MAGNETOOPTIC STRUCTURE FOR SYNCHROTRONS WITH NEGATIVE MOMENTUM COMPACTION FACTOR

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INTRODUCTION

In synchrotrons the transition energy $W_{tr}=m_0c^2(\gamma_{tr}-1)$ is fundamentally important because it determines the maximum attainable accelerated currents. From this point of view it is desirable that the momentum compaction factor $\alpha= 1/\gamma_{tr}^2$ be as small as possible or even negative, which makes γ_{tr} imaginary and accordingly rules out transition energy crossing by particles under acceleration.

On the basis of the theory of "resonant" lattices for synchrotrons with complex transition energy developed in [1,2], examples of such lattice with application to various accelerators are proposed.

The "resonant" lattice was first proposed for the Moscow Kaon Factory [3]. This lattice was then adapted for the TRIUMF KAON Factory [4]. Later it was considered as the best candidate for the Low Energy Booster of Superconducting Super Collider [5], then was adopted for the main accelerator of the Neutrino Factory at CERN [6], and ultimately was implemented in the Japan Protons Accelerator Research Center [1,7]. In the Superconducting option of High Energy Storage Ring lattice of the FAIR project the same idea is also accepted [8]. At present the "resonant" lattice is one of the candidates for PS2 in CERN [9].

The "resonant" lattice is appeared to be useful for electron machines as well. In particular, in the electron-positron collider type a small momentum-compaction factor is needed to reduce the synchrotron tune, while keeping the bunch length and momentum spread constant [1]. In synchrotron-light sources the minimum momentum-compaction factor and the minimum modulation of the dispersion function are both required simultaneously to have a small horizontal emittance [10,11].

MAIN PROPOSITIONS OF THE "RESONANT" LATTICE THEORY

With specially correlated modulation of quadrupoles gradient and orbit curvature and a particular choice of betatron oscillation frequencies, the theory of "resonant" lattices makes it possible to get interrelated dispersion variations D(s) and 1/ρ(s) along the equilibrium orbit and a negative momentum compaction factor

$$\alpha=\frac{1}{C}\int^{C}\frac{D(s)}{\rho(s)}ds\le 0 \qquad (1)$$

General principles of construction of "resonant" lattices detailed in [1,2] are based on the solution of the equation for the dispersion D(s) in the biperiodical structure:

$$\frac{d^2D}{ds^2}+[K(s)+\varepsilon\, k(s)]D=\frac{1}{c(s)} \qquad (2)$$

Here the gradient G(s) and the orbit curvature ρ(s) related to each other through the functions

$$K(s)=\frac{eG(s)}{p},\ \varepsilon\, k(s)=\frac{e\Delta G(s)}{p},$$

where $p=m_0 rv$ is the particle momentum, should be modulated resonantly and in correlation with each other. In what follows we will use harmonics of the modulated function of gradients

$$e\cdot k(\phi)=\sum_{k=0}^{\infty}g_k\cos\ k\phi \qquad (3)$$

where

$$g_k=\frac{e}{p}\cdot\frac{1}{\pi}\int_{-\pi}^{\pi}\Delta G\cos k\phi\, d\phi$$

is the k-th harmonic of the Fourier series of gradients function and $\phi=2\pi\cdot s/L_s$ is the longitudinal coordinate normalized to the superperiod length L_s, and harmonics in the expansion of the curvature function

$$\frac{1}{\rho(\phi)}=\frac{1}{\bar{R}}\left(1+\sum_{n=1}^{\infty}r_n\cos n\phi\right) \qquad (4)$$

where

$$r_n=\frac{\bar{R}}{\pi}\int_{\pi}^{-\pi}\frac{\cos n\phi}{\rho(\phi)}d\phi$$

is the n-th harmonic of the Fourier series of the orbit curvature function and

$$\bar{R}=L_s\cdot S/2\pi$$

is the average curvature radius of the equilibrium orbit in the superperiod, S is the total number of superperiods.

