CORRECTING THE ROUND BEAM LATTICE OF VEPP-2000 COLLIDER USING ORBIT RESPONSE TECHNIQUE

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Abstract

Round colliding beams option in VEPP-2000 [1] puts a number of strict requirements on the collider lattice. The orbit Response Matrix (ORM) technique is a versatile tool for lattice analysis and correction. For linear optical function study and correction, the orbit response to the dipole correctors is collected and processed, while for the orbit correction the quadrupole trimming is used. Theoretical and experimental responses of closed orbit to the same perturbations are compared to determine the most probable deviations of chosen parameters from its design values.

ORBIT CORRECTION

Introduction

Commonly, the perfect unperturbed orbit passes all magnet multipoles through their magnetic centers, so it is useful to know offsets of the real orbit. There are no calibrated absolute beam position monitors at VEPP-2000, so to determine offsets of closed orbit the method is used that allows to measure orbit displacement in lenses using relatively small number of beam position monitors, that can precisely measure orbit shifts.

The method is based on the fact that in case of displaced crossing of multipole magnet variation of the multipole field will change the closed orbit, because of additional dipole field that appears on the particles way.

Orbit Offset Measurement

It easy to show that only the qudrupole has different field in different points. Dipole has uniform field in all space, and $2n$-pole has identical field on the same radii in points that have $\Delta\theta = 2\pi m/(n-1)$ difference in polar angel.

If a particle has an offset in the quadrupole lens $\delta\vec{l} = (\delta x, \delta y)$, then changing of gradient in this lens by δG will shift the closed orbit the same way as a dipole corrector with field $\delta\vec{H} = (\delta x \delta G, \delta y \delta G)$.

So by changing gradients in the lenses one by one and measuring the orbit shifts on BPMs one can construct the response vectors $\delta\vec{X}_{\exp,i}$.

If the structure of accelerator is known then respective theoretical response vectors $\delta\vec{X}_{\mathrm{mod},i}$ for dipole correctors in tested lenses can be calculated. To find absolute shifts $\vec{X}_{\mathrm{err},i}$ of lenses relative to the ideal closed orbit, one should minimize the functional:

$$F(\lambda_i) = (\vec{\mathcal{X}}_{\mathrm{mod},in}\lambda_i - \vec{\mathcal{X}}_{\exp,in})^2 \to \min \quad (1)$$

Here $\vec{\mathcal{X}}_{\mathrm{mod},in}$ and $\vec{\mathcal{X}}_{\exp,in}$ are measured and modeled response vectors normalized by the measurement precision σ_{in}

$$\begin{aligned}
\vec{\mathcal{X}}_{\mathrm{mod},in} &= \left\{ \frac{\delta x_{\mathrm{mod},i1}}{\sigma_{i1}}, \ldots, \frac{\delta x_{\mathrm{mod},iN}}{\sigma_{iN}} \right\} \\
\vec{\mathcal{X}}_{\exp,in} &= \left\{ \frac{\delta x_{\exp,i1}}{\sigma_{i1}}, \ldots, \frac{\delta x_{\exp,iN}}{\sigma_{iN}} \right\}
\end{aligned} \quad (2)$$

The functional (1) has a minimum if:

$$\lambda_{\min,i} = \frac{(\vec{\mathcal{X}}_{\mathrm{mod},i} \cdot \vec{\mathcal{X}}_{\exp,i})}{\vec{\mathcal{X}}_{\mathrm{mod},i}^2} \quad (3)$$

Now the absolute coordinates of the beam in the lenses can be obtained from the following formulas:

$$\delta x_{\mathrm{err},i} = \frac{\delta H_{y,i}\lambda_{\min,i}}{\delta G_i}, \quad \delta y_{\mathrm{err},i} = \frac{\delta H_{x,i}\lambda_{\min,i}}{\delta G_i} \quad (4)$$

Accuracy

To measure the accuracy of obtained displacements one can use functional (1). If the minimal value of this functional is $F_{\min,i} = F(\lambda_{\min,i}) = (\vec{\mathcal{X}}_{\exp,i}^2 - (\vec{\mathcal{X}}_{\mathrm{mod},i} \cdot \vec{\mathcal{X}}_{\exp,i}))$, then let the accuracy $\delta\lambda_i$ of $\lambda_{\min,i}$ be defined by condition $F(\lambda_{\min,i} \pm \delta\lambda_i) = 2F_{\min,i}$. Then:

$$\delta\lambda_i = \frac{\vec{\mathcal{X}}_{\exp,i}^2 - (\vec{\mathcal{X}}_{\mathrm{mod},i} \cdot \vec{\mathcal{X}}_{\exp,i})}{\vec{\mathcal{X}}_{\mathrm{mod},i}^2} \quad (5)$$

Errors in determined orbit can be obtained by combining (4) and (5).

