FEASIBILITY OF PRE-BUNCHED FEL BASED ON COHERENT DIFFRACTION RADIATION

L.G. Sukhikh*, G.A. Naumenko, and A.P. Potylitsyn, Tomsk Polytechnic University, Tomsk, Russia
J. Urakawa and A.S. Aryshev, KEK, Tsukuba, Japan
S. Boogert and P.V. Karataev, John Adams Institute at Royal Holloway, University of London, Egham, UK

Abstract

Feasibility of pre-bunched Free Electron Laser (FEL) based on Coherent Diffraction Radiation (CDR) generated in an open resonator formed by two semi-parabaloidal mirrors was examined. It was shown that in such resonator a significant increase of the radiation photon yield might be achieved.

INTRODUCTION

A scheme of a pre-bunched FEL based on Coherent Transition Radiation (CTR) generated in a closed resonator formed by two flat mirrors (see Figure 1a) has been investigated in [1].

(a)

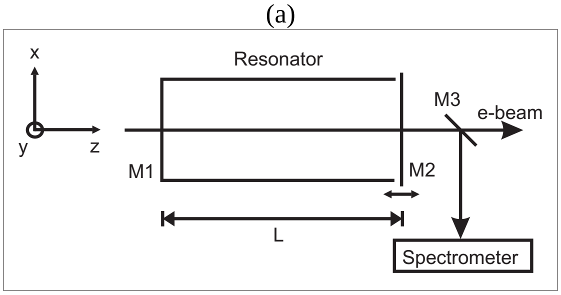

Figure 1a. The scheme of the pre-bunched FEL based on the CTR generated in a closed resonator formed by the flat mirrors. The based distance between mirrors is equal to L_0 =456 mm [1].

(b)

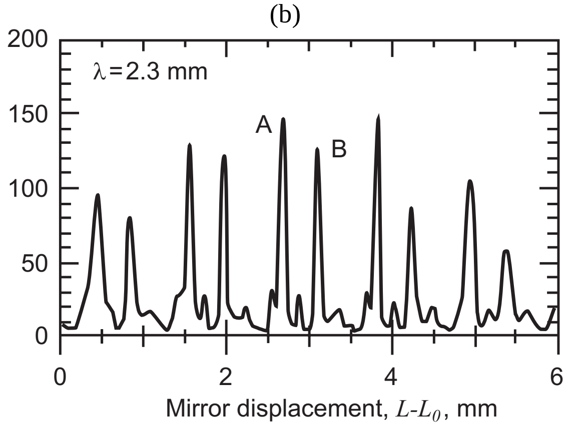

Figure 1b. Detuning curve observed in [1].

Figure 1b shows an experimental detuning curve. Authors did not explain the origin of so-called “main” and “secondary” peaks. However, one might assume that the main peak corresponds to the stimulated forward CTR from the first mirror (M1), and the secondary peak corresponds to the backward CTR from the second mirror (M2) with a coupling window in the center of it. We assume that Figure 1b illustrates the fact that because of the difference between the speed of light and the speed of electrons the forward CTR does not stimulate the backward CTR and vice versa.

The second disadvantage of the scheme is the use of the CTR. It might result in such problems as: interaction of low-emittance high-brightness electron beam with the mirrors might cause their serious damage; multiple scattering of the electrons in the mirror material might cause significant change of the electron beam parameters; and, also, as is well known the angular distributions of CTR have a minimum along the beam direction where the coupling window is situated (see for example [2,14]). In this case the integral photon yield propagating along the electron trajectory is small in comparison with the overall photon yield generated in the process. Therefore, one can conclude that the efficiency of the scheme is rather poor.

In [13] the authors proposed to use a spherical open cavity with a flat radiator with round hole to generate CDR. The disadvantage of the scheme is that the CDR spatial distribution from the flat radiator with the round hole has a minimum along its axis [2] and that would cause problems similar to the one mentioned above.

We propose to use an open resonator consisting of two rectangular semi-parabaloidal mirrors with focal distance $f = L_0/2$ as it is shown in Fig. 2. The main advantages of the scheme are: the resonator is not destructive for the electron beam; the mirrors will not be damaged by the beam; the spatial distribution of diffraction radiation (DR), which would be generated by the electron beam passed near the mirrors, has a maximum along the resonator axis (see for example [3,4]); by choosing proper resonator parameters (impact parameter, mirror tilt angles, etc.) one might expect that the distance traveled by the light at some angles of propagation might be larger than by the electron and, therefore, one might anticipate that the forward CDR would partially stimulate the backward CDR and vice versa. In future experiments we plan to use electron

* e-mail: Sukhikh@interact.phtd.tpu.edu.ru

bunch with the rms length of about σ = 0.3 mm, which allows to generate CDR in millimeter wavelength range.

