RESEARCH OF PHOTON EMISSION OF 120 GEV CHANNELING POSITRONS

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Abstract
The motion of positrons in the interplanar nonlinear potential of a straight thin Si crystal and radiation spectra are calculated.

INTRODUCTION

By this time the considerable number of experimental and theoretical works is devoted to researching the radiation at plane channeling of high energy positrons in monocrystals [1, 2, 3]. This radiation arises during the motion of a charged particle under a small angle in relation to a crystallographic plane and for positrons with energies up to ~20 GeV is monochromatic enough and is characterized by high intensity. At energies of positrons more than ~20 GeV monochromaticity of the radiation strongly degrades. In September, 2009 in CERN the experiment INSURAD devoted to research of radiation at various orientations of bent monocrystals has been made at energy of positrons of 120 GeV. The radiation type of a relativistic particle depends on the value of multipole parameter ρ. When $\rho << 1$ it corresponds to the interference type (dipole approximation) of the radiation formed along sufficiently large length of the crystal. The case with $\rho >> 1$ is close to the synchrotron radiation. At energies of positrons 100 GeV and more the parameter ρ can exceed 20 units for a considerable part of the particles.

In the given work we wish to receive the following results: to define characteristic parameters of motion of an ultrarelativistic particle in real plane potential of a monocrystal and to study the influence of its nonlinearities on ensemble of particles captured in a mode channeling; to calculate radiation spectrums of positrons with energies an order 100 GeV at their different entry initial conditions on an input in a monocrystal.

INTERPLANAR ONE–DIMENSIONAL MOTION OF CHANNALED POSINRONS

The motion of a charged ultrarelativistic particle in the interplanar electric field D of a monocrystal can be described by the following system of equations

$$\frac{E}{c^2}\frac{d^2x}{dt^2}=e\mathrm{D}(x),\quad \frac{d^2y}{dt^2}=0\ ,\quad \frac{ds}{dt}=c\left(1-\frac{1}{2\gamma^2}-\frac{1}{2c^2}\left(\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right)\right),$$

where: x, y, s - the Cartesian co-ordinates of a particle (the electric field D is directed along the axis x); E, e, γ - energy, charge and gamma factor of a particle, accordingly; t - time, c - velocity of light. The first equation describes periodic motion along x, the third equation reflects the influence of transverse motion on longitudinal one. From the above equations it is seen that the problem of finding the trajectory of a particle in three-dimensional space is reduced to finding the function $x(t)$.

We will consider periodic (generally nonharmonic) motion of positrons with energy of $E = 120$ GeV in the interplanar potential of a straight crystal Si with orientation (011). The interplanar potential is calculated for silicon at a room temperature as it is described in work [4]. The interplanar potential of interaction of a positron in a straight crystal is defined by expression [4, 5]

$$U(\xi)=-\frac{d}{2}\sum_{k=1}^{7}\frac{\alpha_k}{2k}\xi^{2k}\quad ,\qquad (1)$$

where:: $\xi = 2x/d$ - normalized interplanar coordinate, $\xi\in[-1,+1]$; $d = 1.92$ Å - interplanar distance in (011) channel; $\vec{\alpha}=(-32.21\ \ 13.86\ \ -443.78\ \ 2340.52\ \ -5315.05\ \ 4811.79\ \ -1375.13)$ in [eV/Å]; such values of $\vec{\alpha}$ provide $dU/d\xi = 0$ at $\xi=\pm1$.

