# THEORY OF THERMAL FATIGUE OF A COPPER SURFACE UNDER THE ACTION OF PULSED MICROWAVE HEATING

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*Abstract*

The normal conducting electron-positron Linear Collider projects imply that accelerating structures and other RF components will undergo an action of extremely high RF fields. Except for the RF breakdown threat, there is an effect of the copper surface damage due to cyclic heating of the skin-layer under the action of currents, induced by microwave irradiation.

In this paper we would like to introduce a new "grain" model of the processes responsible for the fatigue of the copper surface [1]. This model is based on the quasi-elastic interaction between neighbouring grains in the copper due to the thermal expansion of the skin layer. This mechanism of fatigue is compared with another, where stresses are generated by the temperature gradient towards the bulk of the material. With the proposed formalism one can estimate the total number of the RF pulses required to fracture the surface depending on the temperature rise, pulse length and steady state temperature. The parameters necessary to finalize the proposed approach were found through the comparison of experimental data obtained at 11.424 GHz.

## INTRODUCTION

Thermal fatigue phenomena were first discovered in electron beam collectors of high-power microwave devices [1]. Contrary to the well-known RF breakdown, thermal fatigue can occur without normal to the metal surface the *E*-field component; however, a great number of RF pulses are necessary to bring about this effect.

Following the growing interest in electron-positron colliders [2], the thermal fatigue has been intensively investigated in experimental studies. In general, these effects could be responsible for the life time of these extremely expensive machines. The first systematic experimental study of thermal fatigue caused by RF pulsed heating, was undertaken several years ago [3]. The experiments at 11.424 GHz were carried out with millions of RF pulses and showed that the *Q*-factor of the tested copper cavities drops dramatically, if the total number of RF pulses exceeds some critical values. These values could be interpreted as the life time of an RF device. It was also proven that the observed degradation of the *Q*-factor was caused by formation of micro cracks on the copper surface. The cracks formed on the surface near the maximum temperature rise (maximum of magnetic field) and appeared in most cases between the copper grains. This allows one to suppose that the polycrystalline structure of copper is essential in thermal fatigue.

Two reliable experimental data points were established following these tests: 1) under the pulsed temperature rise $\Delta T$ = 120 °K the life time corresponds to $55\cdot10^6$ pulses, 2) under $\Delta T$ = 80 °K this number increases to $80\cdot10^6$ pulses.

This paper presents the new theoretical model of thermal fatigue [4], where the polycrystalline structure of the copper surface is evaluated. The main goal of this study was to create the tool with which the material life time can be scaled by using the experimental data received in a reasonably cost-effective way.

## BASIC THEORY OF HEAT LOAD

Ohmic heating of metal in the RF skin layer $\delta_{ohm}$ naturally causes an increase in temperature which is accompanied by heat diffusion deep into the metal bulk. To start with, we will consider a one-dimensional heat conduction equation:

$$\frac{\partial(\Delta T)}{\partial t} - D\frac{\partial^2(\Delta T)}{\partial z^2} = 0, \qquad (1)$$

where $\Delta T$ — is the temperature rise, $t$ — time, $z$ — coordinate normal to the metal surface, $D$ — thermal diffusivity. Here the boundary conditions are given by:

$$\Delta T(z, t=0) = 0, \quad \left.\frac{\partial(\Delta T)}{\partial z}\right|_{z=0} = -\frac{q(t)}{\lambda_T}. \qquad (2)$$

In (2), $q$ is the power of the heat flow. The heat conductivity $\lambda_T$ is connected with $D$ as $D = \lambda_T / c\rho$, where $c$ is the specific heat capacity and $\rho$ — density. The model is shown in Fig. 1, where the expansion of hotter upper layers into the metal is indicated.

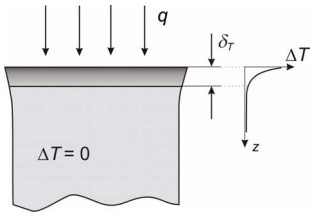


Figure 1: One-dimensional model of heat load.

