ITERATIVE METHOD FOR MODELING OF STEADY OPERATION MODES OF MULTIPURPOSE ISOCHRONOUS CYCLOTRONS

I.V.Amirhanov, G.A.Karamysheva, I.N. Kiyan*, JINR, Dubna, Russia
J.Sulikowski, R.Taraszkiewicz, INP PAS, Krakow, Poland

Abstract

The mathematical and computer modeling of operation modes of multipurpose isochronous cyclotrons is based on calculation of currents in trim coils of correction of the basic magnetic field at a certain level of current in the main coil. The calculation is made for a given extraction energy or rotation frequency of particles. The results of the calculation allow a required magnetic field to be formed with a certain accuracy in the range from the ion source to the extraction radius of particles. The method derives its name from iterative calculation of extraction energy, current in the main coil, and isochronous magnetic field with allowance for influence of the trim coil contributions on the flutter of the basic magnetic field. The essentially new algorithm of excluding some of the involved trim coils from the calculation is based on consecutive selection them. The input of solution in frameworks of the given boundary conditions are carried out step by step. The use of interpolation both inside and between the measured maps of magnetic fields allows interpolating the resulting maps of magnetic fields instead of measuring them. In turn, it allows to replace calculating-experimental iterations for formation of the required magnetic field only calculating. A series of physical experiments carried out in 2007 at the multipurpose isochronous cyclotron AIC144 (INP PAS, Krakow) confirmed legitimacy of this replacement. The essentially new algorithm of excluding some of the involved trim coils from the calculation has allowed stability of the resulting magnetic field to be improved; That is, the influence on the quality of the calculated operation modes of a lot of the factors (the accuracy of the measurement of the source magnetic field maps, the features of the start-up algorithm of the cyclotron at a certain operation mode, the stability of the used power supplies, etc.) became less significant. A series of numerical experiments on calculation of the basic operation mode of AIC (p, 60 MeV / 26.25 MHz) confirmed the necessity of including the evaluation of the solution stability into the calculation.

INTRODUCTION

The mathematical and computer modeling of operation modes of multipurpose isochronous cyclotrons is based on calculation of currents in trim coils of correction of the basic magnetic field at a certain level of current in the main coil. The operation mode of a multipurpose isochronous cyclotron comprises a set of currents in the main coil and trim coils and value of the RF oscillator frequency and dee potential. The operation mode is simulated for a particular particle acceleration task and allows the desired magnetic field to be formed with a given accuracy within the range of working radii from the injection system to the ion beam extraction system. The initial data include the type of particles to be accelerated, the kinetic energy of particles at the certain radius, the harmonic number, the magnetic structure periodicity, the diapason of the magnetic field formation, the radius of the working point (the crossing the mean basic and isochronous magnetic fields), and the mask of the mean magnetic field for the bump and edge magnetic field formation. Calculation is carried out with a set of measured or calculated magnetic fields.

PHYSICAL EXPERIMENTS

The use of interpolation both inside and between the measured maps of magnetic fields allows interpolating the resulting maps of magnetic fields instead of measuring them. In turn, it allows to replace calculating-experimental iterations for formation of the required magnetic field only calculating. A series of physical experiments carried out in 2007 at the multipurpose isochronous cyclotron AIC144 (INP PAS, Krakow) confirmed legitimacy of this replacement. The Figure 1 and Figure 2 show the operation mode parameters and the measured beam currents in the two last physical experiments (13.02.2007 and 19.02.2007).

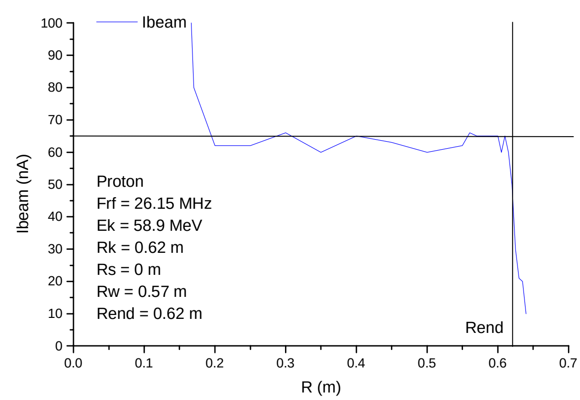

Figure 1: The measured beam current (13.02.2007).