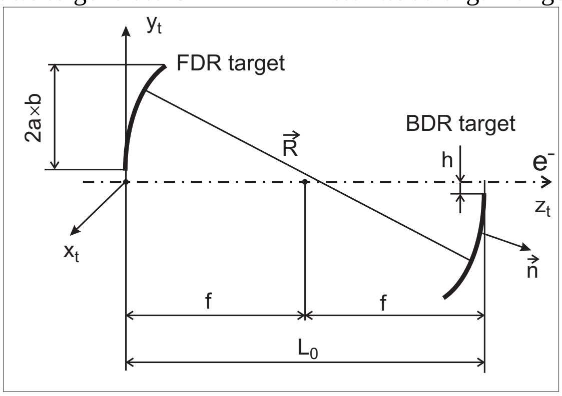

Figure 2. The scheme of the pre-bunched FEL based on the CDR generated in an open resonator formed by two semi-paraboloidal mirrors.

THEORETICAL MODEL

For theoretical simulations of the CDR process for paraboloidal targets we used a simple formula based on Kirchhoff integral for Dirichlet boundary conditions (the Green function on the target surface (S_1) is equal to zero) [5]. The theoretical approach is valid assuming an ideal reflectivity of the target, which is quite applicable for the millimeter wavelength range of the radiation:

$$E^d_{x,y}(\vec{x}) = \frac{1}{\lambda i}\int_{S_t} \frac{e^{i\frac{2\pi}{\lambda}R}}{R}\left(1+\frac{i\lambda}{2\pi R}\right)\frac{\vec{n}_t\cdot\vec{R}}{R}E^0_{x,y}(\vec{x}_t)dS_t \qquad (1)$$

Here λ is the radiation wavelength, R is the distance between two arbitrary points on both target surfaces, $\vec{n}_t$ is the external vector normal to the target surface S_t, $E^0_{x,y}(\vec{x}_t)$ is the initial field. The intensity of radiation can be written in a conventional way:

$$\frac{dW}{d\omega d\Omega} = 4\pi^2 L^2\left(\left|E^d_x\right|^2 + \left|E^d_y\right|^2\right) \qquad (2)$$

Here L is the distance between the target and the observation point.

The formula (1) can be used both for simulation of reflection and diffraction of electron electromagnetic field virtual photons converting into real ones at the first target (forward DR production) and for reflection and diffraction of real photons on the second target.

For conditions of the proposed scheme (the radiation wavelength is about 1 mm and the distance between targets is L_0 =840 mm) the term $\frac{i\lambda}{2\pi R}$ in the formula (1) can be neglected.

According to [6] we can represent the virtual photon electromagnetic field of the electron as:

$$E^0_{x,y}(x_t,y_t) = \frac{2e}{\gamma\lambda}\frac{x_t,y_t}{\sqrt{x_t^2+y_t^2}}K_1\left(\frac{2\pi}{\beta\gamma\lambda}\sqrt{x_t^2+y_t^2}\right)e^{\frac{2\pi i}{\beta\lambda}z_{FDR,BDR}} \qquad (3)$$

Here e is the electron charge, K_1 is the modified Bessel function of second kind (MacDonald function), β is the electron speed in units of light speed. Here $z_{FDR,BDR}$ are defined by the form of the target surface. For the forward DR (FDR):

$$z_{FDR} = \frac{x_t^2+y_t^2}{4f}$$

For the backward DR (BDR):

$$z_{BDR} = -\frac{x^2+y^2}{4f}$$

The distance R is equal to

$$R = \sqrt{(x_t-x_d)^2+(y_t-y_d)^2+\left(L-\frac{x_t^2+y_t^2}{4f}-\frac{x_d^2+y_d^2}{4f}\right)^2}$$

Since L is much larger than the transverse sizes of the target we can introduce the following expansion:

$$\frac{Exp[\frac{2\pi i}{\lambda}R]}{R} \approx \frac{Exp\left[\frac{2\pi i}{\lambda}\left(L+\frac{x_t^2+y_t^2}{2}\left(\frac{1}{L}-\frac{1}{2f}\right)+\right)\right]}{L}\times$$
$$\times Exp\left[\frac{2\pi i}{\lambda}\left(\frac{x_d^2+y_d^2}{2}\left(\frac{1}{L}-\frac{1}{2f}\right)-\frac{x_t x_d+y_t y_d}{L}\right)\right] \qquad (4)$$

