ALTERNATING WAKE FORCE IN RECTANGULAR WAVEGUIDE WITH PERIODIC PERTURBED WALLS

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Abstract

The wake field excited by a relativistic charged bunch in a periodic waveguide can be expressed as a spatial harmonic Floquet series. Usually spatial harmonics of the wake force synchronous with the bunch are of interest due to their action on particles that results in the well known beam loading and beam break up effects in rf structures. However, an alternating transverse wake force which consists of nonsynchronous harmonics can give rise to undulating the particles with alternating transverse velocity that can result in no less important effect such as the wake field undulator radiation [1]. Therefore there exist an interest in developing of methods of calculation of alternating wake forces in a periodic rf structures. In this work a perturbation method for calculation of wake field in rectangular periodic waveguides was considered. A possible usage of the wake field excited by an electron bunch passing through a sub-millimeter planar periodic waveguide for both ultra-high gradient acceleration and generation of the hard wake field undulator radiation is discussed.

INTRODUCTION

The wake field (WF) induced by a relativistic charge particle bunch in a periodic corrugated waveguide and the corresponding wake force can be expanded into Floquet series in spatial harmonics. The spatial harmonics synchronous with the bunch usually are of interest due to their constant action on the particles that results in the well-known beam loading and beam breakup instability effects in the rf structures.

However, the alternating transverse wake force which consists of the nonsynchronous harmonics can give rise to undulating off–axis particles that should result in no less important phenomena such as the wake field undulator radiation (WFUR) [1,2], and the pondermotive focusing. Therefore there exists an interest in developing the such rf structures in which the alternating transverse wake force would be appreciably more than the longitudinal synchronous wake force. These properties are inherent for the rf waveguides with periodic perturbed walls. Just for such periodic perturbed axially symmetrical waveguides the authors [3] have developed a perturbation method for calculating wake fields. As follows from this method amplitudes of nonsynchronous harmonics of the wake field turn out to be of the first order of smallness whereas synchronous ones appear in the second order of smallness. However, the round structures with sub-millimeter sizes, which need for generation of the hard WFUR [2], are difficult to construct and very expensive. So, in connection with development of the micromechanic technology of fabrication of planar sub-millimeter structures, the analytical methods on calculating wake fields in the rectangular periodic waveguides are a relevant topic. In Ref. [4] the approach [3] has been generalized and expanded on the rectangular waveguides. The goal of this paper consists in calculation of alternating wake forces by the method [4] in a rectangular waveguide with periodic perturbed walls, and study a possible usage of the wake fields excited by an electron bunch passing through a sub-millimeter planar periodic waveguide for generation of the hard wake field undulator radiation is discussed.

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CALCULATION OF WAKE FORCES

Let a bunch of N ultrarelativistic electrons with Gaussian distribution

$$\rho = \frac{eN}{\sigma_x \sigma_y \sigma_z \left(\sqrt{2\pi}\right)^3} \exp\left(-\frac{\mathrm{v}^2 (t - z/\mathrm{v})^2}{2\sigma_z^2} - \frac{(x - x_0)^2}{2\sigma_x^2} - \frac{(y - y_0)^2}{2\sigma_y^2} \right) \quad (1)$$

moves along a planar waveguide with weakly-corrugated metallic both upper and lower surfaces. Here e is the electron charge; x, y are transverse coordinates; z is a longitudinal coordinate; x_0 and y_0 are the transverse coordinates of the charge center; v is velocity of the electrons; t is a time; σ_x, σ_y, σ_z are the root-mean-square bunch dimensions. The wake field and the corresponding wake force have to be found. We will apply the method of Green's functions, and so, firstly find the wake fields excited by a point bunch, the current density of which may be written as

$$j_x = 0,\ \ j_y = 0,\ \ j_z = eN\delta(x - x_0)\,\delta(y - y_0)\,\delta(t - z/\mathrm{v}), \quad (2)$$

Consider a planar periodic waveguide sketched in Fig.1.