Orbit Correction

To correct obtained displacements of the closed orbit one should calculate the response matrix M that contains responses of the closed orbit in lenses for each corrector. Here is a tricky place of this method, because there is no "coordinate" of the closed orbit in a thick lens. One way to solve this problem is to use coordinate in the lens' center, but a more convenient method is to use "virtual" BPMs placed in centers of lenses that measure coordinate averaged across lens' length.

Next equation contains the necessary correctors' strengths $\delta\vec{I}_{corr}$:

$$\vec{X}_{\mathrm{err}} = M\delta\vec{I}_{\mathrm{corr}} \quad (6)$$

Commonly, matrix M is not square so to get the best set of correctors values, SVD inversion technique is used. The

SVD technique allows controlling of the correction precision by choosing the number of truncated singular values:

$$\delta \vec{I}_{\text{corr}} = (M)^{-1}_{SVD} \vec{X}_{\text{err}} \qquad (7)$$

Uncertainties in the theoretical model cause the errors in correction of the orbit, so several iterations are commonly needed to get the corrected orbit.

Orbit Correction Results

Figure 1 shows improvement of the closed orbit position relative to the magnet centers of quadrupoles.
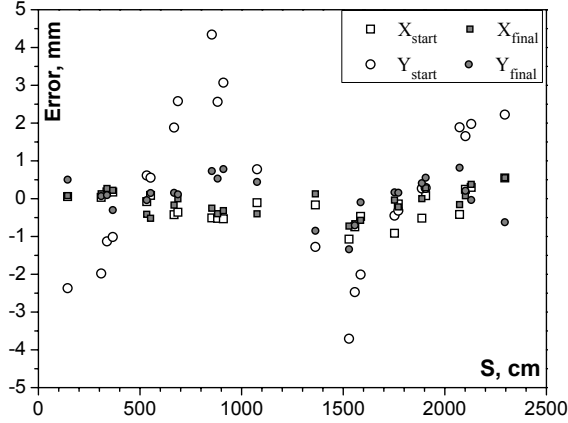

Figure 1: Example of orbit correction.

In some cases of large physical displacement of a certain lens the orbit can not be corrected only by described method, and repositioning of the identified source of problems must be performed.

LATTICE CORRECTION

Introduction

One of the main problems during commissioning and running of circular accelerator is determining and elimination of errors of optical parameters in the real lattice. To correct the lattice of VEPP-2000 the program was written that implements algorithms discussed in [2, 3, 4, 5]. The main idea of the correction method is to minimize χ^2, by varying a set of parameters:

$$\chi^2 = \sum_{i,j} \frac{(M_{\text{mod},ij} - M_{\text{mes},ij})^2}{\sigma_{ij}^2} = \sum_{i,j} V^2_{k(i,j)} \qquad (8)$$

where $M_{\text{exp},ij}$ and $M_{\text{mod},ij}$ – experimental and theoretical closed orbit responses on variation of j-th corrector at i-th BPM; σ_{ij} – precision of corresponding measurement.

The main feature of written code is usage of 6-d formalism for calculation of theoretical responses on dipole correctors. In this formalism vector $X^t = (x, p_x/p_0, y, p_y/p_0, c\Delta t, \Delta p/p_0)$ is used for particle displacements and momenta.