The external vector normal to the surface of the semi-paraboloidal mirror can be written as:

$$\vec{n}_{FDR} = \left\{\frac{x_t}{2f},\frac{y_t}{2f},-1\right\}$$
$$\vec{n}_{BDR} = \left\{\frac{x_t}{2f},\frac{y_t}{2f},1\right\}$$

If f is much larger than the transverse sizes of the target, we can rewrite the upper equation as:

$$\vec{n}_{FDR} = \{0,0,-1\}$$
$$\vec{n}_{BDR} = \{0,0,1\} \qquad (5)$$

Therefore, using Eqs. (1), (3-5) we can find the field of the FDR at the distance L from the first target:

$$E^{FDR}_{x,y}(x_d,y_d,L) = -\frac{1}{\lambda i}\frac{2e}{\gamma\lambda}\frac{1}{L}\int_{-a}^{a}dx_t\int_h^b dy_t\times$$
$$\times Exp\left[\frac{2\pi i}{\lambda}\left(L+\frac{x_t^2+y_t^2}{2}\left(\frac{1}{L}-\frac{1}{2f}+\frac{1}{2\beta f}\right)+\right)\right]\times$$
$$\times Exp\left[\frac{2\pi i}{\lambda}\left(\frac{x_d^2+y_d^2}{2}\left(\frac{1}{L}-\frac{1}{2f}\right)-\frac{x_t x_d+y_t y_d}{L}\right)\right]\times \qquad (6)$$
$$\times\frac{x_t,y_t}{\sqrt{x_t^2+y_t^2}}K_1\left(\frac{2\pi}{\beta\gamma\lambda}\sqrt{x_t^2+y_t^2}\right)$$

The BDR field can be derived in a similar way. The differences from the FDR are: an additional phase term $e^{i\frac{2\pi}{\beta\lambda}L_0}$, corresponding to the phase advance of the electron field moving from one target to another must be added; the normal vector $\vec{n}_{FDR}$ must be changed on $\vec{n}_{BDR}$; and z_{BDR} should be used instead of z_{FDR}. The BDR field can be written as:

$$E_{x,y}^{BDR}(x_d, y_d, L) = \frac{1}{\lambda i}\frac{2e}{\gamma\lambda}\frac{1}{L}\int_{-a}^{a}dx_t\int_{-b}^{-h}dy_t \times$$
$$\times Exp\left[\frac{2\pi i}{\lambda}\left(\frac{L_0}{\beta}+L+\frac{x_t^2+y_t^2}{2}\left(\frac{1}{L}-\frac{1}{2f}-\frac{1}{2\beta f}\right)+\right)\right]\times$$
$$\times Exp\left[\frac{2\pi i}{\lambda}\left(\frac{x_d^2+y_d^2}{2}\left(\frac{1}{L}-\frac{1}{2f}\right)-\frac{x_t x_d+y_t y_d}{L}\right)\right]\times \quad (7)$$
$$\times\frac{x_t, y_t}{\sqrt{x_t^2+y_t^2}}K_1\left(\frac{2\pi}{\beta\gamma\lambda}\sqrt{x_t^2+y_t^2}\right)$$

From the Eq. (6) one can see that for $\beta\to 1$ the term $\left(\frac{1}{L}-\frac{1}{2f}+\frac{1}{2\beta f}\right)$ can be written as $\left(\frac{1}{L}\right)$ and the angular distribution of the FDR from the semi-paraboloidal target in the pre-wave zone [15] is similar to the flat target one in the pre-wave zone, i.e. the radiation is not focused. And from the Eq. (7) one can see that for $\beta\to 1$ the term $\left(\frac{1}{L}-\frac{1}{2f}-\frac{1}{2\beta f}\right)$ can be written as 0 if ($L=f$) and the spatial distribution of the BDR from the semi-paraboloidal target in the pre-wave zone is similar to the flat target one in the wave zone, i.e. the radiation is focused. That was the main difference between FDR and BDR from the semi-paraboloidal target. The backward TR and BDR focusing by concave targets of different shape were discussed earlier in [8,11-12].

For the simulations the following parameters were used: $\gamma = 90$, $\lambda = 1$ mm, $f = 420$ mm, impact parameter $h = 2$ mm, transverse sizes of the targets $2a\times b = 200\times 100$ mm, $L = 840$ mm, $e^2=\alpha$ is the fine structure constant. The integration was made using the Monte-Carlo method with accuracy of about 10%. Throughout the paper the system of units $\hbar = m = c = 1$ was used.

