METHOD TO ESTIMATE THE BEAM AND STRUCTURE PARAMETERS FOR THE DISPERSION ACCELERATOR PARTS

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Abstract

The beam dynamics parameters as the transverse emittance information, momentum spread and structure characteristics are determined by treatment of the small number of profile measurements for the dispersion accelerator parts. The reliability of measurements is estimated.

INTRODUCTION

For dispersion accelerator parts the qualitative transverse profile measurement treatment is important both for circular accelerators and aside beam lines of the linacs. On a measuring area the dispersion distorts the standard phase space ellipse description [1], where the particles with different longitudinal momentum have other transverse central orbits. For the dispersion plane the off momentum particles have a transverse moving over the central orbit defined by

$$x_c(s) = D(s)\cdot\delta \tag{1}$$

where $D(s)$ – the dispersion function at an accelerator point s; δ – the relative particle longitudinal momentum deviation from the nominal value.

Later for simplicity it will be supposed that there is dispersion in the horizontal plane only. The calculations will be presented for this plane.

ASSUMPTIONS AND MATHEMATICAL FORMALISM

Let is proposed the measurement area is placed between the longitudinal coordinates s_0 and s_1 of an accelerator (Fig.1). At the end point a transverse beam profilometer is located.

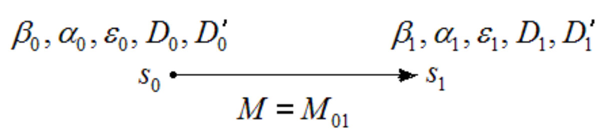

Figure 1: Measurement area

In Fig.1 at points s_0 and s_1 the $\beta_0, \alpha_0, \beta_1, \alpha_1$ are characteristic functions [1] for the beam particles with $\delta \sim 0$; ε_0 , ε_1 - rms unnormalized beam emittances of the beam particles with $\delta \sim 0$; D_0, D_0', D_1, D_1' – the values of dispersion and its derivations; M_{01} – (2×2) transfer matrix of a measurement beam line.

The basic formulas for further calculations are:

$$\overline{x^2}(s) = \overline{\xi^2}(s) + D^2(s)\cdot\overline{\delta^2} \;\; ; \;\; \overline{x}(s) = \overline{\xi}(s) \tag{2}$$

where $\overline{x^2}(s)$ – the square of the rms beam profile measurements for the total beam particles; $\overline{\xi^2} = \beta(s)\cdot\varepsilon(s)$ is the standard beam phase space characteristic [1] for particles with $\delta \sim 0$. For Eq.2 the followed assumptions were done:

- the particles with deviation from nominal longitudinal momentum have the identical normalized distribution functions in transverse planes;
- the momentum distribution function is symmetrical.

For simplicity further we suggest that there are no accelerator elements and there are variation elements, for example quadrupole lenses, to change the transfer matrix M_{01} of the measurement area (Fig.1). Therefore the followed equations are valid [1]:

$$\begin{aligned}
&\varepsilon_0 = \varepsilon_1 \\
&\beta_1 = m_{11}^2\beta_0 - 2m_{11}m_{12}\alpha_0 + m_{12}^2\cdot\frac{1+\alpha_0^2}{\beta_0} \\
&D_1 = m_{11}D_0 + m_{12}D_0' \\
&M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}
\end{aligned} \tag{3}$$

By using the above formulas for the N_m profile measurement results the Eq.2 may be written in the followed form:

$$\overline{x_i^2} = \beta_i\varepsilon_0 + D_i^2\cdot\overline{\delta^2} \quad , \quad i = 1 \div N_m \tag{4}$$

where index i is referred to the values at point s_1 for each measurement. In Eqs.4 there are six variables $\beta_0, \alpha_0, D_0, D_0', \varepsilon_0, \delta$. To exclude from any two Eqs.4 the variables ε_0, δ , for example by followed formulas

$$\varepsilon_0 = \frac{x_i - D_i^2\overline{\delta^2}}{\beta_i} \quad ; \quad \overline{\delta^2} = \frac{\overline{x_i^2}\beta_j - \overline{x_j^2}\beta_i}{D_i^2\beta_j - D_j^2\beta_i} \quad , \quad i \neq j \tag{5}$$

and replace these values to any third equation from Eqs.4 the next form may be derived

$$\overline{x_i^2}(\beta_j D_k^2 - \beta_k D_j^2) + \overline{x_j^2}(\beta_k D_i^2 - \beta_i D_k^2) + \\ + \overline{x_k^2}(\beta_i D_j^2 - \beta_j D_i^2) = 0 \;\; , \;\; i \neq j \neq k \tag{6}$$

In Eqs.6 there are only four independent variables $\beta_0, \alpha_0, D_0, D_0'$. Because of the independence and equality of measurements to combine from N_m by three we get $N_1 = C_{N_m}^3$ equations like Eq.6. In Table 1 this number is presented. The equations like Eq.6 are strongly nonlinear due to the coupling in Eqs.3.

Practically this system may be solved only by numerical methods [2].

