# RECOVERY PROCESS STABILITY STUDY IN ENERGY RECOVERY ACCELERATOR

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*Abstract*

Energy recovery technique in rf accelerator based applications allows to save rf power and reduce radioactive background as well. In this operation mode used beam is directed back to the accelerator in decelerating rf phase where it returns back its kinetic energy to rf field. Thus, rf generator that feeds linac covers cavities walls rf losses only and those part of beam kinetic energy that used for useful effects production as well. The sum of three fields – induced in the linac by an external rf source, accelerated and decelerated beams – determines energy and phase of the beam at linac exit, and together with beam return path optics amplitude and phase of decelerated bunches and hence third component of mentioned sum. In the case of positive sign of this feedback and sufficient amplification in the closed loop just described instability takes place.

The main equations that determine beam-rf cavity interaction in energy recovery rf accelerator are derived, single mode approximation being used. Expressions for small deviation from steady state are obtained followed by stability analysis. Results of calculations for increments of instability are presented and discussed.

## INTRODUCTION

In some electron accelerator applications, only small fraction of kinetic beam energy is used, high brightness light sources of the next generation and free electron lasers being the typical examples. Keeping in mind large value of the beam energy in similar applications (it may be as large as hundreds megawatts) very fruitful idea to recover beam energy is widely discussed and already used in all over the world [1,2,3]. In recovery process used beam is guided back to the same accelerator in decelerating phase and for this reason reduces its kinetic energy along the accelerator. In other words, accelerated and decelerated bunches are spaced by half period of rf field, and in the case of lossless beam recirculation the first harmonic of total current is equal to zero and the total radiation field is equal to zero as well. The question arises whether the recovery process just described is stable in the sense that small perturbations of steady state result in such behaviour of the system that such perturbations tend to zero with the laps of time. Among the many one mechanism of feed back in the system beam – cavity may take place. Any change in beam energy results in phase shift of decelerated bunches if the longitudinal dispersion of return path is not equal to zero. This phase shift in turn results in phase shift of the voltage induced by decelerated bunches in accelerating cavity and thus in amplitude and phase of the total voltage at accelerating gap changing and as the result in energy changing acquired by the accelerated bunches. This feed back may result in instability in the case of its positive sign and sufficient amplification in the closed feed back loop.

Following is quantitative analysis of the processes just described. Single mode approximation is used in beam-cavity interaction equations. We limit ourselves by linear approach in stability analysis.

## THE EQUATIONS OF BEAM-CAVITY INTERACTION

Fig.1 represents the main features of energy recovery linac. Electron beam from injector directed into the main accelerator consisting of rf cavities. Being accelerated electron bunches are rotated by two arcs consisting of bending magnets and entered the same linac in decelerating phase. Thus, electron bunches in two beams are shifted by the angle close to 180 degrees to each others.

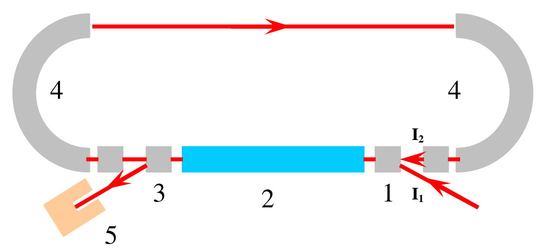


Fig.1. General layout of energy recovery linac. 1– magnet inflector, 2– main linac, 3 – magnet deflector, 4 – bending magnets, 5 – beam absorber.

While passing the main linac in decelerating phase the secondary beam looses its energy and with the help of deflector at the linac exit leaves accelerator and is directed into the beam absorber. In analysis that follows we will assume all voltage and currents being time dependent as complex exponent function $\exp(i\omega t)$ with the appropriate amplitudes which are slow functions of time $t$. Fig.2 represents these values on complex plane.

