RF QUADRUPOLE FOCUSING LATTICES

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Abstract

Spatial homogeneity of a conventional RFQ allows estimating parameters of the lattice easily. Hybrid-RFQ structures with spatially periodic RFQ lenses are more complicated in respect of beam dynamics. Transverse stability of beam motion is defined by lattice parameters. Basically parameters of RF focusing lattices are influenced by longitudinal emittance of a bunch in contrast to static focusing lattices. The paper presents a method which allows evaluating parameters of a very wide class of RF and static quadrupole lattices. Transverse acceptance and acceleration rate could be determined. The method is useful to compare Hybrid-RFQ structures with a conventional RFQ.

INTRODUCTION

The main parameter of periodical focusing lattices is acceptance. It is extremely important for high intensity beams. It is known from the Hill`s equation theory that the acceptance A of any periodical lattice can be defined as [1]:

$$A = \frac{\gamma}{c}\omega_{r\,\min} a^2 , \quad (1)$$

γ –Lorentz factor, c – the speed of light, $\omega_{r\,min}$- minimum frequency of transverse oscillations of a particle, a – aperture radius of a focusing channel. Another lattice parameter, which describes transverse motion of particle, is a phase advance μ.

$$-1 < \cos\mu < 1 \quad (2)$$

should be performed to provide stability of transverse motion. The phase advance can be represented as

$$\mu = \int_t^{t+T_f} \omega_r(t)dt , \quad (3)$$

where $\omega_r(t)$ – frequency of transverse oscillations of a particle, t – time, T_f – the period of time for which a particle passes one period of focusing lattice. Thus phase advance μ means the averaged frequency of transverse oscillations for one period.

The accelerating structure of ion linac can be studied with quasi-static approach usually. Estimations for transverse motion of particles in static periodical lattices are generally performed by matrix method or smooth approximation. However range of application of these methods for RF focusing lattices is limited. There is an experience of application of these methods for a conventional RFQ and spatially periodic quadrupole focusing [1]. The limitation consists of low phase advance $\mu << 2\pi$, thin lens approximation and simplicity mainly for FODO lattices [1]. Unfortunately transverse motion of particles in a Hybrid-RFQ is too complicated to be considered by these methods [2].

The focusing lens of a Hybrid-RFQ is too long to use thin lens approximation in the scale of focusing period. Strength of RF focusing lens dependent on time and long focus length don't allow us to use smooth approximation easily.

The paper presents a method which allows studying a very wide class of RF and static quadrupole lattices.

HYBRID-RFQ

A Hybrid-RFQ structure is combined from accelerating gaps and RF quadrupole lenses [3]. Simplified scheme of the structure and its placement with respect to the first RFQ section are shown in Figure 1.

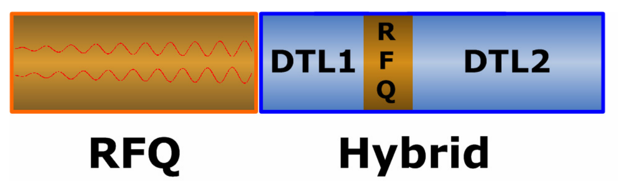

Figure 1: Scheme of Hybrid-RFQ structure.

Focusing regions are formed by vanes with quadrupole symmetry - RFQ-lenses [4]. This part of Hybrid-RFQ structure acts as a conventional triplet of quadrupole lenses. Required focusing strength of RFQ-lenses is defined by distance from axis to vanes.

THE METHOD

The method proposed in this paper is based on several stages:

1. RFQ lens is approximated by electrostatic thin lens with effective gradient $G_{ef} = G_{ef}(\psi, \beta_z)$ dependent on arriving phase ψ and particle velocity β_z. Arriving phase ψ is phase of RF field when a particle arrives at an entrance of RF lens.
2. Effective gradient of the lens and RF defocusing effects of accelerating gaps forms a lattice gradient function $G_{lat} = G_{lat}(z, \psi, \beta_z)$ as a "rectangular" function.
3. Smooth approximation [5] is used to describe the transverse motion of a particle in the gradient G_{lat}. An effective potential function U_{eff} is defined.
4. Study of effective potential function U_{eff} is carried out.