If for system of Eqs.6 the measurement results $\overline{x_i^2}$ and matrix elements of M_i are ideal, without errors, it follows that only four equations are needed in system of Eqs.6 to determine variables $\beta_0, \alpha_0, D_0, D_0'$ and as result the variables ε_0, δ from Eqs.5.

Table 1: Characteristic numbers

N_m	4	5	6	7
$N_1 = C_{N_m}^3$	4	10	20	35
$N_2 = C_{N_1}^4$	1	210	4845	52360
$N_3 = C_{N_1}^{N_m}$	1	252	38760	6724520
$N_4 = C_{N_m}^2$	6	10	15	21

REAL MEASUREMENT TREATMENT

Unfortunately the errors are presented both in the profile measurements and elements of transfer matrices for the measurement area. As result the Eqs.6 may be rewritten in form:

$$\overline{\tilde{x}_i^2}(\tilde{\beta}_j\tilde{D}_k^2 - \tilde{\beta}_k\tilde{D}_j^2) + \overline{\tilde{x}_j^2}(\tilde{\beta}_k\tilde{D}_i^2 - \tilde{\beta}_i\tilde{D}_k^2) + \overline{\tilde{x}_k^2}(\tilde{\beta}_i\tilde{D}_j^2 - \tilde{\beta}_j\tilde{D}_i^2) = b_{ijk} \quad , \quad i \neq j \neq k \qquad (7)$$

where $\tilde{\beta}_i, \tilde{D}_i$ – values calculated by Eqs.3 with errors in the elements of matrix M ; $\overline{\tilde{x}_i^2}$ – square of rms beam profile measurements.

Solution of Basic System

To determine the solution to the basic system consisting from N_1 (Table 1) equations like Eq.7 for undefined variables $\tilde{\beta}_0, \tilde{\alpha}_0, \tilde{D}_0, \tilde{D}_0'$ it may be used the method [2] to minimize function

$$\Phi_m(\vec{y}) = \Phi(\tilde{\beta}_0, \tilde{\alpha}_0, \tilde{D}_0, \tilde{D}_0') = \sum_{m=1}^{N_1} \left| f_m(\vec{y}) \right|^2 \qquad (8)$$

where from Eq.8 $\qquad f_m(\vec{y}) = b_{ijk} \quad . \qquad (9)$

However it was proposed another method to find vector $\vec{y}$ in Eq.8. From N_1 equations like Eq.8 it was combined the systems of four equations. The total number of these systems will be $N_2 = C_{N_1}^4$ (Table 1). Each system is solved by minimization algorithm for Eq.8 and Eq.9 where $m = 1 \div 4$. The gradient method [2] and regularization algorithm [3] were used. From Table 1 it follows that for $N_m \geq 5$ the statistic treatment can be applied for solution vectors $\vec{y}_n, n = 1 \div N_2$ to get the average and rms deviations for parameters $\tilde{\beta}_0, \tilde{\alpha}_0, \tilde{D}_0, \tilde{D}_0'$. If the local minimum for Eq.9 is stable for the different initial approach to vector $\vec{y}$ the results are accepted as final. If for high number of systems there are equalities $\Phi_4(\vec{y}) = 0$ it means there is not local minimum for Eq.8 and the exact solution was found for disturbed data of measurements. This is not correct results because in practice the experimental profile data have accuracy $\sim 10\%$. In this case one of the followed restriction algorithm is used:

- for numerical calculations the limited minimum of $\Phi_4(\vec{y})$ was entered. This limit is determined by the minimum for rms deviation for statistic treatment of solution vectors $\vec{y}_n, n = 1 \div N_2$;
- the limited minimum of $\Phi_4(\vec{y})$ is determined by the calculation of the expected exact measurement results (next section). The deviations of measurements and calculated results have to be less then 10%.

Expected Exact Measurement Results

The algorithm to estimate the quality of measurements was proposed. From previous section the averaged values $\hat{\beta}_0, \hat{\alpha}_0, \hat{D}_0, \hat{D}_0'$ for variables may be introduced to Eqs.7:

$$\overline{\hat{x}_i^2}(\hat{\beta}_j\hat{D}_k^2 - \hat{\beta}_k\hat{D}_j^2) + \overline{\hat{x}_j^2}(\hat{\beta}_k\hat{D}_i^2 - \hat{\beta}_i\hat{D}_k^2) + \overline{\hat{x}_k^2}(\hat{\beta}_i\hat{D}_j^2 - \hat{\beta}_j\hat{D}_i^2) = \hat{b}_{ijk} \quad , \quad i \neq j \neq k \qquad (10)$$

Now the undefined variables are the expected exact measurement data $\overline{\hat{x}_i^2}$, $i = 1, N_m$. The Eqs.10 are linear equations. From Eqs.10 the systems with N_m equations may be designed. The total number of systems is $N_3 = C_{N_1}^{N_m}$ (Table 1). For calculations the reasonable number of measurements is $N_m = 5 \div 6$. Every system is solved by minimization of the function

$$\Phi_{N_m}(\vec{z}) = \Phi(\overline{\hat{x}_1^2}, \ldots\ldots, \overline{\hat{x}_{N_m}^2}) = \sum_{m=1}^{N_m} \left| \xi_m(\vec{z}) \right|^2 \qquad (11)$$

where from Eq.10 $\qquad \xi_m(\vec{z}) = \hat{b}_{ijk} \qquad (12)$

To solve Eq.11 and Eq.12 the regularization algorithm [3] for the unstable systems of linear equations must be used.