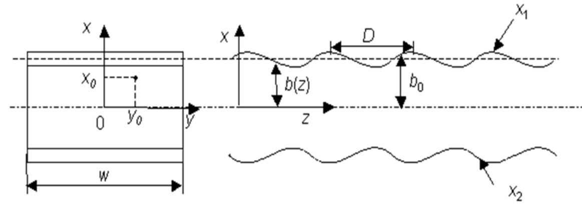

Figure 1: A rectangular waveguide with the plane *yz*; $x_{1,2}=\pm b(z)$ are the surface contours, D is the period.

The periodic shape of the corrugated surface can be represented by a Fourier series expansion

$$b(z) = b_0 \left[1 + \varepsilon \sum_{p=-\infty}^{\infty} C_p e^{i\frac{2\pi p}{D} z} \right], \quad C_0 = 0, \quad (3)$$

where ε is a small parameter ($0<\varepsilon<<1$), b_0 is the average half-distance between the upper and lower surfaces. Expanding the electromagnetic fields in the Fourier's integral over frequencies ω and Floquet series in spatial harmonics, the spatial harmonics of the transverse wake force may be found from the Maxwell equations as:

$$F_{\omega,p,x} = \frac{ie}{\alpha_p^2}\left\{\left[\frac{2\pi p}{D} - \frac{\omega}{\mathrm{v}\gamma^2}\right]\frac{\partial E_{\omega,p,z}}{\partial x} - \frac{2\pi p}{D}\frac{\mathrm{v}}{c}\frac{\partial H_{\omega,p,z}}{\partial y}\right\}, \quad (4)$$

$$F_{\omega,p,y} = \frac{ie}{\alpha_p^2}\left\{\left[\frac{2\pi p}{D} - \frac{\omega}{\mathrm{v}\gamma^2}\right]\frac{\partial E_{\omega,p,z}}{\partial y} + \frac{2\pi p}{D}\frac{\mathrm{v}}{c}\frac{\partial H_{\omega,p,z}}{\partial x}\right\}, \quad (5)$$

where we have introduced the following definitions: $\gamma = 1\Big/\sqrt{1-(\mathrm{v}/c)^2}$; $k = \omega/c$; $k_p = \frac{2\pi p}{D} - \frac{\omega}{\mathrm{v}}$; $\alpha_p^2 \equiv k^2 - k_p^2$, c is the velocity of light. The harmonics of the longitudinal field components satisfy the wave equations

$$\Delta_\perp E_{\omega,p,z} + \alpha_p^2 E_{\omega,p,z} = \frac{4\pi i}{\mathrm{v}}\left(\frac{2\pi p}{D} - \frac{\omega}{\mathrm{v}\gamma^2}\right) j_{\omega,p,z}, \quad (6)$$

$$\Delta_\perp H_{\omega,p,z} + \alpha_p^2 H_{\omega,p,z} = 0, \quad (7)$$

where $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$; $j_{\omega p,z}$ is a current density spatial harmonic of the point bunch Eq.(1)

$$j_{\omega,p,z} = \frac{eN}{2\pi}\delta(x-x_0)\delta(y-y_0)\delta_{0,p}. \quad (8)$$

Here $\delta_{0,\mathrm{p}}$ is the Kronecher's symbol.

The Eqs. (6), (7) should be complemented with the boundary conditions for conducting walls:

$$E_z\left(x, y = \pm\frac{w}{2}, z\right) = H_y\left(x, y = \pm\frac{w}{2}, z\right) = 0. \quad (9)$$

A tangential component of electric field E_τ and a normal component of magnetic field H_n are expressed in terms of E_x, E_z i H_x, H_z, accordingly:

$$\pm E_x(x_{1,2}, y, z)\sin\alpha + E_z(x_{1,2}, y, z)\cos\alpha = 0, \quad (11)$$

$$\pm H_x(x_{1,2}, y, z)\cos\alpha - H_z(x_{1,2}, y, z)\sin\alpha = 0, \quad (12)$$

where $\mathrm{tg}\alpha = db(z)/dz$.

