SIMULATION STUDY OF STABLE AND MOVING BARRIER BUCKETS USING BETACOOL CODE

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Abstract

A moving barrier RF bucket is an effective ion beam accumulation method used, for instance, in Fermilab's Recycler and proposed for NESR at FAIR project. A possible application of a stationary RF bucket is to compensate an ionization energy loss in experiments with an internal target. The ionization energy loss is the main physical effect limiting the experiment duration. The barrier bucket application permits to sufficiently decrease of a required power of a cooling system when a high resolution in an experiment is necessary[1].

Recently a new program was developed for barrier RF bucket simulation for FAIR rings [2]. To compare predictions of different models and to estimate efficiency of the barrier bucket application in internal target experiments the new algorithms were implemented into Betacool program [3] also.

INTRODUCTION

The certain model of ion synchrotron motion in a stationary square wave barrier bucket was implemented into both main algorithms of the Betacool program: RMS Dynamics and Model Beam algorithm[4]. In the frame of RMS Dynamics the calculation of the cooling and heating rates was modified in accordance with the analytic expression for the ion phase trajectory in the longitudinal phase plane. In the Model Beam algorithm the generation of the matched stationary particle array and the simulation of the synchrotron motion were developed.

The principle of the proposed accumulation method is as follows. The ion beam bunch is injected in the longitudinal gap prepared by two moving barrier pulses. The injected beam becomes coasting after switching off the barrier voltages. After the momentum spread is well cooled by electron cooling, the barrier voltages are switched on and moved away from each other to prepare the empty space for the next beam injection. This process is repeated to attain the required intensity.

The general goal of the Betacool program is to simulate long term processes (in comparison with the ion revolution period) leading to the variation of the ion distribution function in six dimensional phase space. Therefore the Betacool is not a tracking code, and simulation of transverse and longitudinal ion motion is based on analytical expressions for the phase trajectories. The investigation of the beam dynamics at arbitrary shape of the distribution is performed using multi particle simulation in the frame of the Model Beam algorithm. In this algorithm the ion beam is represented by an array of model particles. The heating and cooling processes involved into the simulations lead to a change of the particle momentum components and particle number.

RF voltage and particle trajectories in the longitudinal phase space at square wave barrier-bucket

General behaviours of the synchrotron motion are determined by integrated RF pulse strength, and essential physics is independent on the exact shape of the barrier RF wave. Simplest analytical solution for the phase trajectory can be obtained at square wave stable barrier bucket; therefore this model was implemented into the program at the first step. Also it was developed the algorithm for synchrotron motion simulation moving barrier bucket. Arbitrary RF shape can be set up by series of rectangular barriers (Fig.1).

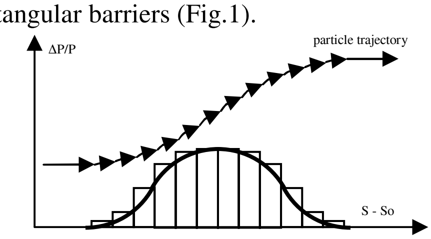

Figure 1: Sinusoidal shape of RF.

The RF voltage time dependence at the square wave barrier bucket can be written as

$$V(t)=\begin{cases} sign(\eta)V_0 \quad if \quad -(T_2+T_1)/2 \le t \le -(T_2-T_1)/2 \\ -sign(\eta)V_0 \quad if \quad (T_2-T_1)/2 \le t \le (T_2+T_1)/2 \\ 0 \quad otherwise \end{cases}$$

(1)

where V0 is the voltage height, T1 is the pulse width, T2 is the gap duration, η is the ring off-momentum factor (Fig.2).

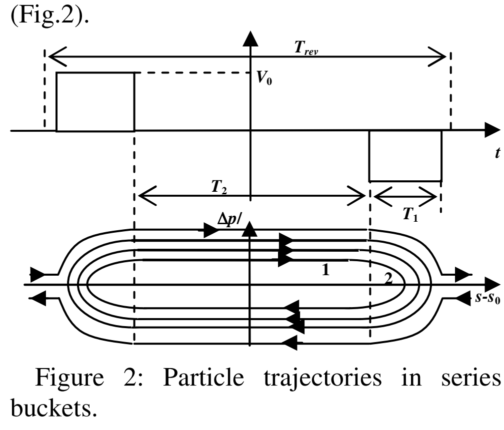

Figure 2: Particle trajectories in series of barrier buckets.