In a Fig. 1 the dependence of normalized frequency $\Omega(\xi_{\mathrm{m}})=\omega(\xi_{\mathrm{m}})/\omega_{\mathrm{o}}$ on the amplitude of periodic motion is shown, where: $\omega_{\mathrm{o}}=(2|\alpha_1|c^2/Ed)^{1/2}\cong 5.013\times10^{13}/\mathrm{sec}$ - frequency of oscillations with small (zero) amplitudes in the potential hole $U(\xi)$. The maximum displacement of periodic motion is interpreted as amplitude ξ_{m}. The motion of a positron in normalized potential well $\widetilde{U}(\xi)=2U(\xi)/|\alpha_1|d$ is described by the canonical equations

$$d\xi/d\tau = p \quad\text{и}\quad dp/d\tau=-d\widetilde{U}(\xi)/d\xi\ ,\qquad (2)$$

where: $\tau=\omega_{\mathrm{o}}\,t$ - the dimensionless time (phase); $p^2/2+\widetilde{U}(\xi)=\varepsilon$ - the transverse energy (integral of motion). Maximum deviation (amplitude) $\xi_{\mathrm{m}}(\varepsilon)$ is defined from equation $\widetilde{U}(\xi_{\mathrm{m}})=\varepsilon$. The dependence of normalized frequency Ω on amplitude ξ_{m} is determined by

$$\Omega(\xi_{\mathrm{m}})=1/\frac{4}{2\pi}\int_0^{\xi_{\mathrm{m}}}d\xi/\sqrt{2(\widetilde{U}(\xi_{\mathrm{m}})-\widetilde{U}(\xi))}\ .$$

The multipole parameter ρ is expressed through parameters of plane periodic motion of a particle as follows [1]: $\rho=2\gamma^2<(\upsilon_{\mathrm{x}}/c)^2>$, where the averaging is taken over the motion period. For the channeled positron with the given ξ_{m} we have

$$\rho(\xi_{\mathrm{m}})=\gamma^2\kappa^2\Omega(\xi_{\mathrm{m}})\frac{4}{\pi}\int_0^{\xi_{\mathrm{m}}}\sqrt{2(\widetilde{U}(\xi_{\mathrm{m}})-\widetilde{U}(\xi))}\,d\xi\ ,$$

where $\kappa=d\omega_{\mathrm{o}}/2c\cong16.052\cdot10^{-6}$. In Fig. 2 (the continuous line) the exact dependence of multipole parameter on amplitude $\xi_{\rm m}$ is shown. Thus, at the given potential both dipole and magnetic bremsstrahlung radiation types can be realized.

The received frequencies correspond to nonlinear (not harmonic) oscillations. The closer $\xi_{\rm m}$ to 1, the stronger difference of periodic motion from the harmonic one. Comparison at given $\xi_{\rm m}$ of the exact numerical decision of the equation of motion with approximating harmonic oscillation $\quad \xi = \xi_{\rm m}\cos\big(\Omega(\xi_{\rm m})\cdot\tau\big) \quad (3)$

with the same $\xi_{\rm m}$ and frequency $\Omega(\xi_{\rm m})$ shows that practically in all range $0 \le \xi_{\rm m} \le 0.980$ we can consider the motion of channeled positrons to be the harmonic one. In harmonic approximation (3) the expression for multipole parameter simplifies to $\rho(\xi_{\rm m}) = \left(\gamma\,\kappa\,\Omega(\xi_{\rm m})\,\xi_{\rm m}\right)^2$ and is shown in Fig. 2 (dashed line). It is seen that the harmonic approximation of periodic motion of positrons with energy of $E = 120$ GeV is quite acceptable for calculation of the radiation spectrum of channeled particles.

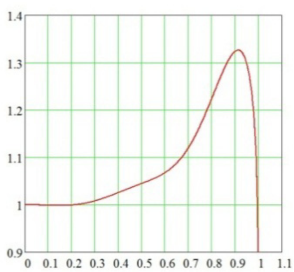

Fig.1: Dependence $\Omega(\xi_{\rm m})$.

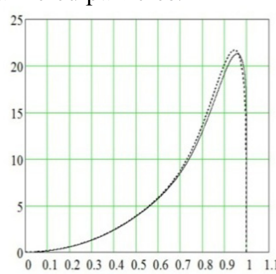

Fig.2: Dependences $\rho(\xi_{\rm m})$.