6-D Dipole Corrector

To calculate the closed orbit response for a dipole corrector, let's consider lattice element A with a steering magnet. If the coordinate of particle at the entrance point is X_{in}, then at the exit point it can be written as:

$$X_{\text{out}} = M_A X_{\text{in}} + \delta, \ \delta^t = (\delta x, \delta x', \delta y, \delta y', \delta L, 0) \qquad (9)$$

were δ describes the steering effect from the corrector; δL is the path elongation in element A for the particle with $X_{\text{in}} = 0$. Hence, using one turn matrix M_{turn}, $Det(I - M_{\text{turn}}) \neq 0$, one can easily calculate the closed orbit parameters at the exit point of element A:

$$X_{\text{exit}A} = (I - M_{\text{turn}})^{-1}\delta \qquad (10)$$

Since the fifth component of X reflects particle's retarding, then the condition of synchronism with RF accelerating field is automatically satisfied.

As was mentioned above the formula (10) works if $Det(I - M_{\text{turn}}) \neq 0$, but even if there is no RF accelerator cavity in the structure, Eq. (9) gives some usefull information, for example ΔE, the energy shift of the synchronous particle caused by steering dipole.

Let's consider a short dipole corrector (that, according to (9) can describe a long one) in a lattice with no coupling between transverse degrees of freedom (fig. 2). In further discussion notations will be used: β, α, γ are the Twiss parameters; η is the periodic dispersion; M is one turn matrix; V is the coordinate vector of the perturbed closed orbit; δ is the vector that describes steering magnet; M_β, V_β, δ_β are mentioned values presented in the betatron frame; T_β is the transformation matrix between common and betatron frames.
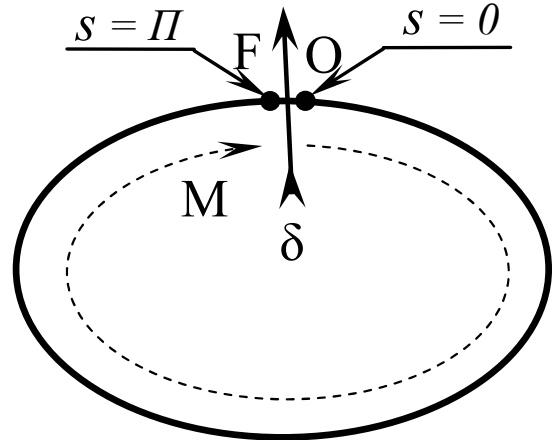

Figure 2: Structure with corrector

$$M_\beta = \begin{pmatrix} M_{x\beta} & & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & 1 & -\alpha\Pi \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (11)$$

$$M_{x\beta} = \begin{pmatrix} \cos\phi + \alpha\sin\phi & \beta\sin\phi \\ -\gamma\sin\phi & \cos\phi - \alpha\sin\phi \end{pmatrix} \quad (12)$$

$$T_\beta = \begin{pmatrix} 1 & 0 & 0 & \eta \\ 0 & 1 & 0 & \eta' \\ -\eta' & \eta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (13)$$

$$V(s) = \begin{pmatrix} x(s) \\ x(s)' \\ \Delta L(s) \\ \Delta\varepsilon \end{pmatrix}, \quad V_\beta(s) = \begin{pmatrix} x_\beta(s) \\ x_\beta(s)' \\ \Delta L_\beta(s) \\ \Delta\varepsilon \end{pmatrix} \quad (14)$$

$$\delta = \begin{pmatrix} \delta x \\ \delta x' \\ \delta L \\ 0 \end{pmatrix}, \quad \delta_\beta = \begin{pmatrix} \delta x_\beta \\ \delta x'_\beta \\ \delta L_\beta \\ 0 \end{pmatrix} \quad (15)$$

Transformation between the common and betatron frames can be performed using next rules:

$$M = T_\beta M_\beta T_\beta^{-1}, \quad V = T_\beta V_\beta, \quad \delta = T_\beta \delta_\beta \quad (16)$$

In particular, relations of δL and δL_β can be obtained:

$$\delta L_\beta = \delta L + \delta x \eta'(0) - \delta x' \eta(0) \quad (17)$$

Third line of the V_β periodicity condition $M_\beta V_\beta(0) + \delta_\beta = V_\beta(0)$ leads to:

$$\Delta L_\beta(0) - \alpha\Pi\Delta\varepsilon + \delta L_\beta = \Delta L_\beta(0) \Rightarrow$$
$$\delta\varepsilon = \frac{\delta L_\beta}{\alpha\Pi} = \frac{\delta L + \delta x \eta'(0) - \delta x' \eta(0)}{\alpha\Pi} \quad (18)$$