The equations of the synchrotron motion of the ion at charge eZ written in the variables (s-s0, $\delta = \Delta p / p$) are

$$\begin{cases} \dfrac{d(s-s_0)}{dt} = |\eta|\beta c\delta_A \\ \dfrac{d\delta}{dt} = 0 \end{cases} \quad (2)$$

inside the gap of the RF voltage, and

$$\begin{cases} \dfrac{d(s-s_0)}{dt} = |\eta|\beta c\delta \\ \dfrac{d\delta}{dt} = -\dfrac{ZeV_0}{Cp_0} \end{cases} \quad (3)$$

when the particle crosses the cavity during the voltage pulse. Here δA is the amplitude of the momentum deviation; C is the ring circumference, p0 – the synchronous momentum.

From a solution of the motion equations one can obtain main parameters of the bucket. The maximum momentum deviation (the barrier height) can be calculated from the following equation

$$\delta_{A,\max}^2 = \frac{2T_1}{T_0}\frac{ZeV_0}{\beta cp_0|\eta|}, \quad (4)$$

where T_0 is the revolution period. Inside the bucket the period of the synchrotron oscillations is equal to

$$T_s = \frac{2T_2}{|\eta|\delta_A} + \frac{4Cp_0\delta_A}{ZeV_0}, \quad (5)$$

and the trajectory length is given by

$$S_b = T_2\beta c + \frac{|\eta|\beta cCp_0\delta_A^2}{ZeV_0}. \quad (6)$$

The rms momentum deviation σ_δ relates to the amplitude as follows

$$\sigma_\delta^2 = \frac{(2/3)|\eta|Cp_0\delta_A^4 + ZeV_0T_2\delta_A^2}{2|\eta|Cp_0\delta_A^2 + ZeV_0T_2}. \quad (7)$$

Rate calculation

Modifications in the rms dynamics algorithm are related mainly to the calculation of the IBS heating and electron cooling rates. At the simulation of an interaction with an internal target the mean energy loss is ignored and the characteristic growth time of the momentum spread is calculated using expression for energy loss fluctuations.

The IBS process in the first approximation can be simulated using a formulae for the coasting beam at the substitution of the bucket length instead the ring circumference. The bucket length is calculated in the following steps. For the given rms momentum spread the amplitude of the rms particle oscillations is to be found from the equation (7). Introducing

$$\xi^2 = \frac{ZeV_0T_2}{2|\eta|Cp_0} \quad (8)$$

one can obtain

$$\delta_A^2 = \frac{\sqrt{9(\xi^2-\sigma_\delta^2)^2 + 12\xi^2\sigma_\delta^2} - 3(\xi^2-\sigma_\delta^2)}{2}. \quad (9)$$

For this amplitude the bucket length is calculated in accordance with (6).

The cooling rate calculation is based on an averaging of the friction force acting on the "rms ion" in the cooling section over phases of the betatron and synchrotron oscillations [7]. To realize this procedure one needs to recalculate the ion co-ordinates (s-s_0, δ), obtained at the exit of the cooling section, into its rms momentum deviation. For the particle at a given longitudinal co-ordinates (s-s_0,δ) the amplitude of oscillations is calculated in accordance with the phase trajectory equation. The result is obvious, when (s-s_0) lies in the gap between RF waves: δA =δ. Inside the wave the amplitude is equal to

$$\delta_A^2 = \delta^2 + \frac{2ZeV_0}{|\eta|\beta cCp_0}(s-s_0), \quad (10)$$

where (s-s0) is measured from the beginning of the wave. The corresponding rms momentum deviation is given by:

$$\langle\delta^2\rangle = \frac{\delta_A^4 + 3\xi^2\delta_A^2}{3(\delta_A^2+\xi^2)}, \quad (11)$$

where ξ is determined by (5).

Simulation of longitudinal motion

The Model Beam algorithm realizes a Monte-Carlo method for a solution of a Langevin equation based on an assumption, that the integration step over time is sufficiently longer than a decoherence period (a few millions of revolutions). In this case the phase advance of the betatron and synchrotron oscillations during the integration step is an arbitrary number. In the transverse phase space the betatron motion is simulated using linear transformation maps.

In the case of a barrier bucket application the period of synchrotron oscillation is determined by the particle momentum deviation. At a low momentum spread it can be of the order of a few seconds. Correspondingly, the decoherence time can be compared or even longer than the step of the integration over time. To take into account this peculiarity of the synchrotron motion two algorithms were developed: at random and at regular phase advances during the integration step.