* e-mail: kin@nusun.jinr.ru

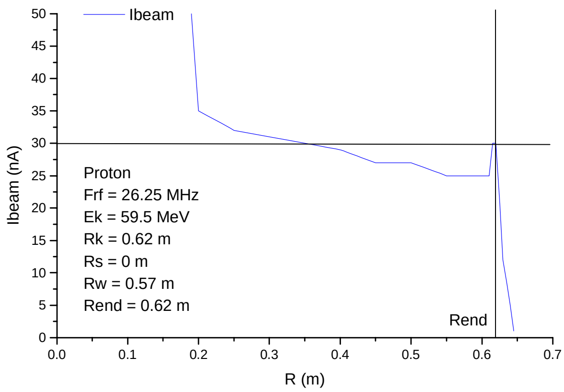

Figure 2: The measured beam current (19.02.2007).

Functional and analytical formulas

The interpolation inside of the measured maps of magnetic fields is realized by cubic spline method and between them by Lagrange method. The following functional is minimized by least-square method. The corresponding inhomogeneous system of linear algebraic equations is resolved by Gauss with pivoting method. The integration of the coefficient matrix elements and free member vector elements is realized by Simpson method. Isochronous field is calculated by Gordon method [1].

$$F(I_1, I_2 ... I_k) = \frac{1}{B_0^2} \cdot \left\{ \int_0^{R_{ext}} \left[\sum_{j=1}^{k} \left(\overline{B}_{tc,j,r,\max} \cdot \frac{I_j}{I_{j,\max}} \right) - \Delta\overline{B}_r \right]^2 dr + \lambda^2 \cdot \sum_{j=1}^{k} \left(\frac{I_j}{I_{j,\lim}} \right)^{2p} \right\} \quad (1)$$

$$\Delta\overline{B}_r = \overline{B}_{is,r} + \overline{B}_{bump,r} + \overline{B}_{edge,r} - \overline{B}_{mc,r} - \sum_{l=1}^{m} \left(\overline{B}_{tc,l,r,\max} \cdot \frac{I_l}{I_{l,\max}} \right) \quad (2)$$

$$I_l \Rightarrow const$$

where $n = k + m$ is number of the involved trim coils, B_0 is coefficient of the proportionality, $\overline{B}_{tc,j,r,\max}$ is maximal contribution of the trim coil, I_j is trim coil current, $\overline{B}_{is,r}, \overline{B}_{bump,r}, \overline{B}_{edge,r}, \overline{B}_{mc,r}$ is isochronous magnetic field, mean magnetic field of the bump and edge, and mean basic magnetic field. The penalty function with variable λ parameter is used for step by step input of the solution in certain frameworks of the given boundary conditions.

The next analytical formula is used for the calculation of the conditionality number of the inhomogeneous system of linear algebraic equations:

$$cond(A) = \|A\| \cdot \|A^{-1}\| \quad (3)$$

$$\|A\| = \max_{1 \le j \le n} \sum_{i=1}^{n} |a_{ij}|;$$

where A – is coefficient matrix. The next analytical formula is used for the calculation of the particle kinetic energy at the certain radius:

$$E_{kin,out} = \sqrt{\left(\overline{B}_{mn,out} \cdot R_{ev,out} \cdot |q| \cdot c\right)^2 + E_0^2} - E_0 \quad (4)$$

$$E_0 = m_0 \cdot c^2 \qquad R_{ev,out} = r_{ev}(R_{mn,out})$$

$$r_{ev}(\overline{r}) = \overline{r} \cdot \left\{ 1 + \frac{\varepsilon_N^2}{2 \cdot (N^2 - k - 1) \cdot (k+1)} \cdot \left(2 - \frac{N^2}{2 \cdot (N^2 - k - 1)} + k + \overline{r} \cdot \frac{\varepsilon_N'}{\varepsilon_N} \right) + \frac{\varepsilon_N^2 \cdot N^2}{4 \cdot (N^2 - k - 1)^2} \right\} \quad (5)$$

$$\varepsilon_N = \frac{B_N}{\overline{B}_{mn}} \qquad \varepsilon_N' = \frac{d\varepsilon_N}{d\overline{r}}$$

$$k = \overline{r} \cdot \frac{\overline{B}_{mn}'}{\overline{B}_{mn}} \qquad \overline{B}_{mn}' = \frac{d\overline{B}_{mn}}{d\overline{r}}$$

where q is particle charge, m_0 is particle mass, c is speed of the light , r_{ev}, $\overline{r}$ is equivalent and mean radii of the closed equilibrium orbit, B_N is main harmonic of the basic magnetic field, $\overline{B}_{mn}$ is mean magnetic field and N is magnetic structure periodicity [2].

NUMERICAL EXPERIMENTS

The essentially new algorithm of excluding some of the involved trim coils from the calculation has allowed stability of the resulting magnetic field to be improved. The algorithm is based on the criterion of minimum for the product of functional minimum (divided by integration step) on conditionality number. The next example demonstrates the results of operation mode calculations carried out for AIC144 multipurpose isochronous cyclotron. The operation mode parameters: proton, F_{rf} = 26.25 (MHz), E_k = 59.5 (MeV), R_k = 0.62 (m), R_s = 0 (m), R_w = 0.55 (m), R_{end} = 0.635 (m), R_{bump} = 0.16 (m), R_{edge} = 0.595 (m).

