ANALYTICAL APPROACH FOR BEAM MATCHING

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Abstract

Charge particle beams transportation with small cross-sections and low energies is an actual problem for a gantry. That beams are used actively for isotope therapy. Beam emittance is its quality factor, and it should be matched with a facility channel acceptance. The method for beam dynamics analysis in lattice is developed in terms of non-coherent particle oscillation study. Nonlinear beam dynamics is investigated by using this method. It is shown that this technique allows one to realize effective beam emittance control. Analytical results obtained are verified by means of numerical simulation.

INTRODUCTION

One of the most interesting problems of accelerator engineering to date are the design and development of high-performance high-current compact systems for an injection and acceleration of low-velocity heavy-ion beams. This problem as well as others cannot be solved without taking into account problem solution on beam emittance matching with an acceptance of an accelerator channel. Effective acceptance evaluation for the resonance accelerator channel depends on a mathematical model used for describing a beam dynamics. Effective acceptance evaluation of the resonance accelerator channel was performed previously on basis of charged particle beam oscillation as a whole [1] – [4], that is under the assumption of coherent oscillations of individual particles. It is of particular interest to consider a model, which is taking into account non-coherent particle oscillations in the beam, and analyse results based on it.

BEAM DYNAMICS

It is difficult to analyse a beam dynamics in a high frequency polyharmonic field. Therefore, we will use one of methods of an averaging over a rapid oscillations period, following the formalism presented in [1] – [4]. One first expresses RF field in an axisymmetric periodic resonant structure as Fouriers representation by spatial harmonics of a standing wave assuming that the structure period is a slowly varying function of a longitudinal coordinate z

$$E_z = \sum_{n=0}^{\infty} E_n I_0\left(k_n r\right) \cos\left(\int k_n\, dz\right) \cos\omega t,$$

$$E_r = \sum_{n=0}^{\infty} E_n I_1\left(k_n r\right) \sin\left(\int k_n\, dz\right) \cos\omega t,$$

where E_n is the nth harmonic amplitude of RF field on the axis; $k_n = (1+2n)\pi/D$ is the propagation wave number for the nth RF field spatial harmonic; D is the resonant structure geometric period; ω is the RF frequency; I_0, I_1 are modified Bessel functions of the first kind.

As it was stated above, we will take into account non-coherent particle oscillations in the beam being accelerated. To this end, one introduces a notion of a reference particle, i.e. a particle moving on the channel axis. A magnetic force can be neglected for low-energy ions. We will assume that $dr/dz \ll 1$. Then, one passes into the reference particle rest frame. There is a differentiation over longitudinal coordinate in the beam motion equation. Thus, the motion equation together with an equation of particle phase variation can be presented in a view of a system of the first order differential equations as follows

$$\begin{cases} \dfrac{d\Gamma}{d\xi} = e_z(\xi, 0, \tau^*) - e_z(\xi, \rho, \tau), \\ \dfrac{d\beta_r}{d\xi} = \beta_z^{-1} e_r(\xi, \rho, \tau). \end{cases} \tag{1}$$

Here we introduced the following dimensionless variables: $\Gamma = \gamma^* - \gamma$; γ^* and γ are the Lorentzs factors for the reference and given particles respectively; $\xi = 2\pi z/\lambda$ is dimensionless longitudinal coordinate; $e_{z,r} = eE_{z,r}Z\lambda/2\pi m_0 c^2$; e is the elementary charge; Z is a charge state of an ion; λ is a wave length of RF field; m_0 is an ion rest mass; c is the light velocity in free space; $\beta_{z,r}$ is normalized velocity component.

Let us introduce a new dynamical variable $\psi = \tau - \tau^*$ ($\tau = \omega t$, τ^* is a normalized motion time of the reference particle at the laboratory coordinate system). Note, that

$$\frac{d\psi}{d\xi} = \beta_s^{-3}\Gamma, \tag{2}$$

β_s is normalized synchronous particle velocity, s is the field harmonic number.