For a short corrector $\delta x = 0, \delta L = 0$ this comes to the well known formula, see e.g. in [3]

Incomplete rank of (9) in the considered case leads to uncertainties in determining of ΔL. It is obvious that in absence of RF acceleration cavity this value can be arbitrary. Let for uniqueness put this value to zero on exit from the corrector. It is easy to show periodicity of V_β by tracking it through one turn, for example starting from exit point of the corrector:

$$M_\beta V_\beta(0) + \delta = \quad (19)$$
$$(x_\beta(F), x'_\beta(F), -\alpha\Pi\Delta\varepsilon, \Delta\varepsilon)^t + \delta_\beta = V_\beta(0)$$

Results

During commissioning of VEPP-2000 permanent attempts were performed to apply the program for improvement of lattice. Difficult way of eliminating of all "bugs" in the code and understanding of some "tricky" moments of method precedes the success. Figure 3 shows the dispersion measured before and after applying of calculated corrections for quadrupoles' gradients and solenoids' fields.

Figure 4 shows the beta-functions and dispersion obtained from the experimental response matrix. The asymmetry of about 10% can be easily controlled in operation using a few sets of correctors basing on shape and behavior of the beam. Improvements done in the lattice of VEPP-2000 resulted in high peak luminosity obtained in the round beam mode.
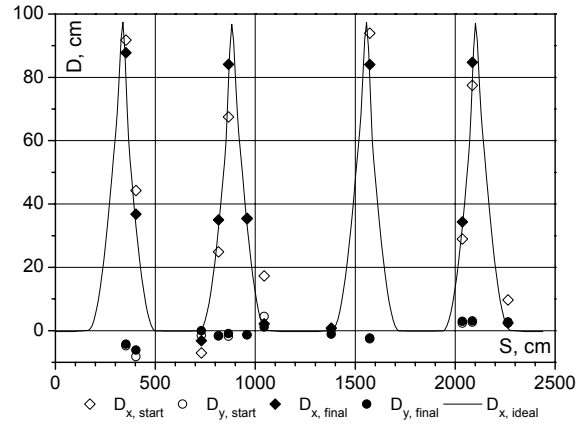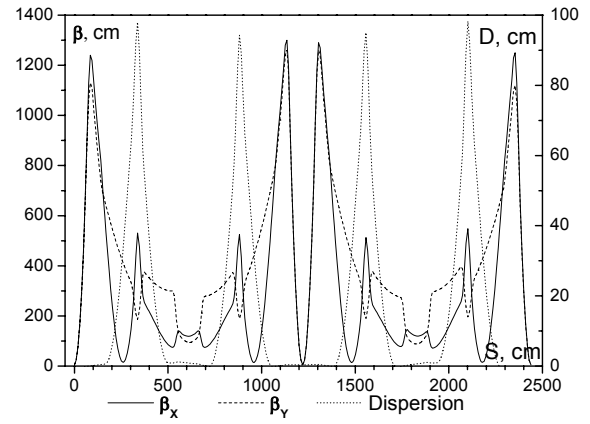

Figure 3: Example of dispersion correction.

Figure 4: Beta-functions and dispersion obtained from experimental response matrix.

REFERENCES

[1] *Yu. M. Shatunov et al.*. Project of a New Electron-Positron Collider VEPP-2000, in: Proc. 7th European Particle Accelerator Conf. (EPAC 2000), Vienna, Austria, 439-441

[2] *J. Safranek*. Experimental Determination of Storage Ring Optics Using Orbit Response Measurements, Nucl. Instr. and Meth. A388, (1997) p. 27.

[3] *J. Safranek*. Beam-based lattice diagnostics, in: Beam Measurements. Proc., Joint US-CERN-Japan-Russia School on Particle Accelerators, S. Kurokawa et al. (eds.), Singapore: World Scientific (1999), p. 298.

[4] *V. Sajaev, L. Emery*. in Proce. of EPAC'02, Paris, France, p.742, 2002

[5] *V. Sajaev, V. Lebedev, V. Nagaslaev, A. Valishev*. in Proc. of PAC'05, Knoxville, Tennessee, 3662-3664, 2005