Table 1: The trim coil excluding.

In-ed trim coils (total)	**Exc-ed trim coils (N)**	**Functional minimum (s)**	**Cond-ty number (cond)**	$\min(s \cdot cond)$ (Cs^2)
20	-	116.928	216771	25346599
19	4	116.974	148986	17427488
18	17	189.215	36727	6949299
17	6	193.448	14827	2868253
16	1	194.201	13559	2633171
15	3	199.024	13082	2603632
14	8	257.196	11539	2967785

The Figure 3 shows the criterion of exit from the iterative cycle of trim coil excluding (minimum of $\min(s \cdot cond)$).

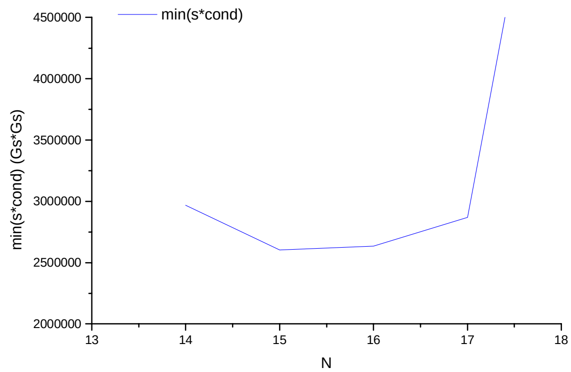

Figure 3: The criterion of exit from the iterative cycle.

Table 2: The operation mode with 20 (variant 1) and 15 (variant 2) involved trim coils.

Parameter	Variant 1	Variant 2	Unit
F_{rf}	26.25	26.25	MHz
U_d	50	50	kV
I_{mc}	583.092	583.092	A
I_{tc1}	0.3	0	A
I_{tc2}	293.3	292.5	A
I_{tc3}	2	0	A
I_{tc4}	-4.9	0	A
I_{tc5}	143.3	149.2	A
I_{tc6}	21.1	0	A
I_{tc7}	75.3	93	A
I_{tc8}	30.7	23.7	A
I_{tc9}	-94.9	-91.9	A
I_{tc10}	-157.4	-157.9	A
I_{tc11}	-249.6	-251.2	A
I_{tc12}	-215.6	-207.7	A
I_{tc13}	-128.3	-146.3	A
I_{tc14}	-145.5	-119.5	A
I_{tc15}	-352.4	-376.5	A
I_{tc16}	-387	-390	A
I_{tc17}	-61.5	0	A
I_{tc18}	-126.5	-243.1	A
I_{tc19}	-340.1	-234.4	A
I_{tc20}	239.7	197.4	A

The Figure 4 shows the error of mean magnetic field for two cases: first – with using 20 trim coils, and second – with using 15 trim coils.

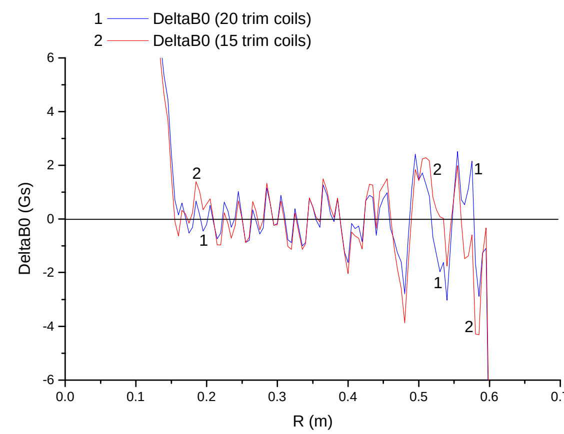

Figure 4: The error of mean magnetic field

CONCLUSION

The errors of the mean magnetic field both in the first and in the second variants are similar. The conditionality number in the second case at ~10 times better than at the first case. The influence on the quality of the calculated operation mode of a lot of the factors (the accuracy of the measurement of the source magnetic field maps, the features of the start-up algorithm of the cyclotron at a certain operation mode, the stability of the used power supplies, etc.) became less significant. The further researches have shown the necessity of the regularization of received solution.

REFERENCES

[1] M.M.Gordon. “Calculation of Isochronous Fields for Sector-Focused Cyclotrons”, Particle Accelerators, 1983, Vol.13, pp.67-84.

[2] V.V.Kol’ga. “Research of the charged particle movement in the relativity cyclotron”. The auto abstract of the dissertation, 2138, Dubna, 1965.