DISTRIBUTION OF CHANNELED PARTICLES ON AMPLITUDES OF MOTION

For determining the full spectrum of radiation from all captured in the channeling positrons it is necessary to know: N - a relative part of particles of the beam, captured into the channeling; $f(\xi_{\rm m})$ - density distribution of channeled positrons on amplitudes $\xi_{\rm m}$. We suppose that at the entry to the straight crystal positrons are distributed uniformly along x, and hence along ξ, and with the angular distribution $g(\vartheta)$. In normalized variables (ξ, p) according to (2) we have the following relation between ϑ and p: $p = \dfrac{d\xi}{d\tau} = \dfrac{\upsilon_{\rm s}}{\omega_{\rm o}(d/2)}\dfrac{dx}{ds} \cong \dfrac{2c}{\omega_{\rm o} d}\vartheta = \dfrac{\vartheta}{\kappa}$, where $s = \upsilon_{\rm s} t \cong ct$ is the longitudinal coordinate along the channel. From here the distribution of particles at the entry to the crystal on variable p becomes $\tilde{g}(p) = \kappa\, g(\kappa p)$. Closed phase curve $p = p(\xi, \xi_{\rm m})$ in the plane (p, ξ) with a fixed $\xi_{\rm m}$ for a channeled particle (see Fig. 3) is given by expression $\quad p(\xi, \xi_{\rm m}) = \pm\sqrt{2(\tilde{U}(\xi_{\rm m}) - \tilde{U}(\xi))} \quad (4)$

with $\xi \in [-\xi_{\rm m}, \xi_{\rm m}]$. Separatrix is (the phase curve, separating the channeled and over-barrier particles) described by expression $p_{\rm c}(\xi) = p(\xi, 1)$. The maximum value of $p_{\rm L}$ corresponds to the Lindhard angle $\vartheta_{\rm L}$, achieved at $\xi = 0$ and equal to $p_{\rm L} = |p_{\rm c}(0)| = \sqrt{2\tilde{U}(1)}$. At the considered parameters of the crystal and the magnitude of the positrons energy we have: $p_{\rm L} \cong 1.189$ and $\vartheta_{\rm L} \cong 19.093\times10^{-6}$. Thus, the portion of particles captured in the channeling mode, i.e. moving inside the separatrix, is given by

$$N = \int_0^1 d\xi \int_{-|p_{\rm c}(\xi)|}^{|p_{\rm c}(\xi)|} \tilde{g}(p)\,dp \;. \qquad (5)$$

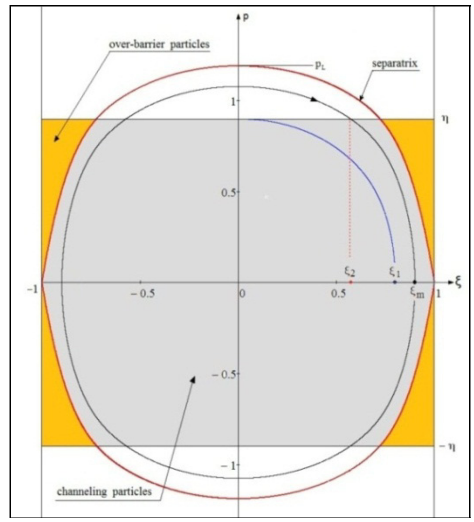

Fig.3: Phase portrait of capture of particles in the channeling.

Now we find the density function $f(\xi_{\rm m})$ of particles distribution on the amplitudes only for the particles occurring in channeling. Hereinafter we mean $p(\xi, \xi_{\rm m})$ to be a positive branch of the definition (4). The relative number of channeled particles N with amplitude $\le \xi_{\rm m}$ is equal to

$$F(\xi_{\rm m}) = \frac{1}{N}\int_0^1 d\xi \int_{-p(\xi,\xi_{\rm m})}^{p(\xi,\xi_{\rm m})} \tilde{g}(p)\,dp \;. \qquad (6)$$