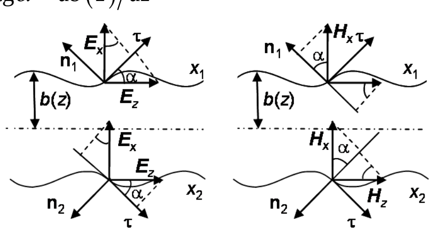

Figure 2: The boundary conditions.

Expanding the fields on the surfaces $x_{1,2}$ in terms of a Taylor's series close to the imaginary planes $x=\pm b_0$ (see Figs.1) and taking into account Eq.(2), we get new boundary conditions on the planes $x=\pm b_0$ (see [4]). Then we will solve Eqs.(6, 7) by a successive approximation method expanding the fields in terms of the powers of ε:

$$E_{\omega,p,z} = E_{\omega,p,z}^{(0)} + E_{\omega,p,z}^{(1)} + E_{\omega,p,z}^{(2)} + ...,$$
$$H_{\omega,p,z} = H_{\omega,p,z}^{(0)} + H_{\omega,p,z}^{(1)} + H_{\omega,p,z}^{(2)} + ..., \quad (13)$$

where $E_{\omega,p,z}^{(n)}, H_{\omega,p,z}^{(n)} \propto \varepsilon^n$.

Omitting suitable calculations, recovering the time dependence of the fields by the inverse Fourier transform, and using the wake forces for a point charge as Green's functions, we obtain the wake force. The p^{th} spatial harmonic (where $p\neq 0$) of the transverse components is of the first order of ε

$$F_{p,x}^{(1)}(x,y,\tau) = \varepsilon p C_p \frac{4\pi^2 e^2 N}{wD} e^{-\frac{(\mathrm{v}\tau)^2}{2\sigma_z^2}} \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} W\left(\frac{\omega_{m,p,n}\sigma_z}{\mathrm{v}\sqrt{2}} - i\frac{\mathrm{v}\tau}{\sqrt{2}\sigma}\right)$$
$$\times\frac{(-1)^n}{1+\delta_{0,n}} e^{-\frac{(\pi m)^2(\sigma_y{}^2+\sigma_x{}^2)}{2w^2}} G_{n,m}(x_0, y_0, x, y), \quad (14)$$

$$F_{p,y}^{(1)}(x,y,\tau) = -i\varepsilon\frac{C_p}{p}\frac{e^2 ND}{\gamma^2 w} e^{-\frac{(\mathrm{v}\tau)^2}{2\sigma_z^2}} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} W\left(\frac{\omega_{m,p,n}\sigma_z}{\mathrm{v}\sqrt{2}} - i\frac{\mathrm{v}\tau}{\sqrt{2}\sigma}\right)$$
$$\times(-1)^n e^{-\frac{(\pi m)^2(\sigma_y{}^2+\sigma_x{}^2)}{2w^2}} \frac{\partial^2}{\partial x\partial y} G_{n,m}(x_0, y_0, x, y), \quad (15)$$

where $\tau = t - z/\mathrm{v}$, $\omega_{m,p,n}$ are the eigenfrequencies

$$\omega_{m,p,n} = \frac{\mathrm{v}D}{4\pi p}\left[\left(\frac{\pi m}{w}\right)^2 + \left(\frac{2\pi p}{D}\right)^2 + \left(\frac{\pi n}{2b_0}\right)^2\right],$$

$W(z) = e^{-z^2}\mathrm{erfc}(-iz)$, and

$$G_{n,m}(x_0,y_0,x,y) = \frac{\sin\left[\frac{\pi m}{2} + \frac{\pi m y_0}{w}\right]}{sh\left[\frac{2\pi m}{w}b_0\right]} \sin\left[\frac{\pi m}{w}\left(y + \frac{w}{2}\right)\right]$$
$$\times\left\{sh\left(\frac{\pi m(x_0+b_0)}{w}\right)\cos\left(\frac{\pi n}{2b_0}(x+b_0)\right) + sh\left(\frac{\pi m(x_0-b_0)}{w}\right)\cos\left(\frac{\pi n}{2b_0}(x-b_0)\right)\right\},$$

It should be noted that there is essential anisotropy of transverse components of the alternative wake forces. Thus, the y-component $F_{p,y} \sim 1/\gamma^2$ is strongly depressed comparatively with $F_{p,x}$., and can be neglected.