Suppose that $|\beta_z - \beta_s| \ll 1$ one can obtain

$$\frac{d^2\psi}{d\xi^2} + 3\varkappa\frac{d\psi}{d\xi} = \frac{1}{\beta_s^3}\frac{d\Gamma}{d\xi} \tag{3}$$

upon differentiation of Eq. 2. The second equation of Eq. 1 can be rewritten as

$$\frac{d^2\delta}{d\xi^2} + \varkappa\frac{d\delta}{d\xi} = \frac{e_r}{\beta_s^3}, \tag{4}$$

where $\delta = \rho/\beta_s$, $\rho = 2\pi r/\lambda$, $\varkappa = \ln'_\xi \beta_s$.

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On averaging Eq. 3 and Eq. 4 over rapid oscillation period one can present the motion equation in the smooth approximation with the restrictions mentioned above in the following matrix form

$$\ddot{\Upsilon} + \Lambda\dot{\Upsilon} = -L\Phi_{\rm ef}, \qquad (5)$$

where the dot above stands for differentiation with respect to the independent longitudinal coordinate and

$$\Upsilon = \begin{pmatrix}\psi\\ \delta\end{pmatrix}, \qquad \Lambda = \begin{pmatrix}3\varkappa & 0\\ 0 & \varkappa\end{pmatrix}, \qquad L = \begin{pmatrix}\frac{\partial}{\partial\psi}\\ \frac{\partial}{\partial\delta}\end{pmatrix}.$$

Now, ψ and δ mean its averaged values. $\Phi_{\rm ef}$ plays role of an effective potential function (EPF) describing a beam interaction with the polyharmonical field of the system subject to the incoherent particle oscillations.

For example, we consider there are two spatial harmonics at the linac. One of it is the synchronous harmonic with $s = 0$, and another one is the nonsynchronous (focusing) with $n = 1$. In this case we have

$$\begin{aligned}\Phi_{\rm ef} = {} & \frac{e_0}{2\beta_s}\left[I_0(\delta)\sin(\psi+\varphi^*) - \psi\cos\varphi^* - \sin\varphi^*\right]\\ & + \frac{e_0^2}{64}\left[I_0^2(\delta) + I_1^2(\delta) - 1\right]\\ & + \frac{5e_1^2}{256}\left[I_0^2(3\delta) + I_1^2(3\delta) - 1\right]\\ & - \frac{e_0^2}{32}\left[I_0(\delta)\cos\psi - 1\right] - \frac{5e_1^2}{128}\left[I_0(3\delta)\cos\psi - 1\right]\\ & - \frac{e_0e_1}{32}\{[I_0(\delta) + I_0(3\delta)]\cos(\psi + 2\varphi^*) - 2\cos 2\varphi^*\}\\ & + \frac{e_0e_1}{32}\{[I_0(\delta)I_0(3\delta) + I_1(\delta)I_1(3\delta)]\cos 2(\psi+\varphi^*)\\ & - \cos 2\varphi^*\},\end{aligned}$$

where $e_n = eE_nZ\lambda/2\pi\beta_s^2 m_0c^2$.

To define eigenfrequencies of small system vibrations, EPF is expanded in Maclaurins series

$$\Phi_{\rm ef} = \frac{1}{2}\Omega_{0\psi}^2\psi^2 + \frac{1}{2}\Omega_{0\delta}^2\delta^2 + o(\Upsilon^{\rm T}\Upsilon),$$

and the coefficients in which are given by

$$\Omega_{0\psi}^2 = -\frac{e_0}{2\beta_s}\sin\varphi^* - \frac{e_0e_1}{16}\cos 2\varphi^* + \frac{e_0^2}{32} + \frac{5e_1^2}{128},$$
$$\Omega_{0\delta}^2 = \frac{e_0}{4\beta_s}\sin\varphi^* + \frac{3e_0e_1}{64}\cos 2\varphi^* + \frac{e_0^2}{128} + \frac{45e_1^2}{512}.$$