At the random phase advance the amplitude of the particle momentum deviation is calculated from its actual momentum and longitudinal co-ordinate using formulae (9) and (10). Thereafter the phase of synchrotron oscillations is generated uniformly between 0 and 2π and the new co-ordinate and the momentum are calculated as functions of the phase.

At the regular phase advance the phase of the synchrotron oscillations of the particle is calculated from its co-ordinate and momentum deviation. The phase advance for each particle is calculated from the period of its synchrotron oscillations as $\Delta\varphi = 2\pi\cdot\Delta t/T_s$, where Δt is the integration step. New particle co-ordinates are obtained from the phase trajectory equation.

Simulation with stable barrier buckets

Possibilities of the new algorithms can be illustrated by example of the simulation in NESR of FAIR project (GSI). One of the main goals of barrier buckets application at the NESR is to reach the high intensity of RIBs required by the internal experiments in the NESR [5, 6] and in particular by the electron-ion collider [7], it is planned to stack the RIBs longitudinally at injection energy i.e. in the range 100-740 MeV/u [8]. The stacking will be supported by electron cooling. A stacking cycle time, i.e. the time between 2 successive injections, below 2 s would be optimal because of the short RIB lifetimes and in order to profit from the planned cycle time of 1.5 s of SIS100, where the primary heavy ion beam is accelerated. In this frame, two options of longitudinal beam accumulation have been investigated by beam dynamics simulations and by experiments in the existing ESR at GSI.

The initial distribution of the particle is generated in the accordance with the analytical model with two barriers or can be read from an external file. The numerical stationary model of the barrier bucket can include any number of barriers with any integration step in BETACOOL. The position and voltage stay constant during the simulation.

Table 1. Parameters of barrier RF bucket used in the simulations.

Pulse width, T_1/T_0		0.1
Gap duration, T_2/T_0		0.6
Voltage amplitude	V	200
The barrier height, $\delta_{A,max}$		$8\cdot10^{-4}$
Synchrotron period at $\delta = 2\cdot10^{-5}$	s	3.93

The initial momentum spread ($dp/p=1.5\cdot10^{-4}$) is chosen in the correspondence with barrier height when all particles are circulated between barriers (Fig.3).

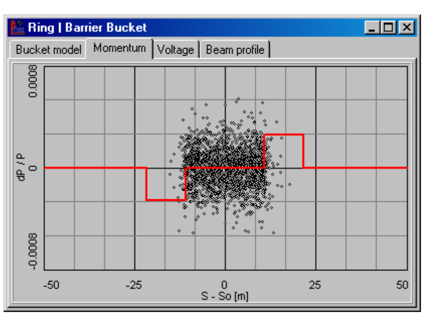

Figure 3:initial distribution of particles in longitudinal phase space, red line corresponds to width and height of bucket.

Simulation is going under action of electron cooling and IBS effects. Particles start motion according phase trajectories at (Fig.4). Momentum spread of particles is decreased by electron cooling.

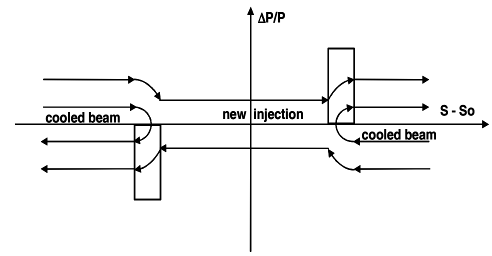

Figure 4:Particle trajectories in series of barrier buckets.

Particles with low momentum spread cannot cross RF barriers and collect outside its (Fig.5). Step by step particles leave injection region.

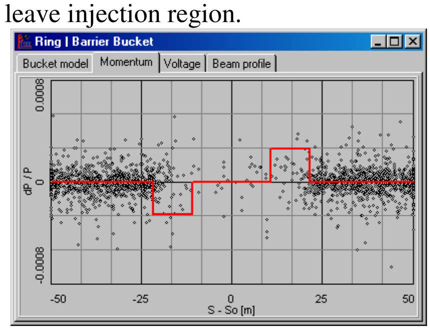

Figure 5: The final distribution of the particle

Finally, when equilibrium between cooling and heating rates is reached, all particles collect in RF gap with low momentum spread and injection region is kept free. That scheme allows to use space between barrier buckets for injection of new part of particles.

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