Since mirror symmetry of the superperiod is one of the conditions for the construction of the "resonant" lattice, expansions of the functions $\varepsilon\, k(\phi)$ and $1/\rho(\phi)$ in the Fourier series involve only terms with cosines.

According to (1), the momentum compaction factor is the average value of the function $D(\phi)/\rho(\phi)$. In the general form, the dispersion $D(\phi)$ and the orbit curvature $\frac{1}{\rho(\phi)}$ can be represented in terms of the averages $\bar{D}$ and $\bar{R}$ and the functions $\tilde{D}(\phi),\tilde{r}(\phi)/\bar{R}$ oscillating about these averages:

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$$D(\phi) = \overline{D} + \tilde{D}(\phi) \text{ and } \frac{1}{\rho(\phi)} = \frac{1}{\overline{R}}(1 + \tilde{r}(\phi)) .$$

Then the momentum compaction factor can be written as the sum

$$\alpha = \frac{\overline{D}}{\overline{R}} + \frac{\overline{\tilde{D}(\phi) \cdot \tilde{r}(\phi)}}{\overline{R}} \tag{5}$$

In an ordinary regular FODO lattice without gradient and orbit curvature modulation the oscillating components are equal to zero, $\tilde{D}(\phi) = 0$, $\tilde{r}(\phi) = 0$, and the momentum compaction factor is governed by the first term in (5). Considering that the average dispersion in classical lattices is $\overline{D} = \frac{\overline{R}}{\nu^2}$ we find that the minimum value of the momentum compaction factor

$$\alpha = \frac{\overline{D}}{\overline{R}} = \frac{1}{\nu^2}$$

is limited by the total number of horizontal betatron oscillations ν in the magnetic optical structure. In the "resonant" lattice, the functions of gradients and/or orbit curvature can be modulated jointly or individually. In [2] general expressions were obtained for the momentum compaction factor for one superperiod

$$\alpha_s = \frac{1}{\nu^2}\left\{1 + \frac{1}{4}\left(\frac{\overline{R}}{\nu}\right)^4 \sum_{k=-\infty}^{\infty} \frac{g_k{}^2}{(1 - kS/\nu)[1 - (1 - kS/\nu)^2]^2} \right.$$
$$+ \frac{1}{4}\sum_{k=-\infty}^{\infty} \frac{r_k^2}{1 - kS/\nu} - \frac{1}{2}\left(\frac{\overline{R}}{\nu}\right)^2 \sum_{k=-\infty}^{\infty} \frac{r_k g_k}{(1 - kS/\nu)[1 - (1 - kS/\nu)^2]}$$
$$\left. - \frac{1}{2}\left(\frac{\overline{R}}{\nu}\right)^2 \sum_{k=-\infty}^{\infty} \frac{r_k g_k}{[1 - (1 - kS/\nu)^2]} + O(g_k^i \cdot r_k^j, i + j \geq 3)\right\} \tag{6}$$

where kS is the modulation frequency of the k-th harmonic in the expansion of the gradient and curvature functions, $\tilde{f}$ is the function describing beam envelope oscillations, which is normalized to its average value. We will call the harmonic closest to ν (with the minimum possible difference $kS - \nu$) and producing the maximum effect on the momentum compaction factor the fundamental harmonic. This harmonic has kS oscillations over the entire lattice in question. In most cases under our consideration the frequency of the k-th harmonic coincides with the number of superperiods, i.e., k = 1 and $kS = S$. Indeed, if both the quadrupole gradient function and the orbit curvature function are modulated with an identical frequency (i.e., at k = n in (3) and (4)), the second term in (5) may make an appreciable contribution to the momentum compaction factor provided that the value $1 - kS/\nu$ is small (see (6)).

In addition, from (6) there follows an obvious condition of antiphase modulation of the gradient and curvature function, which allows correlated variation of the momentum compaction factor with the aid of these functions. We call this lattice, based on the resonant and correlated perturbation of the magnetic optical channel parameters, the "resonant" lattice.