If $q(t)$ is represented by rectangular pulse with duration $\tau$, and the heating of the surface is caused by a tangential component of magnetic field $H_\tau$ which penetrated inside the RF skin layer of the metal, the maximum temperature rise $\Delta T_{\max}$ appears as:

$$\Delta T_{\max} = \frac{\delta_T}{\lambda_T}\sqrt{\frac{\mu_0\omega}{2\pi\sigma}}H_\tau^2. \qquad (4)$$

where $\mu_0$ is the permeability of the vacuum, $\omega$ — RF frequency, $\sigma$ — specific conductance of copper and $\delta_T$ — is thickness of the heated layer, referred to as the thermal skin layer and expressed as:

$$\delta_T = \sqrt{D\tau}\,. \tag{5}$$

## GRAIN MODEL OF THERMAL FATIGUE

In order to study an irreversible destruction of the surface due to pulsed heating, we will analyze the evolution of the metal surface's microstructure. The ideal copper crystal has a face-centered cubic lattice, where any atom is linked to the neighboring one. The probability to break any of these links is well-known:

$$p_j = \exp\left(-U_c / k_B T_c\right), \tag{6}$$

Here $U_c$ — is an average energy of the coupling between neighboring atoms, $T_c$ is a steady state temperature and $k_B$ is a Boltzman's constant. If $T_c > 0$, the probability is nonzero and broken links occur every time, however, if the temperature is well below the melting point, the value of $p_j$ is rather small because $U_c / k_b T_c >> 1$.

In reality, the copper surface contains many different defects. The copper surface at a microscopic level is not homogeneous but is composed of many grains. The size of each grain is dependent on the material purity and treatment. For our purposes, each of the grain interiors will be treated as an ideal lattice. The random orientation of the individual grains is also considered. Following the experimental observations, we will assume that the coupling between grains is much weaker compared to that which can be found inside the ideal crystal.

For the pulse duration which is in the range of interest related to the Linear Collider application ($\tau \leq 10$ μs) the thermal skin layer is significantly smaller than the typical size of the individual copper grain. As a result, the expansion in the thermal skin layer brings us to the situation when neighboring grains start to push against each other, thus bringing extra elastic forces to the scene. Altogether, this increases the probability of breaking the links between the atoms, see Fig. 2.

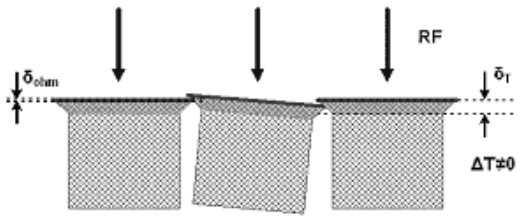


Figure 2: Surface grains interface in the heating process.

Taking into account energy of the elastic forces $\overline{U}$, the probability (6) becomes:

$$p_j = \exp\left(-\left(U_c - \overline{U}\right)/k_B T_c\right). \tag{7}$$

To introduce the effect of the grains interface, we will designate $n_b$ to the number of broken links between neighboring grains, while $n_u$ will remain as the number of unbroken ones. A total balance in each cycle of heating can then be written for reversible links breaking (when the broken link can be repaired after the cycle is finished):

$$\begin{aligned} n_u^{i+1} - n_u^i &= -p_j n_u^i + \left(1 - p_j\right) n_b^i, \\ n_b^{i+1} - n_b^i &= p_j n_u^i - \left(1 - p_j\right) n_b^i, \end{aligned} \tag{8}$$

where $n_b^i$ and $n_u^i$ are the numbers of broken and unbroken links in the steady state (the state with constant temperature) before the cycle $i$ and $n_b^{i+1}$ and $n_u^{i+1}$ are those numbers during the cycle. Following (8) in a steady state $n_b / n_u = p_j / (1 - p_j) \approx p_j$, which can also be written as $n_b = p_j n_u \equiv p_j n_s$, where $n_s$ is the total number of links between a given grain and neighboring grains.

It is natural to expect that some of the links may turn out to be completely broken after a thermal cycle and the number of irreversibly broken links is proportional to the total number of existing links, broken during the cycle:

$$\delta n_{irr} = \tilde{\alpha} \Delta n_b = \tilde{\alpha}\left(p_j(\Delta T) - p_j(\Delta T = 0)\right) \cdot n_s \tag{9}$$

where $\tilde{\alpha}$ is a constant. Expression (9) with regard to (7) allows us to write the probability of an *irreversibly* broken link formation:

$$p_j^{irr} = \frac{\delta n_{irr}}{n_s} = \tilde{\alpha} \cdot \exp\left(-\frac{U_c}{k_B T_c}\right)\left(\exp\left(\frac{\overline{U}}{k_B T_c}\right) - 1\right). \tag{10}$$

Assuming that every link is broken independently, the probability of breaking all $n_s$ irreversibly after $N$ thermal cycles can be written taking into account $U_c >> k_b T_c$ as

$$P \approx N \cdot \prod_{j=1}^{j=n_s} p_j^{irr}\,. \tag{11}$$

Apparently, $P$ can potentially reach some critical values which are large enough to cause the copper surface to fracture [3]. Using (10) and (11) the total number of pulses required to develop cracks on the surface is

$$N_f = P_c \cdot \exp\left(\frac{U_c n_s}{k_B T_c}\right)\left(\exp\left(\frac{\overline{U} n_s}{k_B T_c}\right) - 1\right)^{-1}, \tag{12}$$

where $P_c$ is considered as a certain critical value.