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Stage 1 (Lens approximation)

Focusing effect on transverse motion of a particle along the RFQ lens can be described by expression in Cartesian coordinates:

$$\frac{d\beta_x}{dz} = -\frac{qG(z)x}{\beta_z W_0}\cos(\omega t + \psi), \tag{4}$$

$\beta_x = v_x/c$ – transverse component of particle velocity, q – particle charge, $\beta_z = v_z/c$ – longitudinal component of the particle velocity, W_0 – rest energy of a particle, $\omega = 2\pi f$ – frequency of RF field. Let us equate the effect of RF lens to effect of static lens:

$$\int_0^{L_{lens}} G(z)x(z)\cos\left(\omega\frac{z}{v_z} + \psi\right)dz = G_{ef}(\psi,\beta_z)xL_{lens}, \tag{5}$$

where L_{lens} – lens length. The effective gradient can be presented as a series:

$$G_{ef}(\psi,\beta_z) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \mathrm{G}_{1,n,m}\beta_z^n\cos(m\psi) + \mathrm{G}_{2,n,m}\beta_z^n\sin(m\psi). \tag{6}$$

Stage 2 (Effective lattice gradient)

RF field of accelerating gaps sequence can be written as:

$$E_x = -E_1 I_1(kr)\frac{x}{r}\sin\varphi, \tag{7a}$$

$$E_y = -E_1 I_1(kr)\frac{y}{r}\sin\varphi, \tag{7b}$$

$$E_z = E_1 I_0(kr)\cos\varphi, \tag{7c}$$

where E_1 – amplitude of accelerating harmonic, $r^2 = x^2 + y^2$ – radial coordinate. $I_0(kr)$, $I_1(kr)$ – modified Bessel functions, $\varphi = kz - \omega t$ – phase of RF field when particle passes the center of an accelerating gap.

$$k = \frac{2\pi}{\beta_s\lambda} = \frac{\omega}{v_s}, \tag{8}$$

where $\beta_s = v_s/c$ is a velocity of the synchronous particle, λ – wavelength of RF field. Defocusing effect of RF gaps:

$$\frac{d\beta_x}{dz} = \frac{-qE_1I_1(kr)}{2\beta_z W_0}\frac{x}{r}\sin\varphi. \tag{9}$$

At paraxial approximation:

$$I_1(kr) \approx k\frac{r}{2}. \tag{10}$$

Thus

$$\frac{d\beta_x}{dz} = \frac{-qE_1kx}{4\beta_z W_0}\sin\varphi. \tag{11}$$

Finally the equation of particle transverse motion in the lattice can be written as:

$$\frac{d\beta_x}{dz} = -\frac{qG_{lat}(z,\psi,\beta_z)}{\beta_z W_0}x, \tag{12}$$

here

$$G_{lat}(z,\psi,\beta_z) = \begin{cases} G_{ef}(\psi,\beta_z),\ z\in RFQ\ lens \\ -\frac{E_1}{4}k\sin(\psi-\Delta\psi),\ z\in DTL \end{cases}, \tag{13}$$

and $\Delta\psi = \psi - \varphi$ is a phase difference between the entrance of RFQ lens and center of the gap.

Figure 2 shows the lattice gradient function $G_{lat}(z,\psi,\beta_z)$ along the focusing period of a Hybrid-RFQ as a sum of RFQ focusing effect of lens and defocusing effect of accelerating gaps.

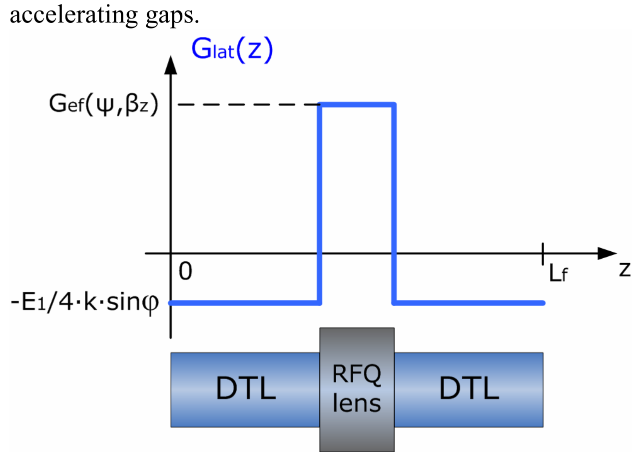

Figure 2: Lattice gradient along the focusing period L_f of a Hybrid-RFQ.

