown that for the initial normalized emittance  $\varepsilon_{\rm T} = 0.1\pi \cdot \rm{mm} \cdot \rm{mrad}$ and the transverse longitudinal emittance  $\varepsilon_{\rm V} = 0.3\pi \cdot {\rm keV/u} \cdot {\rm nsec}$ this connection can be neglected if maximum beam envelop  $X_m < 3$ -4mm and inner radius of accelerating drift tubes a = 15mm. In this paper the various focusing methods are compared for low ion velocities and for the charge-tomass ratio Z/A = 1/66.

## LONGITUDINAL BEAM DYNAMICS IN SC LINAC

The initial section parameters of the RIB linac was described in [4]. The longitudinal ion dynamics in such periodic structure is complicated. The reference particle velocity  $\beta$  can be represented as a sum of a smooth

motion term  $\beta_c$  and a fast oscillation term, a period of which is equal to period accelerator structure *L*. The fast oscillation term can be neglect if the cavity length and period structure *L* are closely.

Beam focusing can be reached with the help of special lenses. It is suggested the post-accelerator will contain  $N_s = 60$  such lenses. The distance between the cavity and the lens edge  $L_d = 100$  mm. The distance between two cavities equals  $L_1 = 75$  mm if there is no lenses between the cavities. The inner radius of the lenses equals 15 mm and the effective length  $L_s = 200$  mm. The different focusing methods of SC solenoid lenses, magnetic or electric quadruples, and RF fields are proposed and discussed in this report.

The 3D ion dynamics in periodic acceleration structure can be calculated by means of the transfer matrices for longitudinal and transverse motion which are divided into an integral number of transport system elements.

## SOLENOID FOCUSING

Let's consider the beam focusing by means of a periodic sequence of magnetic solenoids and cavities. The solenoid matrices  $M_{sol}$  was found early in [3]. In a simple case, when the effective phases of reference particles in the cavities are identical, the period consists of a superconducting solenoid for transverse focusing and superconducting RF cavity for acceleration and longitudinal focusing. The transfer matrices  $M_z$  and  $M_r$  allow to find the phase advances per period:

$$\mu_{z} = \arccos\left(\frac{1}{2}\left(M_{z11} + M_{z22}\right)\right)$$
(1a)

$$\mu_r = \arccos\left(\frac{1}{2}\left(M_{r_{11}} + M_{r_{22}}\right)\right)$$
(1b)

It is interesting to compute the necessary magnetic field value *B* for a considered interval of beam velocity  $0.01 \le \beta \le 0.06$ . The final choice of the focusing magnetic field and the parameter  $\mu_r$  can be made if the normalized transverse emittance  $V_r$  and maximum size of beam envelope  $X_m$  are fixed. In our case

$$\mu_r = \arcsin\left(\frac{V_r M_{r12}}{\beta \gamma X_m^2}\right) \tag{2}$$

where  $M_{r12}$  is non-diagonal element of the transfer matrix.

The Figure 1 shows the dependence of phase advances  $\mu_r$  from the beam velocity (dash-dot line) for the different maximum values of the beam envelope  $X_m$  when the

magnetic field B = 9 T and the transverse emittance  $V_r = 0.1\pi$ ·mm·mrad. The same Figure shows the function of  $\mu_r(\beta)$  (the solid line) which was found from the formula (1b) at the same magnetic field for the particle phase  $\varphi_c = -20^\circ$ . The beam envelop will not exceed 4 mm in the magnetic field B = 9 T if the beam velocity  $\beta > 0.04$ . In this case the value of  $\mu_r < 10^\circ$ . The magnetic field must be increased in order to provide the same focusing condition when the beam velocity  $\beta < 0.04$ .

It is interesting to find the value of the focusing magnetic field  $B_{\min}$  as function of the ion velocity  $\beta$  if the envelop  $X_m$  is the constant along the linac. The system of two equations (1b) and (2) can be solved for  $X_m = const$  in the accelerator structure which consists of the periodic sequence of the solenoids and the resonators. In this case the value  $B_{min}$  is more 15 T when beam envelop  $X_m = 3$  mm. If the maximum envelop  $X_m = 4$  mm, the magnetic field  $B \le 9$  T only when the beam velocity  $\beta > 0.04$ .



Figure 1: The phase advances  $\mu_r$  for different beam envelops  $X_m$ .



Figure 2: The focusing magnetic field  $B_{\min}$  as a function of the beam velocity for two beam envelops  $X_{\rm m}$ .

## QUADRUPOLE FOCUSING

The magnetic solenoid elements would need to produce axial fields larger 15 T for adequate transverse focusing. More effective focusing can be obtained with either magnetic or electric quadrupoles. The focusing element is employed in the form of a singlet or a duplet lens. The focusing period in these cases consists of F-singlet, accelerating cavity and D-singlet (FODO focusing) or a duplet and accelerating cavity (FDOFDO focusing).

