# SPACE CHARGE SIMULATION USING MADX WITH ACCOUNT OF SYNCHROTRON OSCILLATIONS

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## Abstract

Direct space charge forces can be simulated with the 6D beam dynamics code MADX using a number of 4D BEAMBEAM elements with Gaussian transverse profile for charge density. To take into account effects of synchrotron oscillations on space charge (s.c.) forces, the amplitude of BEAMBEAM elements is modulated according to the distance between a particle and the bunch center assuming Gaussian longitudinal profile. Parameters of every BEAMBEAM element (charge and sizes) are defined by local values of beta-function and dispersion, while they are updated according to the beam intensity and beam emittances at every turn. MADX script accomplishing this method has been written for the lattice of the existing Debuncher ring. The slow extraction at the 3rd order resonance with simultaneously varying the horizontal tune and the sextupole strength is considered as one of the options for Debuncher to be used in the "mu2e" project. Our MADX simulation results are compared with results obtained by V.Nagaslaev (FNAL) using the particle-in-cell ORBIT-code. The evolutions of the phasespaces and the beam intensity within ten thousands turns have shown a good agreement between the MADX and ORBIT results.

## **INTDRODUCTION**

Direct s.c. forces in beam dynamics codes can be represented analytically by the bunch with elliptical crosssection. This approach has been already used with several codes, e. g. FRANKENSPOT [1], MICROMAP [2] and MAD8 [3], where the beam with Gaussian distribution is usually assumed. With the MADX code [4], which is a successor of MAD8 code, the s.c. forces can be simulated using an arbitrary number of 4D BEAMBEAM elements with Gaussian transverse profile for charge density [5,6].

In Ref. [5], the MADX with 4D s.c. has been implemented for coasting beam. In Ref. [6], effects of synchrotron oscillations on s.c. forces have been simulated assuming a prescribed Gaussian modulation of the longitudinal profile for 4D-BEAMBEAM elements.

In this report, the MADX 6D-tracking is implemented and the amplitude of 4D-BEAMBEAM is a function of the distance between an arbitrary particle and the bunch center. The example MADX-script is written for the FNAL Debuncher taking into considerations turn-to-turn variations of the lattice parameters, beam intensity and emittances during a slow extraction process.

## S.C. SIMULATIOS WITH MADX

A numerical realization of the s.c. calculations with MADX deals with the s.c. force created by thin elements,

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e.g., 4D-BEAMBEAM kicks, which are inserted around the ring according to some integration method. The method had been already implemented in other beam dynamics codes [1-3]. Our task is a step-by-step adaptation some of them to MADX, which is presently one of the most advanced code for nonlinear beam dynamics simulations without space-charge. The most of work is done using a language of MADX input scripts.

In our realization, s.c. kicks are inserted within every lattice thick element (e.g., BEND, QUADRUPOLE, DRIFT, etc.), according to the 2<sup>nd</sup> order ray tracing integrator [7]. At the beginning, the linear s.c. kicks represented with MATRIX elements are used for calculations of the beam sizes at the given transverse emittances, while an iteration procedure with TWISS command is used. Then, the nonlinear s.c. kicks represented with 4-D BEAMBEAM elements are inserted along the ring instead of s.c. MATRIX elements. The beam sizes of every BEAMBEAM are derived from calculations with MATRIX elements. The number of particles in every particular BEAMBEAM element is set up according to the formula presented in Ref. [3].

The benchmarking for the intense coasting beam in SIS-18 lattice has shown a good consistency with MICROMAP and SIMPSON codes [8]. The benchmarking with MADX assuming a prescribed Gaussian modulation of the longitudinal profile of 4D-BEAMBEAM elements is also showed good resuts [6,8].

## 6D-MADX tracking with 4D-BEAMBEAMs

To take into account effects of synchrotron oscillations on s.c. forces, the more realistic model is implemented in this report. 6D-MADX tracking determines the distance between an arbitrary particle and the bunch center. The amplitude of BEAMBEAM elements is modulated according to Gaussian longitudinal profile with a variance derived from the longitudinal emittance  $\mathcal{E}_{L}$  calculated according to a special fitting procedure [9] for the integral of distribution function  $F(I) = 1 - \exp(-I/\mathcal{E}_{L})$ , where I is the action variable.

The beam sizes of 4-D BEAMBEAMs in bending magnets are corrected according to the relation  $\sigma_{\text{tot}}^2 = \sigma_{\beta}^2 + [D(s)\sigma_p]^2$ , where D(s) is the dispersion.

## MADX-tracking in varying lattice

Normally MADX is used for multi-turn tracking in the lattice with constant parameters. The multi-turn tracking in the lattice with turn-by-turn varying parameters can be realized via multi-runs of the one-turn tracking (TURNS=1), while changing the lattice and beam

parameters after every turn. For example, the MADX synopsis is the following one:

## TRACK, RECLOSS, APERTURE; START, X, PX, Y, PY, T, PT; RUN, TURNS=1;

#### ENDTRACK;

MADX script accomplishing this method has been written for the lattice of the FNAL Debuncher ring. The developed MADX script has all attributes of a simple particle tracking code. At the beginning, the script accepts the coordinates of the injected particles, and then, after every turn it updates the lattice parameters according to the prescribed law, collects surviving and lost particles, calculates some "integral" beam parameters, updates the charge and sizes of every BEAMBEAM element according to new beam intensity and emittance, reads, fills and saved the tables with temporary parameters. Totally, the MADX-script consists of about 12 hundreds lines without counting the lattice file. The detailed blockdiagram of the algorithm can be found in the Ref.[10].