Then for the density function is determined as $f(\xi_{\rm m}) \equiv dF(\xi_{\rm m})/d\xi_{\rm m}$ i.e.

$$f(\xi_{\rm m}) = \frac{1}{N}\frac{d\tilde{U}(\xi_{\rm m})}{d\xi_{\rm m}} \int_0^{\xi_{\rm m}} d\xi \frac{\tilde{g}(p(\xi,\xi_{\rm m})) + \tilde{g}(-p(\xi,\xi_{\rm m}))}{p(\xi,\xi_{\rm m})} . \qquad (7)$$

We confine ourselves to the simplest case of uniform and symmetric about zero angle distribution of particles, i.e. $g(\vartheta) = 1/2\vartheta_{\rm o}$ if $\vartheta \in [-\vartheta_{\rm o}, \vartheta_{\rm o}]$, $g(\vartheta) = 0$ if $\vartheta \notin [-\vartheta_{\rm o}, \vartheta_{\rm o}]$. Hence, in the plane of normalized variables we have $\tilde{g}(p) = 1/2\eta$ if $p \in [-\eta, \eta]$ and $\tilde{g}(p) = 0$ if $p \notin [-\eta, \eta]$, here $\eta = \vartheta_{\rm o}/\kappa$ is the boundary of the beam with variable p. For the case when the half-width of the angular spread is less than the Lindhard angle, $\vartheta_{\rm o} < \vartheta_{\rm L}$ and, hence $\eta < p_{\rm L}$ (see Fig. 3), we introduce the amplitude ξ_1, for which the phase curve has a maximum $p(0,\xi_1)=\eta$. In addition, for every phase curve with the amplitude $\xi_1<\xi_m\le 1$ we determine the value ξ_2 which depends on ξ_m. Dependence $\xi_2(\xi_m)$ is determined by $\sqrt{2(\widetilde{U}(\xi_m)-\widetilde{U}(\xi_2))}=\eta$.

According to the formulas (5÷7) for our case of the angular spread in the beam of positrons, we get:

$$N=\xi_2(1)+\frac{1}{\eta}\int\limits_{\xi_2(1)}^{1} d\xi\sqrt{2(\widetilde{U}(1)-\widetilde{U}(\xi))}\;;$$

$$f(\xi_m)=\frac{1}{N\eta}\frac{d\widetilde{U}(\xi_m)}{d\xi_m}\begin{cases}\int\limits_{0}^{\xi_m} d\xi/\sqrt{2(\widetilde{U}(\xi_m)-\widetilde{U}(\xi))} & if \quad 0\le\xi_m\le\xi_1\\ \int\limits_{\xi_2(\xi_m)}^{\xi_m} d\xi/\sqrt{2(\widetilde{U}(\xi_m)-\widetilde{U}(\xi))} & if \quad \xi_1\le\xi_m\le 1\end{cases}.$$

Calculated by these expressions dependence of the relative capture N in the channeling regime on the value ϑ_o is shown in Fig. 4 and density functions $f(\xi_m)$ for some values ϑ_o/μrad are shown in Fig. 5.

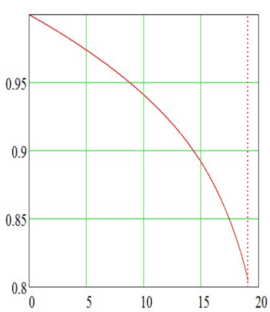

Fig.4: Function $N(\vartheta_o/\mu\text{rad})$.

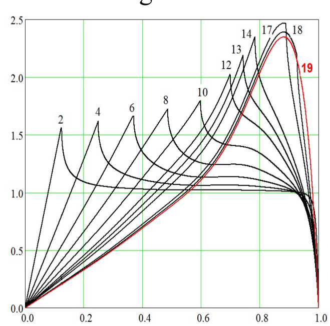

Fig.5: Functions $f(\xi_m)$

From the above analysis it follows the further important conclusion: in the potential (1), where $d\widetilde{U}/d\xi\to 0$ at $\xi\to\pm 1$ the distribution $f(\xi_m)\to 0$ at $\xi_m\to 1$.