According to the superposition principle the total voltage $\hat{U}_\Sigma(t)$ at cavity gap is equal to the sum of three voltages – the first one $\hat{U}_e(t)$ excited by the external rf generator, while the second $\hat{U}_1(t)$ as well as the third one $\hat{U}_2(t)$ induced by the accelerated and decelerated beams:

$$\hat{U}_\Sigma(t) = \hat{U}_e(t) + \hat{U}_1(t) + \hat{U}_2(t)\,, \qquad (1)$$

where $t$ stands for time and "hat" symbols above letters denotes complex values. The primary (being accelerated) beam energy gain in the main linac is:

$$E = eU_\Sigma \cos(\psi) = \mathrm{Re}(e\hat{U}_\Sigma) \qquad (2)$$

Here $U_\Sigma, \psi$ stand for the amplitude and the phase of accelerating voltage and $\mathrm{Re}$ means real part. To simplify calculations, here and in the formulae followed all voltages are understand as energies acquired by unit charge after cavity passage.

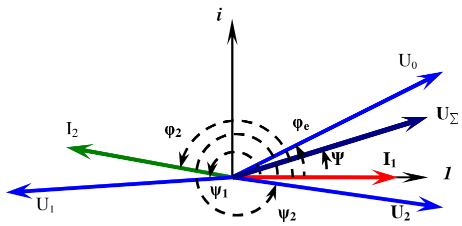


Fig.2. Voltages and currents on complex plane. $I_1$-accelerated beam, $I_2$-decelerated beam, $U_0$-the voltage induced in cavity by external rf generator, $U_1$-the voltage induced by accelerated beam, $U_2$-the voltage induced by decelerated beam, $U_\Sigma$-total voltage at the cavity gap.

It follows from cavity excitation electrodynamics that:

$$\frac{d^2u}{dt^2} + \frac{\omega_0}{Q}\frac{du}{dt} + \omega_0^2 u = -\frac{\omega_0 R}{Q}\frac{dJ}{dt} \qquad (3)$$

Here $u(t)$ stands for voltage of the accelerating mode, $Q$ are cavity quality factor, $J$ is the first harmonic of the accelerated current, $\omega_0$ is the cavity eigen frequency

$$Q = \frac{Q_0}{1+\beta},\ R = \frac{R_0}{1+\beta}\ , \qquad (4)$$

where $R_0$ is cavity shunt impedance and $\beta$ is cavity coupling coefficient respectively.

Representing beam current and induced field in the form

$$J(t) = I(t)\exp(i\omega t),\ u = U(t)\exp(i\omega t)\,, \qquad (5)$$

where $I(t), U(t)$ are slow functions of time one arrives to the end at the differential equation for complex amplitudes:

$$\frac{dU}{dt} + \left(\frac{\omega_0}{2Q} + i\Delta\omega\right)U = -\frac{\omega_0 R}{2Q} I\ . \qquad (6)$$

While deriving this equation we neglect the terms which are small sufficiently compared to the remaining in (6).

## STABILITY ANALYSIS

We will carry out stability analysis in linear approximations. It follows from (6), that

$$\frac{d\Delta\hat{U}}{dt} + \left(\frac{\omega_0}{2Q} + i\Delta\omega\right)\Delta\hat{U} = -\frac{\omega_0 R}{2Q}\Delta\hat{I} \qquad (7)$$

where $\Delta$ means small deviation from steady state. As it has been declared in introduction we study beam phase – cavity voltage instability mechanism and for this reason we assume that

$$\Delta\hat{I}_2 = \Delta(I_2\exp(i\varphi_2)) = iI_2\exp(i\varphi_2)\Delta\varphi_2 \qquad (8)$$

where $I_2$ and $\varphi_2$ are the amplitude and the phase of the decelerated electron bunches at the accelerator entrance. In the analysis followed these are assumed be constants, while $\Delta\varphi_2$ is a function of time. The deviation of $\varphi_2$ originates from the energy deviation $\Delta E$ at the accelerator exit followed by subsequent return path changing due to longitudinal dispersion:

$$\Delta\varphi_2(t) = \frac{2\pi\alpha L}{\Lambda}\frac{\Delta E(t-T)}{E} \qquad (9)$$

The arguments values in formula (9) reflect the fact that there is the delay $T$ between energy changing and following appropriate phase shift of the secondary bunches. Here $T$ is revolution period, $\alpha, L, \Lambda$ are momentum compaction factor, magnetic arcs length and accelerating voltage wavelength respectively.