For every given grain with $n_s$ existing links we can write the elastic forces energy (see [4]):

$$\overline{U} = \frac{E}{1 - \mu} \cdot \frac{\alpha^2 \cdot \delta_T \cdot l_0^2 \cdot \Delta T^2}{2 \cdot n_s}, \tag{13}$$

where $\alpha$ is the coefficient of thermal expansion of the individual grain with a typical size of $l_0$, $E$ is Yung's module and $\mu$ is Poisson coefficient.

We can now express the general equation for the number of pulses necessary to fracture the copper surface, keeping the least number of unknown parameters ($C$, $\zeta$):

$$N_f = \frac{C}{\exp\left(\zeta \cdot \sqrt{\tau} \cdot \Delta T^2\right) - 1}. \qquad (14)$$

Using experimental data points from [3] we can estimate the values of $C$ and $\zeta$, see Fig. 3.

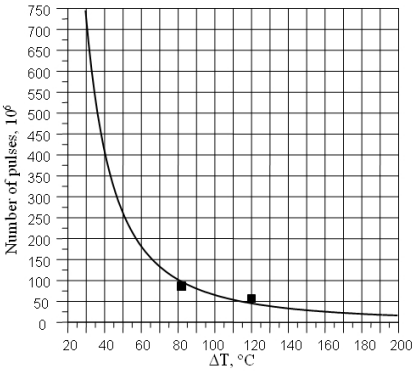


Figure 3: The experimental data points and the theoretical fit of the total number of pulses required to fracture the copper surface at $\tau$ = 1000 ns.

The life time dependence on the absolute value of the pulse length we would consider as a new effect, which was never predicted in former theories. This for example, brings the condition for the "fixed" life time:

$$\Delta T^2 \cdot \tau^{1/2} = \text{const} \qquad (15)$$

which shows that it is possible to match $\Delta T$ and $\tau$ in order to keep a similar life time.

## OTHER FATIGUE MECHANISMS

### *Stresses caused by temperature gradient*

Next, we will consider the stresses caused by temperature gradient on the metal surface. The origin of these stresses is that the upper atomic layer, at temperature $\Delta T_1$ will expand more widely than the underlying layer, heated up to $\Delta T_2 < \Delta T_1$. Assuming that the forces between atomic layers have a quasi-elastic nature, we can find that the energy of elastic forces depends on $\tau$ and $\Delta T$ as follows:

$$\overline{U}_{\nabla T} \propto \left(\nabla T\right)^2_{\max} \propto \left(\Delta T\right)^2 / \sqrt{\tau} \qquad (16)$$

and the critical number of pulses according to (12)

$$N_f = \frac{C_1}{\exp\left(\zeta_1 \cdot \Delta T^2 / \sqrt{\tau} \cdot\right) - 1}. \qquad (17)$$

where $C_1$ and $\zeta_1$ — are the constants to be derived from experimental data.

Equation (17) is analogous to (14), but each of them should be valid in their specific regions of pulse length. Being accepted, for the similar temperature rise ($\Delta T > 0$), the life time should go to zero for both extremities: $\tau \to 0$ (17) and $\tau \to \infty$ (14), even for the second case, the term "pulsed" looses its meaning.

For typical grain size $l_0 < 0.2$ mm the contribution of temperature gradient effects is significant only at pulse durations $\tau < 3$ ns. At longer pulses the life time of the copper surface is conditioned on grain interaction effects.

### *Long pulses approximation*

The formula similar to (14) and (17) can be obtained in one more important case — the case of long pulses, such that the surface grains have fairly even heat load distribution. This corresponds to the condition $\delta_T >> l_0$. and actually describes volume effects. The elastic energy now can be derived from (13) by replacing $\delta_T$ with $l_0$ and $\tau$-dependence of $N_f$ becomes then

$$N_f = \frac{C_2}{\exp\left(\zeta_2 \cdot \Delta T^2 / \tau\right) - 1}. \qquad (18)$$

## CONCLUSION

The theory of the copper surface fatigue induced by pulsed heating due to RF Ohmic losses has been presented. For the fixed absolute temperature rise, the different mechanisms appeared to be responsible for the fracturing of the copper surface, depending on the pulse length of the applied fatigue cycle.

In the range of RF pulse lengths (10–1000 ns) which is of the highest interest for normal conducting Linear Colliders the major contribution to the thermal fatigue comes from the interaction between the copper grains located close to the surface.

This study evaluates a tool which can allow the prediction of the material life time behavior towards an extremely high number of fatigue cycles, provided the experimental data for low statistics is available.

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