Electron energy losses connected with wakfield excitation are defined the synchronous harmonic (p=0) of longitudinal component of electric field which appears in the series (13) in the second order of smallness

$$F_{0,z}^{(2)}(x,\tau) = -\varepsilon^2\frac{2\pi^2 e^2 N b_0}{wD\mathrm{v}} e^{-\frac{(\mathrm{v}\tau)^2}{2\sigma_z^2}} \sum_{q=1}^{\infty} q\left|2C_q\right|^2 \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^n\omega_{m,q,n}}{1+\delta_{0,n}}$$
$$\times e^{-\frac{(\pi m)^2(\sigma_y{}^2+\sigma_x{}^2)}{2w^2}} \mathrm{Re}\left\{W\left(\frac{\omega_{m,q,n}\sigma_z}{\mathrm{v}\sqrt{2}} - i\frac{\mathrm{v}\tau}{\sqrt{2}\sigma}\right)\right\}$$
$$\times\frac{\sin\left[\frac{\pi m}{2} + \frac{\pi m y_0}{w}\right]}{sh^2\left(\frac{2\pi m b_0}{w}\right)} \sin\left[\frac{\pi m}{w}\left(y + \frac{w}{2}\right)\right]$$
$$\times\left\{\left\{(-1)^n sh\left(\frac{\pi m}{w}(x_0+b_0)\right) + sh\left(\frac{\pi m}{w}(x_0-b_0)\right)\right\} sh\left(\frac{\pi m}{w}(x+b_0)\right)\right.$$
$$\left. + \left\{(-1)^n sh\left(\frac{\pi m}{w}(x_0-b_0)\right) + sh\left(\frac{\pi m}{w}(x_0+b_0)\right)\right\} sh\left(\frac{\pi m}{w}(x-b_0)\right)\right\} \quad (16)$$

WAKE FORCE CHARACTERISTICS

Let us consider the electron bunch with $Q=eN=1$ nC which moves along the planar periodic waveguide with the surface contours $b(z)=b_0[1+\varepsilon\cos(2\pi z/D)]$ (where $b_0 = D$, $w = 10b_0$, $\varepsilon = 0.1$) as it is represented in Fig.3.

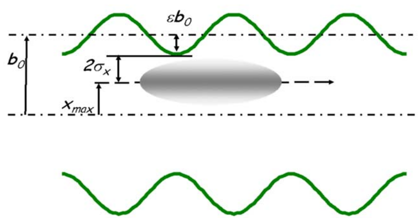

Figure 3: The position of an electron bunch in the waveguide.

As it has been shown in Ref. [4] the distribution of wake field in a cross-section has shape of a surface wave. So, to obtain high alternating wake force as well as high gradient acceleration the bunch should travel off axis of the *sub-millimeter* waveguide at the maximally possible distance x_{max} from the axis

$$x_{\max}/b_0 = 1 - \varepsilon - 2\sigma_x/b_0 \quad (17)$$

The second requirement for generating the wake field undulator radiation consists in forming the maximum of alternating transverse wake force at the pick of charge density. The needed wake field distribution along the bunch can be obtained in a case of a long bunch, $D \le \sigma_z$. In Fig.4 it is presented the distribution of the wake force spatial harmonics ($F_{1,x}$, and $F_{0,z}$) along the bunch with $\sigma_z = D = 100$ μm, $\sigma_x = \sigma_y = 10$ μm at $x_0/b_0 = 0.7$, $y_0 = 0$.