Thus, the following principles underlie the general approach to construction of a "resonant" lattice:

- the fundamental modulation frequency should be identical for the functions of the gradients and the orbit curvature and higher than the horizontal betatron frequency $kS > \nu$ with as minimum a difference $kS - \nu$ as possible;
- modulation of the orbit curvature should be in antiphase with modulation of the quadrupole gradients, $g_k r_k < 0$;
- amplitudes of each of the fundamental harmonics, g_k and r_k, should be as high as possible;
- exact equality of the frequencies $\nu = kS$ and $\nu = kS/2$ at which the dispersion and the β-function increase beyond limits should be eliminated.

"RESONANT" LATTICE WITHOUT γ_{tr} CROSSING

The most important application of the "resonant" lattice is the magneto-optic structure without γ_{tr} crossing. It will be considered on the proton synchrotron example [9] as possible candidate for CERN PS2.

Superperiod structure

The number of cells in a superperiod N_{cell} is dictated by the required phase advance of radial oscillations. Following the theory of resonant lattices, we will try to construct a lattice with the horizontal frequency ν_{arc} as close to the number of superperiods S_{arc} as possible [1,2]. In this case, the phase advance of horizontal oscillations per cell will be about $2\pi \frac{\nu_{arc}}{S_{arc} \cdot N_{cell}}$. At the same time it is known that from the point of view of minimization of the β-functions for a cell the phase advance of radial oscillations should fall within the range 60°–100°. Thus, in a lattice with the fundamental harmonic of the modulation of the superperiod parameters $k = 1$ and with $\nu_{\text{arc}} < S_{arc}$ the number of cells turns out to be 3–4 per superperiod.

As example we take the PS2 parameters [9] with the magnetic rigidity $p/e \approx 170$ *m·T*. To eliminate the transition energy crossing in anew designed PS2 synchrotron the gamma-transition must be moved away from acceleration range $\gamma \approx 5 \div 50$.

Based on the above reasoning, the "resonant" lattice method with simultaneous orbit curvature and quadrupole gradient modulation with an identical frequency of the fundamental harmonics and an approximately identical contribution of both modulations to the final value of the momentum compaction factor is most effective. From (6) it is easy to derive the following equality for an arbitrary fundamental harmonics g_k and r_k giving $\alpha \approx -1/\nu^2$:

$$\left(\frac{\bar{R}}{\nu}\right)^2 \cdot \frac{g_k}{1-\left(1-kS_{arc}/\nu_{arc}\right)^2} - r_k = \pm 2^{3/2}\left(kS_{arc}/\nu_{arc}-1\right)^{1/2} \quad (7)$$

and

$$\left|r_k\right| \le \left(\frac{\bar{R}}{\nu}\right)^2 \left|\frac{g_k}{1-(1-kS_{arc}/\nu_{arc})^2}\right| \quad (8)$$

As was already mentioned, modulation of gradients and modulation of the orbit curvature should be in antiphase and the reasonable location of the missing magnet cell is at the centre of the superperiod. This means that the amplitude of the fundamental harmonic of the orbit curvature modulation should be negative, $r_n < 0$, and therefore the amplitude of the gradient modulation will be positive. At these conditions the γ-transition varies in a wide region from $\gamma_{tr}=\nu$ to $\gamma_{tr}=i\nu$ with quadrupole gradient modulation only. As an example of a lattice with both modulations, you can see figure 1.

