NUMERICAL AND ANALYTICAL METHODS OF MODELLING OF BUNCH DYNAMICS IN DIELECTRIC FILLED ACCELERATING STRUCTURES*

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Abstract

RF waveguide structures are a basis for development of new generation of accelerators on the basis of a wakefield method of the charged particle acceleration, and also free electron lasers. Numerical and analytical calculation methods of Vavilov-Cherenkov radiation generated by relativistic electronic bunches in wave guides with dielectric filling, and also self-coordinated bunch dynamics in own and external fields are considered.

INTRODUCTION

The modern accelerating technique is in search of new methods for ensuring progress in the field of experimental physics of high energy. The developed technologies of dielectric wakefield acceleration of electrons on Vavilov-Cherenkov effect are one of the most perspective directions of creation of high gradient structures of modern linear accelerators for high energy physics [1].

Linear accelerators are considered also as sources of sequence of electronic bunches for the free electron laser which is considered now the major candidate for creation of ultra short impulses (of attosecond range) X-ray radiation. Waveguide structures with dielectric filling (Fig. 1) excited by a high current electronic bunch were investigated intensively for the last years [1] – [5]. The main purpose is of prospects of their use as high gradient linear accelerators.

For linear accelerator the achievement of high accelerating fields in structure where the electronic bunch gained energy ~GeV at extremely short distances is necessary. Increase of wake fields is reached on the basis of increase in a charge of generating bunch creating a wakefield wave, and also optimization of the geometrical sizes and a material of filling of accelerating structure. However along with accelerating fields the high current bunch generates the considerable rejecting fields leading to a bend of the bunch and its deviation from an axis of wave guide structure. In this regard one of the main problems in realization of the wakefield method is control of an intensive electronic bunch in the channel of the wave guide and prevention of subsidence of particles on its wall.

*Work is supported by the Russian Foundation for Basic Research and the Ministry of Education and Science of the Russian Federation in the framework of the Federal Program “Human Capital for Science and Education in Innovative Russia” for 2009–2013.
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Key task in the wakefield acceleration method is modelling of the self-coordinated movement of a relativistic electronic bunch passing through dielectric structure in Vavilov-Cherenkov fields created by it.

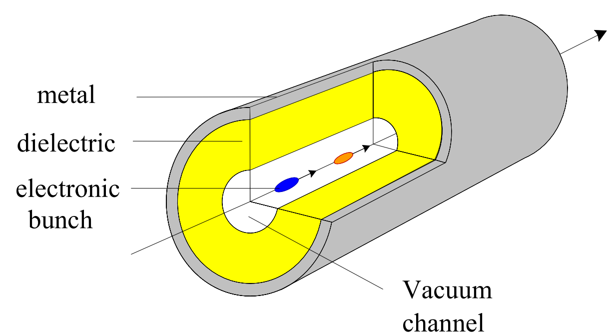

Figure 1: Wakefield waveguide structure.

To research of bunch dynamics the numerical approaches based on modelling of electronic bunches by a method of macroparticles are applied. The method of macroparticles is based on splitting of phase volume of a bunch into a large number of not being crossed elementary volumes; movement of each is identified with movement of one macroparticle with a total charge and mass of particles entered into this volume.

BEAM DYNAMICS EQUATIONS

The description of movement of an electronic bunch was carried out on the basis of the equations of relativistic dynamics [3]:

$$F_z = -eE_z = m_e\, d(V_z\gamma)/dt\,, \qquad F_r = d(m_e V_r \gamma)/dt\,,$$

where

$$E_z = q\sum_{n,m}\Big[\psi_{E_z\, n,m} I_n\big(k_{r\,n,m} r(\zeta,t)\big)\cdot$$
$$\cdot\int_0^{\zeta} f(\zeta_0)\cos\big(k_{z\,n,m}(\zeta-\zeta_0)\big) I_n(k_{r\,n,m} r(\zeta_0,t))\,d\zeta_0\Big],$$