During the simulations the effect of formation zone (effect of interference between the field of radiation and the field of particle during their joint travel) for the FDR was not taken into account. As it was shown in [7] the influence of this effect is not significant and can be neglected. Also the surface current which appears on the edge of the target was not taken into account.

SIMULATION RESULTS

At first a simple test of the model was made. Figure 3 shows the FDR spatial distribution in the vertical direction in the focal point.

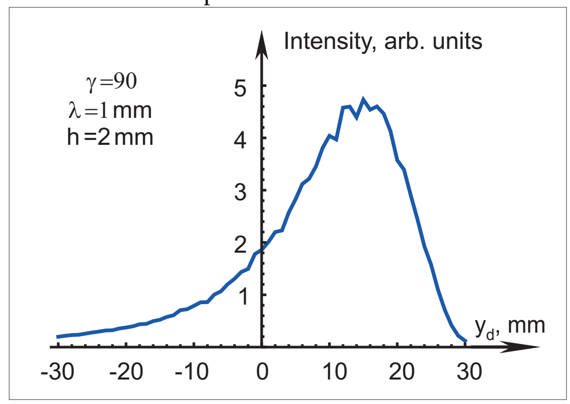

Figure 3. The spatial distribution of the FDR in the vertical direction in the focal point

One may see that the FDR from a semi-paraboloidal target is not focused. Moreover, it is similar to the FDR from a flat target. In papers [8,9] authors have shown that DR can be focused by external optics (for example, by a paraboloidal mirror). Therefore, for the model test purposes the reflection of the FDR from a rectangular paraboloidal mirror with dimensions of $2a\times 2b = 200\times 200$ mm was simulated and compared with the BDR from the semi-paraboloidal target which is focused by the target. Experimentally the fact of the BDR focusing by a semi-paraboloidal target was demonstrated in [10]. Reflected FDR was simulated using the formula (1), where instead of the initial field $E_{x,y}^{0}(x_t, y_t)$ the field of FDR $E_{x,y}^{FDR}(x_t, y_t, L_0)$ represented by Eq. (6) was substituted. The comparison result is shown in Figure 4.

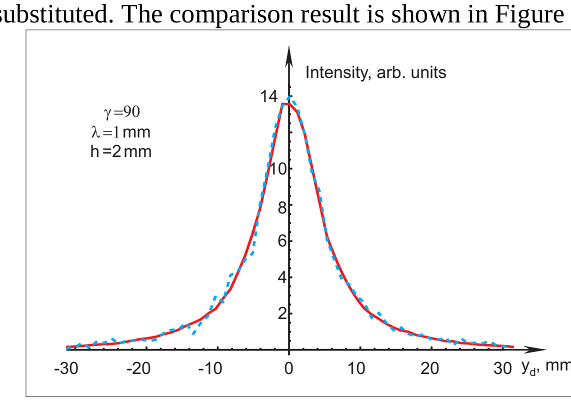

Figure 4. The comparison of the spatial distributions of the BDR (red solid line) and the FDR reflected by the paraboloid (blue dashed line)

From the Figure 4 one can see that both distributions are similar both by the shape and the intensity without using any additional fit parameters. This fact allows to hope that the model is correct.

Figure 5 shows the distribution of the FDR in the vertical direction on the surface of the second target.

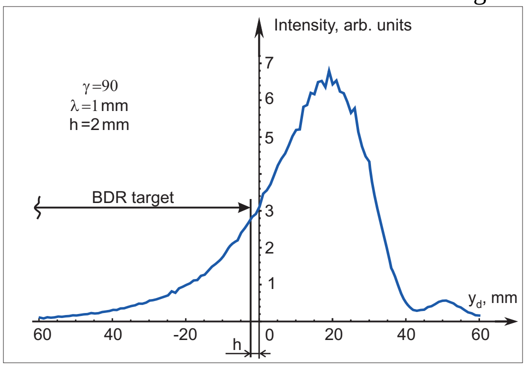

Figure 5. The FDR spatial distribution on the surface of the second target.

The second target is situated in the negative part of the curve. One may see from the figure 5 that the dominant part of the radiation misses the second target. However, the rest of the radiation (about 25%) is reflected by it. At almost the same time the BDR is generated by the electron.