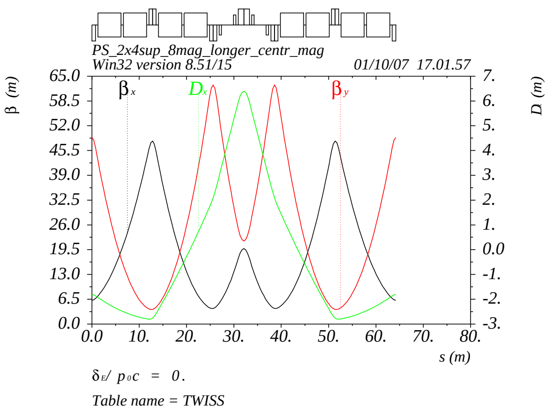


Figure 1: Superperiod of "resonant" lattice with simultaneous orbit curvature and quadrupole gradient modulation.

Figures 2 and 3 show the results yielded by various modifications of the method. In the first case (Fig. 2) the central quadrupole is "cut" in two slices and a sextupole is inserted between the slices, as was done, for example, in the JPARC project [7]. In the second case (Fig. 3) the orbit curvature is varied without a decrease in the total number of magnets, by varying the central cell length alone. This option may be advantageous for a magnetic optical lattice with rectangular magnets because the magnet sagitta is considerably decreased.

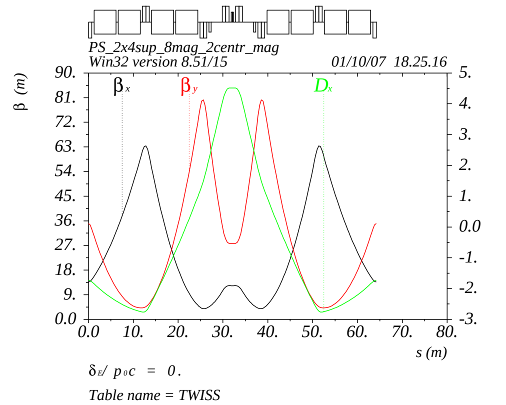


Figure 2: Superperiod with the central quadrupole sliced.

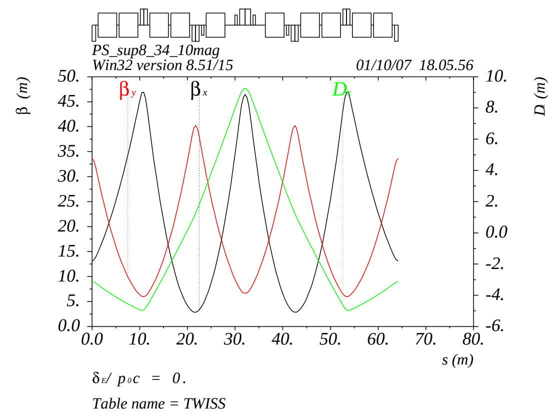


Figure 3: Superperiod with 10 magnets.

Arc structure

The number of arcs and straight sections is dictated by many parameters, first of all by the required architecture of the ring and the projected experiments. For instance, for the PS2 it is two.

For the dispersion in straight sections to be zero, the arc consisting of S_{arc} superperiods should have a phase advance of radial oscillations that is a multiple of 2π, i.e., ν_{arc} should be an integer. This means that the phase advance in one superperiod should be $2\pi\nu_{arc}/S_{arc}$. On the other hand, for the momentum compaction factor to be controlled, the betatron frequency of horizontal oscillations should be smaller than the number of superperiods multiplied by the number of the fundamental harmonic. From this point of view it is reasonable to take the minimum possible difference

$$\nu_{arc} - kS_{arc} = -1 .$$

Thus, there exist many ratios between S_{arc} and ν_{arc}:

(4:3), (6:5), (8:7), (10:9), ….

Besides, there is another possibility, when the arc is divided into many sub-arcs in this ratio within which the above ratios hold, for example, the ratio can be

$$S_{arc} : \nu_{arc} = 8 : 6 = 2 \text{ x } (4:3)$$

As is seen, in all ratios the number of superperiods S_{arc} is taken to be even while the betatron oscillation frequency takes on integral odd values. In this case, the phase advance of the radial oscillations between the cells located in different superperiods and separated by $S_{arc}/2$ superperiods is obviously

$$2\pi \cdot \frac{\nu_{arc}}{S_{arc}} \cdot \frac{S_{arc}}{2} = 2\pi \cdot \frac{\nu_{arc}}{2} = \pi + 2\pi n\,,$$

which corresponds to the condition of first-approximation compensation for the nonlinear effects of sextupoles located in these cells. This remarkable property also applies to higher multipoles in bending magnets and quadrupoles because each of them has a partner in the other quarter of the arc at a distance of odd integral π of radial oscillations (see Fig. 4).