ISBN 978-3-95450-125-0

$$F_r = F_{focus} - eq\sum_{n,m}\Big[\psi_{F_{r\,n,m}} I_n'\left(k_{r\,n,m} r(\zeta,t)\right)\cdot$$
$$\cdot\int_0^{\zeta} f(\zeta_0)\sin\left(k_{z\,n,m}\left(\zeta-\zeta_0\right)\right) I_n(k_{r\,n,m} r(\zeta_0,t))d\zeta_0\Big].$$

where $r(\zeta,t)$ is a bunch deflection from waveguide axes, $\zeta = z - vt$ is a distance behind the bunch, F_f is a focusing force, e and m_e are charge and mass of electron, q and γ are charge and relativistic factor of the bunch, $k_{z\,i,j}$ and $k_{r\,i,j}$ are longitudinal and radial components of wave vector, $\psi_{E_{z\,i,j}}$ and $\psi_{F_{r\,i,j}}$ are coefficients of series, depending of geometry, wave guide filling permittivity and initial charge place, $f(\zeta_0)$ is a function describing longitudinal charge distribution, $I_n(x)$ are modified Bessel function of n-th order.

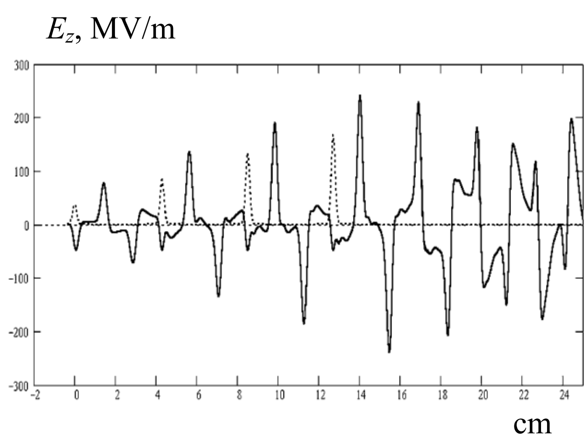

Figure 2: Longitudinal field structure behind four bunch train.

The task of the description of movement of macroparticles is self-coordinated: the mutual provision of particles in ensemble influences a field created by particles which, in turn, leads to change of their position.

Complexes of the BBU'3000 and BeamDyn programs were developed for the solution of the equations of self-coordinated dynamics of a relativistic electronic bunches, allowing to model movement of system of the bunches consisting of a chain of accelerating driver bunches and the accelerated bunch. Also the approximate analytical method of calculation of self-coordinated dynamics of the bunch was developed, presented in the separate bench report [7] at this conference.

Determinative influence on speed and accuracy of the bunch dynamics calculation on each step of algorithm is rendered by procedure of calculation of fields created by ensemble of particles, and procedure of definition of new position of particles under the influence of the found forces.

In the BBU'3000 program [4] calculation of the fields created by ensemble of particles, is carried out by summation of fields of a large number of macroparticles into which three-dimensional bunches with the set function of a charge spatial distribution break. In the BeamDyn [5] program for calculation of radial dynamics, according to [3], it is supposed that the charge is concentrated in the centre of cross-section distribution of the bunch. Longitudinal and radial fields turn out by integration of the function describing a radiation field in a point of z, r from the dot charge being in a point with coordinates of z_0, r_0 convoluted with function of the charge distribution on length of the bunch. On each time step dependence of radial shift of particles of the bunch of r_0 on z_0 which is used for field calculation is known. Its longitudinal splitting into a chain of macroparticles is made for definition of radial shift of the average line of the beam $r(\zeta,t)$. On the basis of integration of the equations of movement there are coordinates of macroparticles along the average line of the bunch during the subsequent moment of time, and then this discrete sequence is interpolated for obtaining again functional dependence.

For definition of new positions of particles in algorithms of bunch dynamics calculation numerical methods of the solution of the dynamics equations are traditionally used. They are transfer of the equations to a difference form with accuracy of the decision of the 1st order [3] and method of Runge-Kutta of the second and the fourth orders [4, 5]. Now the analytical solution of the particle dynamics equations under the influence of constant force [6] in which coordinates and speeds of a particle are received based on expressions:

$$z = z_0 + \frac{c}{a^3}\left[aa_z\left(\gamma_1-\gamma_0\right)+\gamma_0\left(a_r^2\beta_{z0}-a_z a_r\beta_{r0}\right)\ln|\delta|\right],$$