#### Converting the Debuncher lattice for MADX

Two original Debuncher lattices have been used in MADX simulations: 1) the original MAD8-lattice edited by A.Werkema [11]; 2) the ORBIT-style lattice derived from the MAD8-lattice by V. Nagaslaev [12]. Of the two lattices, the latter can be used for the direct comparison of the results obtained with ORBIT and MADX, and the former for the beam dynamics in the non-linear magnets.

Firstly, the ORBIT-style lattice has been prepared for the MADX simulations. In order to accomplish this conversion, the dedicated FORTARN code has been written. It includes the following steps: 1) the acceptance of about two hundreds of the 6-by-6 sectors matrices; 2) the generation of MADX lattice with matrices and special elements (e.g., apertures, driving sextupoles); 3) the insertion of BEMBEAM elements in places between sector matrices ( $N_{BB}$ =213 for Debuncher lattice). Finally, the consistence of the linear optics has been checked with TWISS command of the MADX-code.

Secondly, the MAD8 lattice has been converted into the MADX-lattice, while preserving all relations for elements excitations existed in the MAD8 file. Then, the BEMBEAM elements ( $N_{BB}$ =642) have been inserted in the middle of every thick element according to the 2<sup>nd</sup> order ray-tracing integrator. Again, the consistence of the linear optics has been checked with TWISS commands of the MAD8 and MADX codes.

#### SIMULATIONS RESULTS

#### Simulations with ORBIT code

Beforehand the present study, the beam dynamics simulations of the slow extraction in the Debuncher ring had been performed by V. Nagaslaev with the particle-incell code ORBIT [12]. The slow extraction at the 3rd order resonance with simultaneously varying the horizontal tune and the sextupole strength is considered as one of the options for Debuncher to be used in the "mu2e" project [13]. Figure 1 shows the simulation results with ORBIT code. The prescribed ramp-laws for the driving sextupoles and bare tune  $Q_x$  are shown in Fig.1,a and Fig.1,b, resp. Figure 1,c shows the tune spread used here for the evaluations of the tune-shift. Figure 1,d presents the number of survived particles (or the beam intensity) vs the turn number. Figure 2 shows evolution of the horizontal phase space x - x' vs the turn number.



Fig.1 Simulations with ORBIT:

a) The sextupole strength K2 vs the turn number;

b) The bare tune  $Q_r$  vs the turn number;

c) The horizontal tunes vs the action for every particle ;

d) The number of survived particles vs the turn number.

## MADX simulations for the ORBIT-style lattice

Figure 3 shows the simulation results with MADXcode for the ORBIT-style lattice. Evolutions of the beam intensity shown as the number of survived particles for MADX (Fig.3) and ORBIT (Fig.1,d) are in a good agreement. The beam intensity is equal to 42 % and 40% at the turn 3000 and 11 % and 10% at the turn 9000 for the MADX and ORBIT simulations, respectively. Evolutions of beam emittances and the number of particles involved in the evaluations of the emittance fitting are also shown in Fig.3. Evolutions of other parameter had been presented in Ref.[10].

Let's note a very similar behavior of the horizontal phase space evolutions calculated with ORBIT (Fig.2) and MADX (Fig.4).

## MADX simulations for the MAD-style lattice

MADX simulations have been also performed with MAD-style Debuncher lattice converted from the original MAD8 lattice. In principle, MAD-style lattice includes many additional high-order effects in comparison with the ORBIT-style lattice, which is composed of the 6-by-6 sector matrices combining several elements included in a sector. The simulations have showed that results for MADstyle Debuncher lattice are very similar to the results with ORBIT-style lattice. For example, the beam intensity is equal to 46 % at the turn 3000 and 12 % at the turn 9000. Other results can be found in [10].

## CONCLUSION

MADX can be applied for s.c. simulations for coasting and bunched beams. Benchmarking test showed good agreements with other codes. Developed MADX-script for lattice with variable parameters can be used for slowextraction simulations. Simultaneous usage of MADX and ORBIT allowed to correct possible bugs in data submitted to the codes. The slow-extraction simulations for the intense beams showed a good agreement between results obtained by MADX and ORBIT codes using either ORBIT-style lattice or MADX-style lattice.



Fig.2 The horizontal phase space evolutions by ORBIT: x - x' vs the turn number.



Fig.3 MADX simulations for the ORBIT-style lattice: the dependences on the turn number for the number of survived particles, the number of the emittance particles, and the horizontal and vertical rms emittances.



Fig.4 The horizontal phase space evolutions by MADX: x - x' vs the turn number.

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<sup>a</sup> Published at http://www.JACoW.org

<sup>b</sup> Published at http://beamdocs.fnal.gov