NUMERICAL SIMULATION

The analytical results obtained above were used to investigate the beam matching possibility at the linac output. The beam was the unbunched 2.5 keV/u lead ions Pb^{25+} with charge-to-mass ratio is equal to 0.12. Self-consistent beam dynamics simulations were conducted by means of a modified version of the specialized computer code BEAMDULAC-ARF3 based on CIC technique to calculate beam self-space-charge field. Computer simulation was carried out for the linac structure under the following parameters: $\lambda = 8.88$ m, system length is equal to 2.5 m, channel aperture is equal to 5 mm; input and output values of the equilibrium particle phase are equal to $-\pi/2$ and $-\pi/6$ respectively, synchronous harmonic maximal value at the axis is equal to 16 kV/cm, $e_1/e_0 = 9$. The equilibrium particle phase linearly increases at the bunching length (1.75 m) and plateaus further. Note that the variation of the synchronous harmonic amplitude against longitudinal coordinate (at 1.75 m) was calculated by using the technique described in [1]. Initial beam radius and current were 1 mm and 5 μA respectively. This parameters guarantee a positivity of the eigenfrequency of the small transverse tunes and, therefore, provide beam matching at the linac output. The output beam energy and current transmission coefficient were 100 keV/u and 85% respectively.

4D beam phase volume projection onto $(\psi, \dot{\psi})$ phase plane together with phase paths calculated in keeping with Eq. 5 at linac output is shown in Fig. 1. There are channel longitudinal acceptance in conservative approximation (curve 1) as well as channel dynamical acceptance in Fig. 1 too.

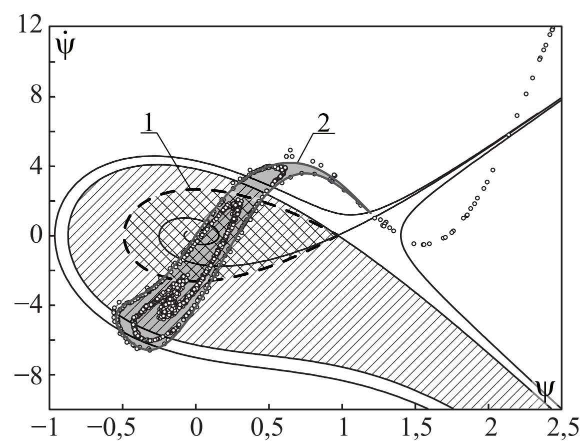

Figure 1: 4D beam phase volume projection and phase paths.

The size of beam envelope and transmission are shown in Fig. 2 and Fig. 3 respectively. The output beam radius is nearly 1.5 times greater than the input one because of this fact. This result is acceptable. It is clear that main linac parameters choice based on proposed technique is rather efficient to realize beam envelope (emittance) control.

SUMMARY

Beam dynamics model with regard for particles non-coherent oscillations was made. Effective acceptance evaluation in terms of this model was evaluated. The necessary restrictions on the linac parameters were imposed to make beam matching at the output. The numerical simulations of the self-consistent low-velocity heavy-ion beam dynamics confirmed the analytical results obtained.

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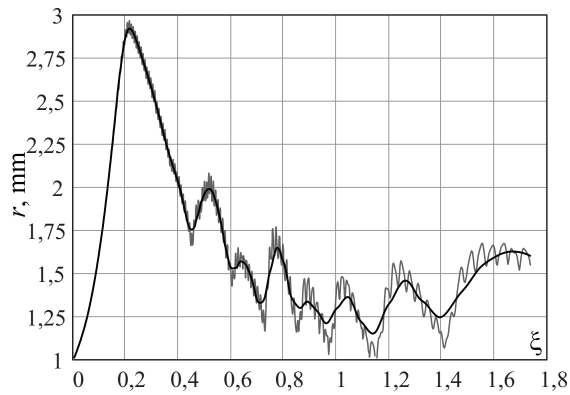

Figure 2: Beam envelope.

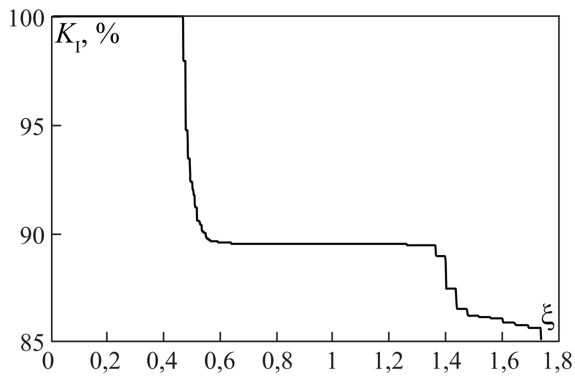

Figure 3: Current transmission.

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