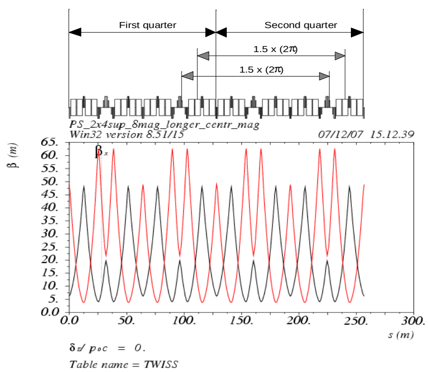
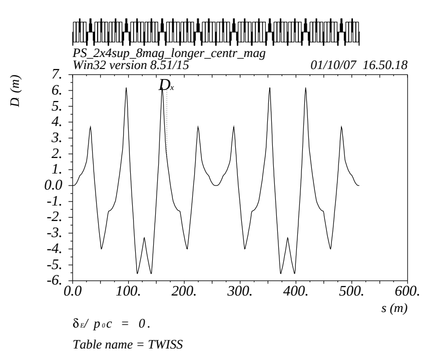


Figure 4: Half of arc with (S_{arc}:ν_{arc})=8:6.

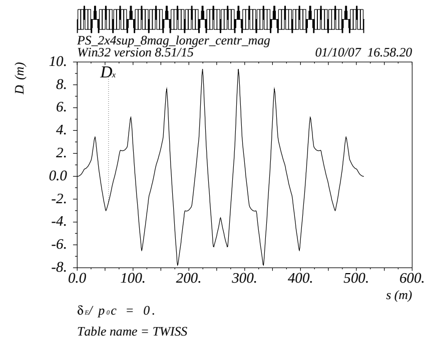
Thus, choosing S_{arc}, k, and ν_{arc}, we determine the lattice of the arc and the number of arcs. On the one hand, we are limited by strict rules for the choice of these parameters, on the other, the choice is quite wide and we may speak about a certain class of accelerators with such arcs.

By way of example, let us consider the lattices for the PS2 accelerator with an identical number of arcs and identical transition energy $\gamma_{tr} = i10$.

In the first version the arc has the number of superperiods S_{arc} = 8 and the frequency of horizontal oscillation in the arc ν_{arc} = 6, in the second version S_{arc} = 8 and ν_{arc} = 7 (see Fig. 5). The behavior of the $\beta_{x,y}$-functions remains unchanged (see fig.1) for both options.



(a)



(b)

Figure 5: Dispersion in the 8-superperiods arc with the horizontal tune ν_{arc}=6 (a) and ν_{arc}=7 (b).

You can see that the first option has smaller dispersion and therefore it is more preferable.

"RESONANT" LATTICE FOR STOCHASTIC COOLING

Another application of the "resonant" lattice is the advanced lattice for the stochastic cooling. It is known to intensify the stochastic cooling process it is desirable to have the mixing factor between the pick-up and kicker as large as possible, and, on the contrary, in the case of mixing between the kicker and pick-up we should try to make it smaller. This option can be realized if the lattice has different local optical features between the pick-up – kicker and the kicker – pick-up. It can be seen that the "resonant" lattice has a remarkable feature: the gradient and the curvature modulation amplify each other if they have opposite sign, and, on the contrary, they can compensate each other when they have the same sign (see exps. 7,8):

$$\frac{1}{4(kS/\nu - 1)} \cdot \left(\frac{g_k}{[1-(1-kS/\nu)^2]} \mp |r_k| \right)^2 \approx 2 \text{ or } 0 \quad (9)$$