Figure 6 demonstrates the spatial distributions of BDR, reflected FDR and their superposition $4\pi^2 L_0^2 \left(\left| E_x^{ref\ FDR} + E_x^{BDR} \right|^2 + \left| E_y^{ref\ FDR} + E_y^{BDR} \right|^2 \right)$ on the surface of the first target.

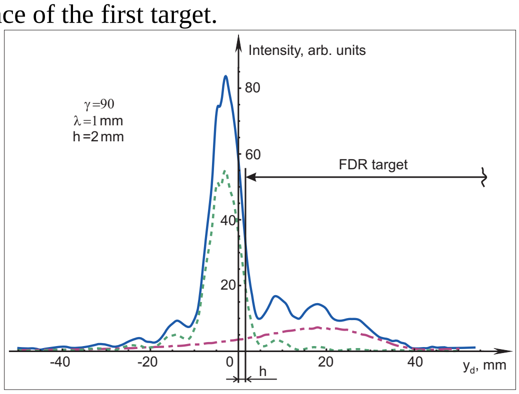

Figure 6. The BDR spatial distributions (green dotted line), the reflected FDR (red dashed-dotted line) and their superposition (blue solid line) on the surface of the first target.

From the Figure 6 one can see that the main part of BDR will be reflected from the first target. The interference of BDR and reflected FDR is constructive, and the increase of intensity is significant. One can say that in spite of the fact that the main part of the FDR is leaked; the rest part of the radiation might stimulate the BDR. One can hope that the stimulation of the FDR by the BDR plus reflected FDR might be stronger. Therefore, one can say that the radiation output might be higher than for the closed resonator based on CTR [1]. Figures 5 and 6 demonstrate that it is possible to detect the radiation in both forward and backward directions. Probably, one can increase the efficiency of this scheme adjusting the tilt angles and position of the mirrors.

CONCLUSION

The theoretical simulations of the proposed scheme of the pre-bunched FEL have shown that there is a constructive interference between BDR and FDR. Therefore one can anticipate that FDR might be stimulated by the BDR and vice versa. Using proposed scheme one can increase the stimulation efficiency by tuning the impact-parameter and the targets tilt angles, and, therefore, one can hope that radiation output would be significantly higher.

In the conclusion one might add that every consequent electron bunch will interact with the stimulated DR in the focal point of the resonator. Due to this interaction the soft x-ray radiation will be generated via inverse Thompson scattering mechanism [11]. This process might be a promising candidate for constructing a compact short-pulse soft x-ray radiation source.

REFERENCES

[1] Y. Shibata et al., NIMA, **528,** (2004),162

[2] A.P. Potylitsyn, in NATO Workshop “Advanced Radiation Sources and Applications”, Springer, New York, p. 149 (2006).

[3] A.P. Potylitsyn, NIMB, 145 (1998), 169

[4] T. Muto, et al., PRL, 90, 10 (2003), 1004801

[5] J.D. Jackson, *Classical Electrodynamics*, John Wiley&Sons, New York, USA, 1998.

[6] M. L. Ter-Mikaelyan, *High Energy Electromagnetic Processes in Condensed Media* (Wiley/Insterscience, New York, 1972).

[7] A. V. Serov and B. M. Bolotovskii, Technical Physics, 42, 9 (1997).

[8] P.V. Karataev, Phys. Letters A **345**, 428 (2005)

[9] B.N. Kalinin, G.A. Naumenko, A.P. Potylitsyn, et al., JETP Lett. **83** 3 (2006)

[10] L.G. Sukhikh, G.A. Naumenko, A.P. Potylitsyn, J. Urakawa, A. Aryshev, S. Boogert, P.V.Karataev, Proceedings of DIPAC 2007, Venice, Mestre, Italy (May 2007), WEPB13, to be published.

[11] A.P. Potylitsyn, L.G. Sukhikh, A.S. Kostousov, J. Urakawa, A. Aryshev, S. Boogert, P.V.Karataev, RUPAC 2006, Novosibirsk, Russia (September 2006) http://cern.ch/AccelConf/r06/PAPERS/MONO07.PDF

[12] A.P. Potylitsyn and R.O. Rezaev, NIMB **252** (2006) 44

[13] C. Settakorn and H. Wiedemann in *Electron-Photon Interaction in Dense Media*, edited by H. Wiedemann, Kluwer Academic Publishers, Dordrecht, 2001.

[14] V.L. Ginzburg and V.N.Tsitovich, "Transition Radiation and Transition Scattering", Adam Hilger, Bristol, 1990.

[15] V.A. Verzilov, Phys. Lett. A 273 (2000) 135