$$r = r_0 + \frac{c}{a^3}\left[aa_r\left(\gamma_1-\gamma_0\right)+\gamma_0\left(a_z^2\beta_{r0}-a_r a_z\beta_{z0}\right)\ln|\delta|\right],$$

$$\beta_z = \xi/\gamma_1,\ \beta_r = \eta/\gamma_1,$$

$$\delta = \frac{a^2 t+\gamma_0\left(a_z\beta_{z0}+a_r\beta_{r0}\right)+a\gamma_1}{\gamma_0\left(a_z\beta_{z0}+a_r\beta_{r0}+a\right)},$$

where $a=\sqrt{a_z^2+a_r^2}$, $a_z=-eE_z/(m_e c)$, $a_r = F_r/(m_e c)$, $\xi = a_z t+\beta_{z0}\gamma_0$, $\eta = a_r t+\beta_{r0}\gamma_0$, $\beta_z = v_z/c$, $\beta_r = v_r/c$, $\gamma_0 = \left(1-\beta_{z0}^2-\beta_{r0}^2\right)^{-1/2}$ is an initial relativistic factor, $\gamma_1=\sqrt{1+\xi^2+\eta^2}$.

Comparison of efficiency of algorithms shows that for calculations of dynamics it is the most preferable to use the algorithm based on the exact analytical solution of the movement equation of a relativistic particle under the influence of constant force, providing a prize till speed of calculation in 2-4 times in comparison with numerical methods of the differential equations of movement solving.

ISBN 978-3-95450-125-0

On fig. 3 as an example radial distribution of a charge and energy distribution in bunches at the moment of a contact by the bunch of wall of the wave guide is shown.

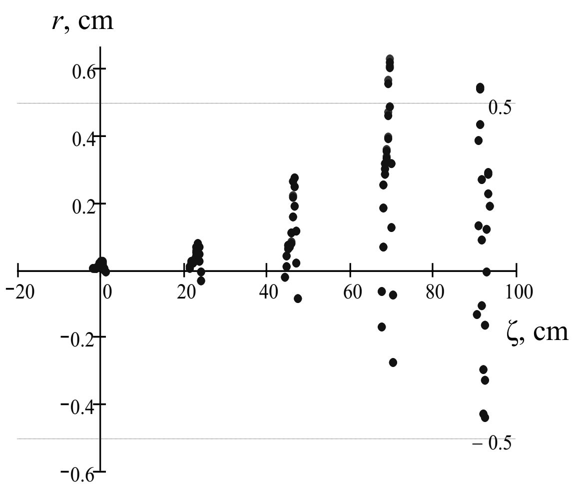

Figure 3: Radial macro particles displacement.

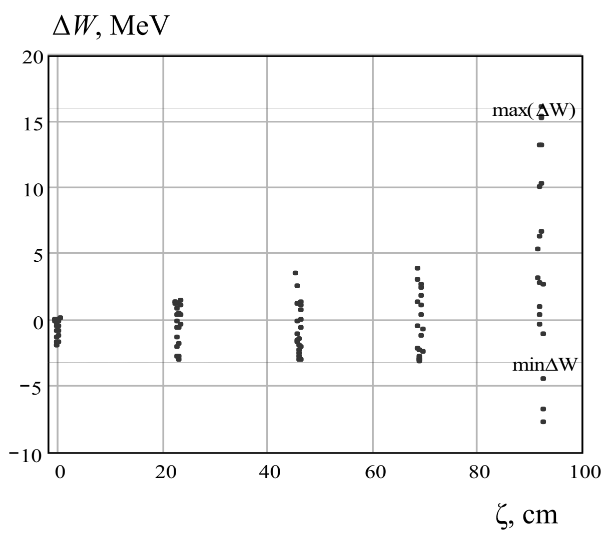

Figure 4: Macroparticle energy distribution.

CONCLUSION

The methods of calculation of Vavilov-Cherenkov radiation fields presented in this work and self-coordinated dynamics of bunches in wakefield accelerating structures open prospects for development of new generation of accelerators for physics of high energy and perspective sources of radiation in THz range of frequencies intensively mastered now.

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ISBN 978-3-95450-125-0