Then the momentum compaction factor in the imaginary arc takes the form $\alpha_{kp} \approx -1/\nu^2$, and the momentum compaction in the real arc is $\alpha_{pk} \approx 1/\nu^2$. Thus, in such a lattice, we can make two arcs with different slip factors: $\eta_{pk} = 1/\nu^2 - 1/\gamma^2$; $\eta_{pk} = -1/\nu^2 - 1/\gamma^2$. In case $\gamma \approx \nu$, one of the arcs is isochronous when the slip factor is $\eta_{pk} \approx 0$, and the other slip factor is $\eta_{kp} \approx -2/\nu^2$.

As example we consider the possible optics for the HESR lattice of FAIR project [12] with different slip factors η_{pk}, η_{kp} in the real arc and the imaginary arc. Both arcs have 4-fold symmetry with superperiodicity S =4. The phase advance per arc is $\nu_{x,y} = 3.0$ in both

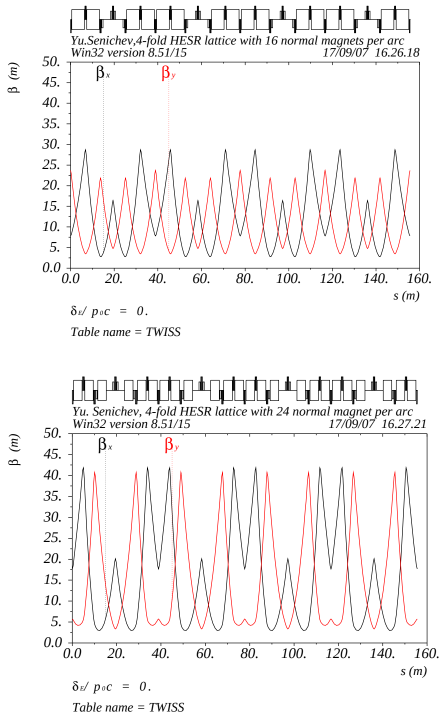


Figure 6: β-functions on the arcs with 16 and 24 magnets for γ_{tr}=i6.

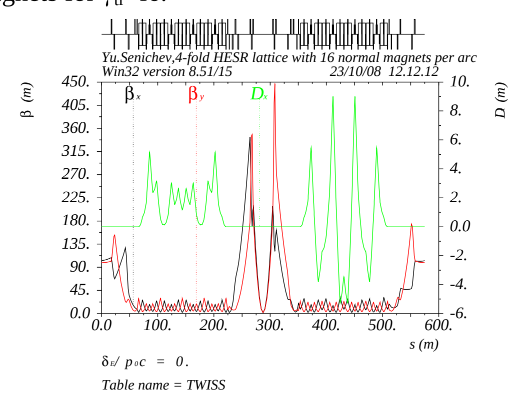


Figure 7: The advanced HESR lattice with two different arcs.

planes. Each superperiod consists of three FODO cells with 4 or 6 bending magnets and 3 families quadrupoles (fig. 6).

As example figure 7 shows the optics of whole ring with two different arcs, where $\alpha_{pk} \approx 1/6^2$ $\alpha_{kp} \approx -1/6^2$. In both arcs the dispersion is suppressed to zero to have the zero-dispersion straight sections. The different momentum compaction factors are reached mainly due to the dispersion function change, and the β-function changes insignificantly. Two families of sextupoles are used for the chromaticity correction, and their non-linear influence is self-compensated inside of each arc. Due to this fundamental advantage in spite of two different arcs the dynamic aperture is not suffered in comparison with option with two identical arcs.