As it follows from (1) and (2)

$$\Delta E = e\,\mathrm{Re}\,\Delta(\hat{U}_e + \hat{U}_1 + \hat{U}_2) = e\,\mathrm{Re}(\Delta\hat{U}_2) \qquad (10)$$

The equations (7), (10) and (10) determine the behavior of the system under discussion for small deviation from steady state. These have to be rewritten in real representation for following analysis:

$$\Delta\varphi_2(t) = \frac{2\pi\alpha L}{\Lambda}\frac{\Delta E(t-T)}{E} \qquad (11)$$

$$\frac{d\Delta U_{2x}}{dt} + \frac{\omega_0}{2Q}\Delta U_{2x} - \Delta\omega\Delta U_{2y} = \sin\varphi_2\frac{\omega_0 R}{2Q}I_2\Delta\varphi_2 \qquad (12)$$

$$\frac{d\Delta U_{2y}}{dt} + \frac{\omega_0}{2Q}\Delta U_{2y} + \Delta\omega\Delta U_{2x} = -\cos\varphi_2\frac{\omega_0 R}{2Q}I_2\Delta\varphi_2 \qquad (13)$$

$$\Delta E = e\Delta U_{2x} \qquad (14)$$

Normalising all voltages by the amplitude $U_\Sigma$ we have the following system after simplifications:

$$\frac{d\Delta u_{2x}}{dt} + \frac{\omega_0}{2Q}\Delta u_{2x} - \Delta\omega\Delta u_{2y} = A\sin\varphi_2\Delta u_{2x}(t-T) \qquad (15)$$

$$\frac{d\Delta u_{2y}}{dt} + \frac{\omega_0}{2Q}\Delta u_{2y} + \Delta\omega\Delta u_{2x} = -A\cos\varphi_2\Delta u_{2x}(t-T) \qquad (16)$$

$$A = \frac{\omega_0 R}{QU_\Sigma}I_2\frac{\pi\alpha L}{\varepsilon\Lambda} \qquad (17)$$

Hear $\Delta u = \Delta U / U_\Sigma$ for all indexes, $\varepsilon = E/eU_\Sigma$ and $\Delta\varepsilon = \Delta E/eU_\Sigma$. We will find solutions of the system (15)-(17) in the form exponential dependence of the variables. Substituting $\Delta u_{2x} = \exp(kt)$ and $\Delta u_{2y} = a\exp(kt)$ into the equations above we have:

$$k + \frac{\omega_0}{2Q} - a\Delta\omega = A\exp(-kT)\sin\varphi_2 \qquad (18)$$

$$ak + a\frac{\omega_0}{2Q} + \Delta\omega = -A\exp(-kT)\cos\varphi_2 \qquad (19)$$

The case $k = 0$ corresponds to threshold of static instability (if it takes place). Substituting this value into the system (18), (19) and resolving the system relative $A$ one has the following expression for instability threshold

$$\frac{2\pi L}{\varepsilon\Lambda}\frac{I_2 R}{U_\Sigma}\left|\alpha(\sin\varphi_2 - \frac{2Q\Delta\omega}{\omega_0}\cos\varphi_2)\right| = 1 + (\frac{2Q\Delta\omega}{\omega})^2 \qquad (20)$$

For instability to take place the expression in brackets has to be positive. The instability does take place at all if longitudinal dispersion of beam return path is equal to zero or, in our notation, momentum compression factor of magnet arcs is equal to zero: $\alpha = 0$. The feed back is broken also if

$$\tan\varphi_2 = \frac{2Q\Delta\omega}{\omega_0} \qquad (21)$$

Keeping in mind that angle $\varphi_2$ (recovery angle) is close to $\pi$ in energy recovery accelerator and representing this in the form $\varphi_2 = \pi + \eta$, $\eta$ being close to zero, one can notice that feed back is broken when $\eta$ is equal to cavity detuning angle.