THE RADIATION OF CHANNELED POSITRONS IN QUASIPERIODIC MOTION

To find the radiation spectrum of channeled positrons, oscillating in the interplanar potential (1), use the formula derived in [1] (p.303) for the quasiperiodic motion of a particle at all values of ρ. The need to consider the radiation spectrum in such a very general way is due to the fact that in the potential (1) multipole parameter (see Fig. 2) covers a wide range of values ρ providing different types of radiation. The radiation spectrum of one positron per unit length of a short crystal is determined by the expression:

$$\frac{d^2E}{dE_\gamma\,ds}=-\frac{\alpha E_\gamma}{c(2\pi\gamma)^2}\sum_{n=1}^{\infty}\Phi\big(n-\zeta(1+\rho/2)\big)\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi}d\varphi_1\,d\varphi_2\,\times$$

$$\times J_o\left(2\int\limits_{\varphi_2}^{\varphi_1}d\varphi\,\mu(\varphi)\sqrt{\zeta\big(n-\zeta(1+\rho/2)\big)}\right)\times\left[1+\frac{A(E_\gamma)}{2}\big(\mu(\varphi_1)-\mu(\varphi_2)\big)^2\right]\times$$

$$\times\cos\left\{(n-\zeta\,\rho/2)(\varphi_1-\varphi_2)+\zeta\int\limits_{\varphi_2}^{\varphi_1}d\varphi\,\mu^2(\varphi)\right\},$$

where: $\alpha=1/137.04$, E_γ - the energy of the emitted photon, step function $\Phi(y)=1$ at $y\ge 0$ and $=0$ at $y<0$, J_o is the Bessel function, $\mu(\varphi)=\gamma(\upsilon_x(\varphi)-<\upsilon_x>)/c$, $\zeta(E_\gamma,\xi_m)=E_\gamma\,E/2\gamma^2(\hbar\omega)(E-E_\gamma)$, $\omega=\omega_o\cdot\Omega(\xi_m)$, $A(E_\gamma)=1+E_\gamma^2/2E(E-E_\gamma)$.

The motion of a positron is presented in the form of a harmonic oscillation (3) with the frequency depending on its amplitude ξ_m. For such an approximation the spectral dependency $(d^2E/dE_\gamma\,ds)$ on E_γ was calculated by the previous formula and shown in Fig. 6 for a single channeled positron with the following values of ξ_m = 0.3, 0.5, 0.7, 0.9.

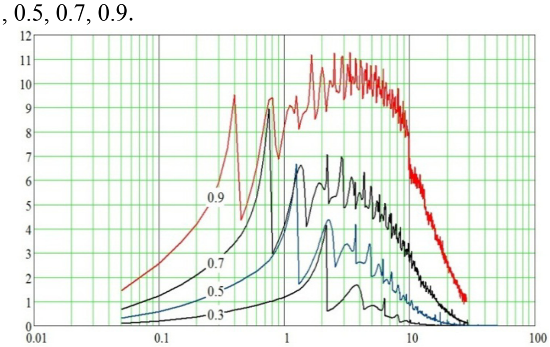

Fig.6: Dependencies $(d^2E/dE_\gamma\,ds)\times cm$ on E_γ /GeV.

CONCLUSION

With the help of above consideration the photon spectra were calculated for the positron energy in the range of 80-120 GeV for (011) and (111) silicon planes. These spectra were inserted into the Monte Carlo program which was used for simulations of differential radiation energy losses of 120 GeV positrons at the INSURAD experiment conditions. The experiment was performed with usage of bent monocrystals with thickness of 1-2 mm. Simple estimations show: influence of crystal bending on a photon emission process is negligible at radii more than 4 m; the mean number of photons emitted by one positron in such thickness is more than 1 several times. Comparison between the energy losses measured in experiment and results of Monte Carlo simulations have shown good coincidence them [6].

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