"RESONANT" LATTICE FOR SYNCHROTRON LIGHT SOURCES

Third application of the "resonant" lattice is the synchrotron light source. Since the horizontal emittance depends upon the horizontal dispersion function η_x, as $\varepsilon_x \propto \langle H \rangle_{dipole}$, where $H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$. Therefore lattices with small electron emittances require smaller dispersion function and, as consequence, stronger sextupoles are needed in order to correct the chromaticity. In the same time, as is well known, the sextupoles dramatically decrease the dynamic aperture due to their nonlinear action.

Now let us consider the average Hamiltonian of the motion in the action-angle variables:

$$H(I_x, \vartheta_x, I_y, \vartheta_y) = \frac{(k_x^2 + k_y^2)^{1/2}}{k_x}\Delta I_x + \frac{(k_x^2 + k_y^2)^{1/2}}{k_y}\Delta I_y + 2\left\langle h_{k_x,k_y,p}\right\rangle I_x^{k_x/2} I_y^{k_y/2}\cos\left(k_x\vartheta_x + k_y\vartheta_y\right) + \varsigma_x I_x^2 + \varsigma_{xy} I_x I_y + \varsigma_y I_y^2 \quad (10)$$

where the coefficients $\zeta_x, \zeta_y, \zeta_{xy}$ determine the non-linear tune shift and

$$\left\langle h_{k_x,k_y,p}\right\rangle = \int_0^C \beta_x^{k_x/2}\beta_y^{k_y/2} S_{x,y}(s)\exp i(k_x\mu_x + k_y\mu_y)ds .$$

In the "resonant" lattices the sextupolar terms are aimed to make smaller, and each pair of sextupoles affect like one octupole. Then the influence of the non-linearity is specified by the discriminant in the expression:

$$\hat{I}_x^{1/2} = -\frac{3h_{30p}\cos 3\hat{\vartheta}_x}{8\varsigma_x} \pm \frac{1}{4\varsigma_x}\sqrt{\frac{9}{4}h_{30p} - 8\varsigma_x(\Delta + \varsigma_{xy} I_y)}$$

The lattice with $\zeta_x >> h_{30p}$ have to be classified as a special lattice [11], since it is a case, when the value of h_{30p} is effectively suppressed, but the non-linearity remain to be under control and strong.

To realize such a lattice the most appropriate values of tune for arc are the integer numbers $n_x = 3$ and $n_y = 2$, giving a phase advance per cell equal to 135° and 90° correspondingly. To a first approximation in this condition, the non-linear action of each $n-th$ sextupole is compensated by $(n+4)-th$ in the horizontal and $(n+2)-th$ in the vertical planes correspondingly. By this method together with smallest emittance we get the large dynamic aperture. As a comparison, the dynamic aperture of a modified circular Chasman-Green lattice with the same number of sextupole families is smaller by a factor of four.

Besides, there is much interest at present in magnetic lattices, which can be operated over a range of momentum compaction factor. This would provide several advantages, the possibility to work without sextupoles, high bunch compression and the high peak current for driving of a free electron laser. This tunability is obtained due to a special modulation of the closed orbit curvature, as we make in the "resonant" lattices. Figure 8 shows the arc with the tuneable momentum compaction factor.

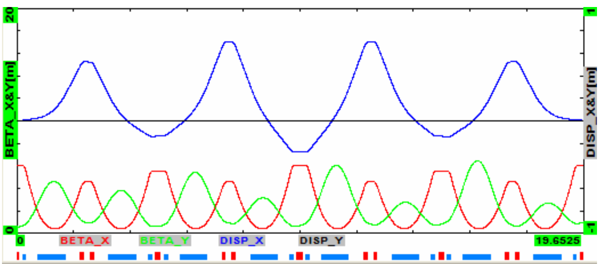


Figure 8: The arc lattice with tunable momentum compaction factor.

In this lattice one cell is formed by two adjoining cells, and each second focusing quadrupole is replaced by a short combined function bend magnet with positive gradient BG(G>0). As result the arc consists of four cells and each half cell has a structure: trim QF+short BG(G>0)+long BG(G<0)+QF. The trim quadrupole is used for fine tuning. The dispersion function is equal to zero at the ends of arc automatically due to an integer tune number for the arc and the matching sections are not needed. Due to the low beta function and zero dispersion function in the middle of each long bending magnet the electron beam emittance has a rather low value of 15 nm at 1.4 GeV (example of "Astrid").