If $\tan\eta > 2Q\Delta\omega/\omega_0$ the feed back becomes negative for the case $\alpha > 0$, and the static instability does not take place. But it is well known from feed back systems theory that negative in static sense feed back may become positive at definite frequency range resulting in unstable state. Let us find the solution of the system (18), (19) for $k = \mathrm{Re}\,k + i\,\mathrm{Im}\,k = 0 + i\Omega$ supposing zero cavity detuning. It follows from (18) that

$$\frac{\omega_0}{2Q} = A\sin\varphi_2\cos\Omega T,\ \Omega = -A\sin\varphi_2\sin\Omega T \qquad (22)$$

This is transcendental system relative the variables $\Omega$ and $A$, but simple considerations deliver us to the approximate solution

$$\Omega T \approx \frac{\pi}{2}, -A\sin\varphi_2 \approx \frac{\pi}{2T} \qquad (23)$$

Upper formulae determine the least roots of the system (22). It follows from (17) and (23), that instability threshold for $\alpha\sin\varphi_2 < 0$ (for zero cavity detuning)

$$\frac{2\pi L}{\varepsilon\Lambda}\frac{I_2 R}{U_\Sigma}\left|\alpha\sin\varphi_2\right| \approx \frac{\pi}{2}\frac{\tau}{T}, \qquad (24)$$

where $\tau = 2Q/\omega_0$ is cavity time constant. As compared with expression (20) instability threshold is $\frac{\pi}{2}\frac{\tau}{T}$ higher provided $\tau >> T$, as it takes place for superconducting cavity for example.

It is worthwhile to note that instabilities just described remind those arising in racetrack microtron [4]. Mutual dependences of the kind voltage - current and injected bunches phase – current inherent to the accelerators of microtron types result in similar system behaviour.

## CONCLUSION

In energy recovery rf accelerator recovered electron beam – cavity interaction takes place resulting in static or dynamics instability. The instability is coursed by mutual dependence of the accelerating voltage and the phase of bunches that entered once again the cavity to be decelerated. The instability mechanism reminds those inherent to high current racetrack microtron.

The mechanism of beam–cavity interaction just explored takes place in rf electron recirculating linac as well. Substantial difference is in shunt impedance $R$ value for recovery and recirculating linacs. Since $R = R_0/1+\beta$ for superconducting cavity case $R_{recirculating} << R_{recovery}$ for the reason that in the first case $\beta >> 1$, while in the second case coupling is mach less (close to unity for ideal machine). That is why the threshold for beam current is shifted to mach higher value for recirculating linac.

The stability problems just described are also discussed in paper [5].

This work had been stimulated by numerous discussion concerning possible schemes of new accelerator complex realisation on the basis of superconducting cavities at Lebedev Physical Institute [6], and the author is obliged to his colleagues for these fruitful discussions.

## REFERENCES

[1] U.Amaldi. Physics Letters, Volume 61B, number 3, 1976, pp. 313-315.
[2] V.E.Balakin, G.I.Budker, A.N.Skrinski. Procedings of the 6-th All union conference on charged particle accelerators, Dubna, October 11-13, 1978, v.1, pp.27-41, in Russian.
[3] ERL09 Workshop Proceedings. June 8-28, 2009. https://wiki.lepp.cornell.edu/web/bin/view/LEPP/Events/ERL09/
[4] V.G.Kurakin. High Current Racetrack Microtron as a Free Electron Laser Driver. Nuclear Instrument and Metods in Physics Research, A341(1994), pp 407-411.
[5] N.A.Vinokurov et al. Critical issues for high power FEL based on microtron recuperator/electron out coupling scheme, in Proceedings of SPIE, Vol.2988, 1997, pp. 221-231.
[6] V.G.Kurakin et al. Superconducting RF Electron Recirculator for Nuclear Physics Research at Lebedev Physical Institute, in Proceedings of the 22 Russian conference on charged particles accelerators RuPAC-2008, Zvenigorod, September 28 - October 3, 2008, pp.107-109.