## Magnetic Quadrupoles

In order to avoid trapped flux and excessive rf loss in the neighboring superconducting cavities it need complete shielding of magnetic field. We will suggest that the distance between the cavity and quadrupole edge is the same as solenoid L=100 mm. Let's consider the beam focusing in FODO system. The transverse phase advances per period  $\mu_r$  is function a magnetic field gradient G and beam velocity  $\beta$ . Fig. 3 shows the dependence of  $\mu_r$  from beam velocity for three gradients 75 T/m, 100 T/m and 200 T/m. In the second case (G = 100 T/m) the focusing condition can will be realized in the whole interval beam velocity  $(0.01 < \beta < 0.06)$ . For other gradients the focusing is possible if  $\beta > 0.019$  for G = 75 T/m or  $\beta > 0.026$  for G = 200 T/m. The choice of field gradient is connected with the maximum value of the beam envelop. For G = 100 T/m and  $\beta$  = 0.01  $X_m$  = 8 mm. The maximum envelop is smaller 4 mm when  $\beta > 0.025$ . It is interesting to find the value of magnetic field gradient as function of the ion velocity if the envelop  $X_m$  is the constant along the linac. The system of two equations (1) and (2) can be solved for  $X_m = 3 \text{ mm}$  and 4 mm in FODO structure. The Fig. 4 shows the function  $G(\beta)$  for these two beam envelops. The peculiarity of beam focusing in FDOFDO system looks like FODO. Here the beam envelop  $X_m < 4$  mm when G = 100 T/m and 0.016 <  $\beta < 0.06$ .



Figure 3: The phase advances  $\mu_r$  for different magnetic field gradient G.



Figure 4: The magnetic field gradient *G* as a function of the beam velocity for two envelops  $X_m$ .

#### Electric Quadrupoles

The investigation beam dynamics in the periodic structure with electric quadrupoles can be accomplished similarly. The electric quadrupole has a bore radius a = 15 mm and vane voltage  $V_0$ . The transverse phase advance per period  $\mu_r$  as a function of beam velocity for different a vane voltage  $V_0$  is shown in Fig. 5. The dependence of  $X_m$  from beam velocity in system FODO for three voltages  $V_0 = 100$ , 150 and 200 kV is illustrated in Fig. 6. The maximum envelop is smaller 3mm when  $\beta > 0.025$  and  $V_0 > 150$ kV. The conditions of beam focusing in FDOFDO system look like FODO. Here the beam envelop X < 3mm when  $V_0 > 150$ kV and  $0.02 < \beta < 0.06$ .



Figure 5: The phase advances  $\mu_r$  for different vane voltage  $V_0$ .



Figure 6: The beam envelop  $X_m$  as a function of the beam velocity  $\beta$  for different vane voltage  $V_0$ .

#### **COMBINED FOCUSING**

The required solenoid and quadrupole fields are mostly defined by the high value of the RF defocusing factor in the SC resonators. A possible alternative focusing method based on combination of low focusing fields and alternating phase focusing (APF). In this case the focusing period contains two SC cavities, in which the effective phases must be alternated between positive and negative values. By adjusting phases  $\varphi_1$  and  $\varphi_2$  of each cavity individually, we can provide both longitudinal and transverse focusing and decrease the focusing field. For accelerator with SC solenoids this method was developed and studied both analytically [3] and with the help of the three-dimensional ray tracing code TRACK [4]. It was shown that a combined focusing structure with solenoid and APF has obvious advantages compared to the reference design and can significantly reduce the cost of the linac. The similar result can be found for SC linac with electric quadrupoles. The value of vane voltage decreases if APF is used. The dependence of the vane voltage from beam velocity is illustrated in Fig.7. The variant which corresponds to case  $\varphi_1 = -\varphi_2$  has smaller voltage, but longitudinal stability and phase advance per focusing period  $\mu_r$  abruptly decrease in this case. The longitudinal acceptance can be increased only with asymmetrically choice effective phases  $|\varphi_1| > |\varphi_2|$ . In every case additional studies of longitudinal beam dynamics are necessary.



Figure 7: The voltage of the electric quadrupoles as a function of the beam velocity for two envelops  $X_{\rm m}$ . The solid lines correspond to case  $\varphi_1 = -\varphi_2 = -20^\circ$ ; the dash lines correspond to case  $\varphi_1 = -30^\circ$ ,  $\varphi_2 = 20^\circ$ .

## CONCLUSION

The various focusing methods are compared for low ion velocities. The focusing fields of SC solenoid lenses, electric or magnetic quadruples are founded. The low charge state beams require stronger transverse focusing. In order to decrease focusing fields the alternative focusing method must be used. This method is based on combination of low focusing fields and alternating phase focusing (APF). The choice of adjusting phases  $\phi_1$  and  $\phi_2$  of each cavity must be provided both longitudinal and transverse focusing.

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