CONCLUSION

We developed the multi-application "resonant" lattice with the distinguishing features:

- ability to achieve the negative momentum compaction factor using the resonantly correlated curvature and gradient modulations;
- gamma transition variation in a wide region from $\gamma_t \approx \nu_x$ to $\gamma_t \approx i\nu_x$ (ν_x is the horizontal tune) with quadrupole strength variation only;
- dispersion-free straight section;
- independent optics parameters of arcs and straight sections;
- two families of focusing and one of defocusing quadrupoles;
- separated adjustment of gamma transition, horizontal and vertical tunes;
- convenient chromaticity correction method using sextupoles;
- first-order self-compensating scheme of multipoles and a large dynamic aperture;

REFERENCES

[1] Yu.Senichev, A «resonant» lattice for a synchrotron with a low or negative momentum compaction factor, submitted in Particle Accelerator, KEK Preprint 97 40, 1997,

[2] Yu. Senichev and A. Chechenin, Theory of "Resonant" Lattices for Synchrotrons with Negative Momentum Compaction Factor, Journal of Experimental and Theoretical Physics, 2007, vol. 105, No. 5, pp. 988–997

[3] N. Golubeva, A. Iliev, Yu. Senichev, The new lattices for the booster of Moscow Kaon Factory, Proceedings of International Seminar on Intermediate Energy Physics, v. 2, p. 290, Moscow, 1989

[4] U. Wienands, R. Servranckx, N. Golubeva, A. Iliev, Yu. Senichev. A racetrack lattice for the TRIUMF Kaon Factory Booster, Proceedings of the 15-th International Conference on High Energy Accelerators, Hamburg, 1992

[5] E. Courant et al., Low Momentum Compaction Lattice Study for the SSC Low Energy Booster, Proceeding IEEE PAC, p. 2829, San-Francisco, 1991

[6] H. Schönauer, B. Autin, R. Cappi, J. Gareyte, R. Garoby, M. Giovannozzi, H. Haseroth, M. Martini, E. Métral, W. Pirkl,C.R. Prior, G.H. Rees, I. Hofmann, Yu. Senichev, A slow-cycling proton driver for a Neutrino Factory, Proceedings of EPAC, p. 966, Vienna, 2000

[7] Y. Ischi, S. Machida, Y. Mori_, S. Shibuya, Lattice design of JHF synchrotrons, Proceedings of APAC, Tsukuba, 1998

[8] Yu. Senichev, S. An, K. Bongardt, R. Eichhorn, A. Lehrach, R. Maier, S. Martin, D. Prasuhn, H. Stockhorst, R. Toelle, Lattice Design Study for HESR, Proceedings of EPAC, Lucerne, 2004, p.653

[9] Yu. Senichev, The lattice with imaginary γ-transition for the CERN proton synchrotron PS2, CERN-2007-005, 2007, Proceedings of BEAM-07, p. 171. (https://care-hhh.web.cern.ch/care-hhh/BEAM07/Proceedings/Proceedings/Session2++/S42-Senichev-letter.pdf)

[10] Yu. Senichev, The proposed racetrack lattice for the synchrotron light source "Astrid II" , Proceeding of EPAC, Stockholm, 1998, pp. 642,

[11] Yu. Senichev, Low periodicity lattice for Third generation synchrotron light sources: avoidance of dynamic aperture reduction by sextupole compensation, Proceedings of PAC, New York, 1999, p. 2442

[12] Yu. Senichev, The advanced HESR lattice for improved stochastic cooling, COOL-07, Bad Kreuznach, Germany, 2007, p.102 (http://bel.gsi.de/cool07/PAPERS/TUA2C07.PDF)