As results for the vectors $\vec{z}_n, n = 1 \div N_3$ statistical treatment may be applied to estimate the measurement reliability.

Emittance and Momentum Spread Estimation

The application of the averaged values $\hat{\beta}_0, \hat{\alpha}_0, \hat{D}_0, \hat{D}_0'$ calculated in the previous section for determination of ε_0, δ by Eqs.5 has the poor statistics (N_4 in Table 1).

Therefore it was proposed to use the solution vectors $\vec{y}_n, n = 1 \div N_2$ for every system from Eqs.7. The statistics for ε_0 and δ will be the same as for basic vectors $\vec{y}$

(N_2 in Table 1). According to the system design, the equation construction and inequality for Eqs.7, the number of measurement results for calculations, which are used, will be

$$N_5 = \begin{cases} 4 \div N_m & , \quad N_m \le 12 \\ 4 \div 12 & , \quad N_m > 12 \end{cases} \qquad (13)$$

For determination of the variables ε_0, δ, when the parameters $\tilde{\beta}_0, \tilde{\alpha}_0, \tilde{D}_0, \tilde{D}'_0$ were found, N_5 measurement results are used. With data errors the Eqs.4 may be rewritten

$$\overline{\tilde{x}_i^2} - \tilde{\beta}_i \tilde{\varepsilon}_0 - \tilde{D}_i^2 \cdot \overline{\tilde{\delta}^2} = c_i \quad , \quad i = 1 \div N_5 \qquad (14)$$

The solution for system Eq.14 is carried out by the minimization of function

$$\Phi_2(\vec{\zeta}) = \Phi(\tilde{\varepsilon}_0, \overline{\tilde{\delta}^2}) = \sum_{m=1}^{N_5} \left| c_m(\vec{\zeta}) \right|^2 \qquad (15)$$

From Eq.13 it is evidence the number of the equations in Eqs.14 is $N_5 > 2$ (the number of variables). The regularization algorithm [3] for unstable systems of linear equations was used to solve Eqs.14 with Eq.15. The results are vectors $\vec{\zeta}_n, n = 1 \div N_2$ and further the standard statistic treatment may be used.

It should be noted that for Eqs.14 the expected exact measurement data $\overline{\tilde{x}_i^2}, \; i = 1 \div N_m$ might be used instead of the experimental data $\overline{\tilde{x}_i^2}, i = 1 \div N_m$. It leads to the results with small differences for the average values $\hat{\varepsilon}_0$ and $\overline{\hat{\delta}^2}$ with essentially less rms deviations from these values. However the additional investigations have to be done.

Beam Centre Dynamics

According to Eq.2 and Eqs.3 the beam center dynamics with measurement errors may be described by

$$\tilde{x}_{ci} - \tilde{m}_{11i} \tilde{x}_{c0} - \tilde{m}_{12i} \tilde{x}'_{c0} = d_i \qquad i = 1 \div N_m \qquad (16)$$

where $\tilde{x}_{c0}, \tilde{x}'_{c0}$ – unknown beam centre phase space coordinates at longitudinal point s_0 (Fig.1); $\tilde{x}_{ci}$ – experimental beam centre at point s_1; $\tilde{m}_{11i}, \tilde{m}_{12i}$ – elements of matrix M for the measurement with number i.

Appling the method from previous sections the systems for two variables were constructed. The number of systems is $N_4 = C_{N_m}^2$ (Table 1). For $N_m = 5$ or 6 the statistics is poor. Therefore the minimization algorithm may be applied to the total system Eqs.16 for function

$$\Phi_2(\vec{\tau}) = \Phi(\tilde{x}_{c_0}, \tilde{x}'_{c_0}) = \sum_{m=1}^{N_m} \left| d_m(\vec{\tau}) \right|^2$$

PECULARITIES OF METHOD

- The measurement area may be very shot, that permits to vary the transfer matrix parameters in the wide range without beam particle losses.
- There are possibilities to make the standard statistic treatment and to estimate the reliability of measurement data.
- The parameters of the beam and accelerator structure may easy compare with another experimental and theoretical results.

However now there are the method disadvantages:

- the number of measurements for calculations have to be not less than 5;
- at present there is not effective interactive procedure for the fast data treatments;
- from the form of Eq.2 and Eq.6 the dispersion have to be with the same sign at the point s_1 (Fig.1) for all measurements which are used for the treatments. However two series of measurements may be done for the different dispersion signs if it is possible to carry out.

CONCLUSIONS

The proposed method is valid for a small number of the transverse beam profile measurements. The information about the main beam parameters could be calculated. Simultaneously the beam line structure functions are determined for the experimental area. These data could be used to define the total beam line operation parameters.

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