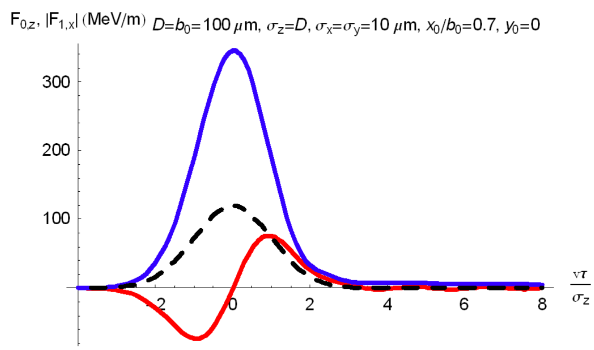

Figure 4: The WF distribution along the bunch. The lines: red is $F_{0,z}$, blue is-$|F_{1,x}|$, dash is the charge density.

From Fig.4 we can see that the wake fields localize within the bunch. The first half-bunch loses energy in the decelerating longitudinal field while the second one adsorbs it fully without storing the radiation in the waveguide after the bunch. The absolute value of the transverse component of the alternating wake force $F_{1,x}$ reaches the maximum at the charge density maximum where the longitudinal component of the synchronous harmonic of the electric field $F_{0,z}$ changes its sign. The wake field "spot" moves with the bunch velocity. In the absence of storing wake fields in the waveguide, and due to the femtosecond range duration of the wake field "spot" $\delta\tau \sim \sigma_z/v$, the breakdown field of metallic surface should be very high.

In the next Fig.5 it is demonstrated what magnitudes of wake fields may be reached by scaling down the beam sizes to the frontier values, $\sigma_z = D = 20$ μm, $\sigma_x = \sigma_y = 1$ μm, at $x_0/b_0 = 0.8$ which satisfy Eq.(17). We can see that $|F_{0,z}| \le 3$GeV/m and $|F_{1,x}| \le 20$GeV/m for Q=1nC.

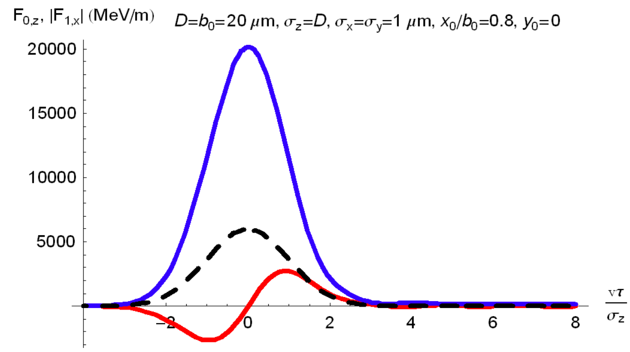

Figure 5: The WF distribution along the bunch. The lines: red is $F_{0,z}$, blue is-$|F_{1,x}|$, dash is the charge density.

The Fig.6 shows what equivalent undulator magnetic field corresponds the first harmonic of the alternating transverse wake force $|F_{1,x}|$.

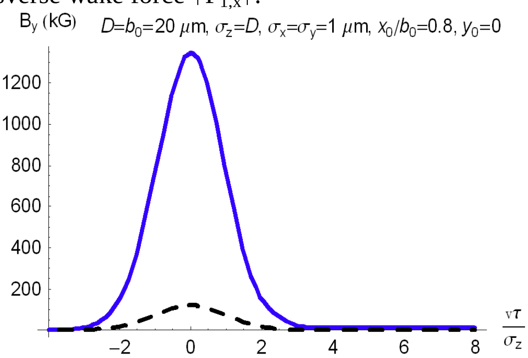

Figure 6: The equivalent undulator magnetic field.

SUMMARY

By perturbation method we calculated wake forces induced by an ultrarelativistic electron bunch in rectangular waveguides with periodically perturbed walls, and obtained the following main results:

- The transverse components of the alternating wake force in a planar waveguide are strongly anisotropic.
- For a long bunch the wake field localizes within the bunch without storing the radiation into the waveguide.
- The transverse components of the alternating wake force are maximal at the maximum of the charge density.
- The wake field excited by an electron bunch passing through the sub-millimeter planar periodic waveguide may be used for both a high gradient acceleration and generation of the wake field undulator radiation.

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