In the case of more detailed analysis we should consider $E_1 = E_1(\beta_z)$. The proposed method allows us to take this dependence into account easily.

Stage 3 (Smooth approximation)

According to the smooth approximation method the component of effective potential function U_{eff} corresponds to the longitudinal motion of a particle can be presented as:

$$U_{eff}{}^* \approx -\frac{qE_1}{m_0}\left[I_0(kr)\cos\varphi + \varphi\sin\varphi_s\right]. \tag{14}$$

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The total 3D effective potential function is:

$$U_{eff}(x,y,z) \approx -\frac{qE_1}{m_0}\left[I_0(kr)\cos\varphi + \varphi\sin\varphi_s\right] + \\ + \frac{q}{m_0}\frac{a_0}{2}x - \frac{1}{2}\left(\frac{q}{m_0}\right)^2 \sum_{n=1}^{\infty} \frac{a_n^2 + b_n^2}{\left(\frac{2\pi n}{L_f}\right)^2}(x^2+y^2). \quad (15)$$

Here

$$a_0 = a_0(\psi, \beta_z), \\ a_n = a_n(\psi, \beta_z), \quad (16) \\ b_n = b_n(\psi, \beta_z),$$

are coefficients of the Fourier-series of G_{lat}-function

$$G_{lat}(z,\psi,\beta_z) = \\ = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{L_f}z\right) + b_n \sin\left(\frac{2\pi n}{L_f}z\right). \quad (17)$$

Stage 4 (Effective potential study)

Motion equations in the smooth approximation can be written as:

$$\frac{d^2x}{dt^2} = -\frac{\partial U_{eff}}{\partial x}, \\ \frac{d^2y}{dt^2} = -\frac{\partial U_{eff}}{\partial y}, \\ \frac{d^2z}{dt^2} = -\frac{\partial U_{eff}}{\partial z}. \quad (18)$$

The averaged frequency of transverse oscillations can be calculated as:

$$\left\langle \Omega_x^2 \right\rangle = \frac{1}{x}\frac{\partial U_{eff}}{\partial x}, \quad (19)$$

and transverse phase advance is:

$$\mu = \left\langle \Omega_x^2 \right\rangle T_f. \quad (20)$$

Figure 3 presents the results of the particle tracking simulation. Trajectories in real field and obtained by smooth approximation are shown. Smoothed trajectory is close enough to the "real" one. The phase advance of oscillation corresponds to the calculated one. It is about to 60° at period length of 2.1m. The lattice acceptance can be estimated with (1) for a given aperture. Energy gain is calculated with (7c).

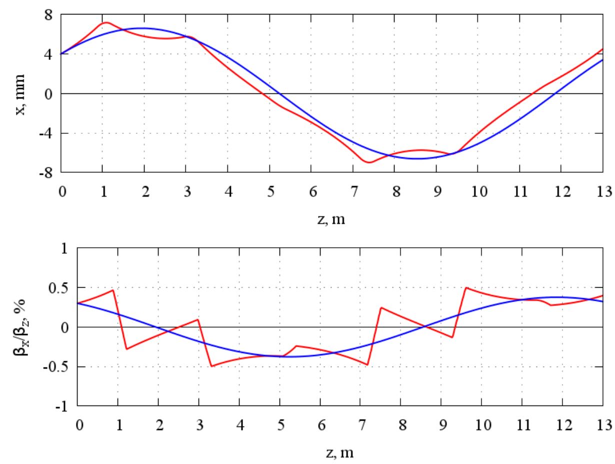

Figure 3: Trajectories of transverse particle motion through the Hybrid structure without accelerating gaps. Red - trajectory in real field, blue - trajectory calculated with the smooth approximation.

CONCLUSION

The method of RF quadrupole focusing studying has been proposed. It has been applied to Hybrid-RFQ analysis. Both averaged frequency of particle oscillations and transverse phase advance have been estimated. The agreement between "real" and "smooth" particle motions has been shown.

The method allows us to compare any types of focusing easily. It gives some advantages for RF focusing analysis. Lattice parameters can be estimated. Nonlinear effects can be studied with proposed method.

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