SEARCH OF THE MOTION INTEGRAL AT LINAC WITH RF FOCUSING

V.S. Dyubkov*, S.M. Polozov, NRNU MEPhI, Moscow, Russian Federation

Abstract

The problem of the effective linac design is of interest to many fields of science, industry and medicine. It is well known that nonsynchronous harmonics of RF field (RF undulator) are focusing the low-energy particles. Analytical beam dynamics investigation can be carried out by means of the averaging method over the rapid oscillations period (the so-called smooth approximation) in the oscillating fields. Motion equation is presented in the form of the Hamilton's equations. Motion integrals are sought by means of Poincare mapping.

INTRODUCTION

The problem of the effective low-energy linac design is of interest to many fields of science, industry and medicine (e.g. nuclear physics, surface hardening, ion implantation, hadron therapy). There are a few problems that lead to beam instabilities at linacs. Nonlinear effects are the most important among it. Certain nonlinear problems of accelerator physics are both important for successful operation of accelerator and interesting as problems in their own right. In this paper we consider nonlinear interaction between beam particles and a field of accelerator structure. In order to accelerate the low-energy ion beams one of the following fruitful rf focusing types can be used: alternating phase focusing (APF), radio frequency quadrupoles (RFQ), focusing by means of the nonsynchronous wave field as well as the undulator rf focusing. Each of mentioned focusing types has its advantages as well as disadvantages. For example, we consider axially symmetric Wideröe type structure with the rf focusing by the nonsynchronous harmonics [1], [2].

BEAM DYNAMICS

It is difficult to analyse a beam dynamics in a high frequency polyharmonic field. Therefore, we will use one of methods of an averaging over a rapid oscillations period, following the formalism presented in [1] – [3]. One first expresses RF field in an axisymmetric periodic resonant structure as Fouriers representation by spatial harmonics of a standing wave assuming that the structure period is a slowly varying function of a longitudinal coordinate z

$$\begin{aligned} E_z &= \sum_{n=0}^{\infty} E_n I_0\left(k_n r\right)\cos\left(\int k_n\,dz\right)\cos\omega t, \\ E_r &= \sum_{n=0}^{\infty} E_n I_1\left(k_n r\right)\sin\left(\int k_n\,dz\right)\cos\omega t, \end{aligned} \tag{1}$$

where E_n is the nth harmonic amplitude of RF field on the axis; $k_n = (1+2n)\pi/D$ is the propagation wave number for the nth RF field spatial harmonic; D is the resonant structure geometric period (depends on z implicitly); ω is the RF frequency; I_0, I_1 are modified Bessel functions of the first kind.

We shall assume the beam velocity (the one-particle approximation) differs from one of the field harmonic phase-velocities strongly except the synchronous harmonic of rf field, the gap-to-gap spacing of rf structure along the beam axis being defined as $D = \beta_s\lambda\,(s+0.5)$, where s denotes the synchronous harmonic number, β_s is the normalized velocity of the synchronous (equilibrium) particle.

It is convenient to introduce the nondimensional variables $\widehat{\mathbf{Q}} = (\xi;\varrho)$ and τ as

$$\widehat{\mathbf{Q}} = 2\pi\mathbf{R}/\lambda, \quad \mathbf{R} = (z;r), \quad \tau = \omega t, \tag{2}$$

then one can write the second Newton's law

$$\frac{d^2\widehat{\mathbf{Q}}}{d\tau^2} = \widehat{\mathbf{e}}\left(\tau,\widehat{\mathbf{Q}}\right), \tag{3}$$

where $\widehat{\mathbf{e}} = q\mathbf{E}\lambda/2\pi mc^2$, q and m are charge and mass of a particle.

The particle path in the rapidly oscillating field (1) we search as a certain sum of a some slowly varying term and a rapidly oscillating one. We assume that the amplitude of the rapid velocity oscillations is much smaller than the slowly varying velocity component for the smooth approximation to be employed.

On averaging Eq. 3 over rapid oscillation period one can present the motion equation in the smooth approximation with the restrictions mentioned above in the following form of the Hamilton's equations

$$\frac{d\mathbf{\Omega}}{d\tau} = \frac{\partial\mathcal{H}}{\partial\mathcal{P}}; \qquad \frac{d\mathcal{P}}{d\tau} = -\frac{\partial\mathcal{H}}{\partial\mathbf{\Omega}}, \tag{4}$$

where $\mathcal{P}$ and $\mathbf{\Omega}$ are the canonically conjugate variables, the canonical coordinates being selected in such a way that the origin in a phase space is an equlibrium point, i.e. $\mathbf{\Omega} = (\overline{\widehat{\mathbf{Q}}} - \widehat{\mathbf{Q}}_s)/\beta_s$ and the beam-wave system Hamiltonian is

$$\mathcal{H}\left(\mathcal{P},\mathbf{\Omega}\right) = \frac{1}{2}\mathcal{P}^2 + U_{\text{ef}}(\mathbf{\Omega}). \tag{5}$$

Here $U_{\text{ef}}(\mathbf{\Omega})$ is the Effective Potential Function which describes the low-energy beam interaction with the polyharmonical field of the system. The EPF depends solely on the averaged variable $\mathbf{\Omega} = (\zeta;\eta)$.

* vsdyubkov@mephi.ru

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EPF can be written as $U_{\rm ef} = U_0 + U_1 + U_2$ [1], introducing notations

$$U_0 = -\frac{1}{2} e_s \left[I_0(\eta) \sin(\varphi_s + \zeta) - \zeta \cos\varphi_s - \sin\varphi_s \right];$$

$$U_1 = \frac{1}{16} \sum_{n \neq s}^{\infty} \frac{e_n^2}{\nu_{n,s}^2} w_{n,s}^{(0)}(\eta) + \frac{1}{16} \sum_{n=0}^{\infty} \frac{e_n^2}{\mu_{n,s}^2} w_{n,s}^{(0)}(\eta);$$

$$\begin{aligned} U_2 = \frac{1}{16} \sum_{\substack{n \neq s \\ k_n + k_p = 2k_s}}^{\infty} \frac{e_n e_p}{\nu_{n,s}^2} & \Big[w_{n,s,p}^{(1)}(\eta) \cos(2\zeta + 2\varphi_s) \\ & + 2\zeta \sin 2\varphi_s - \cos 2\varphi_s \Big] \\ + \frac{1}{8} \sum_{\substack{n \neq s \\ k_n - k_p = 2k_s}}^{\infty} \frac{e_n e_p}{\nu_{n,s}^2} & \Big[w_{n,s,p}^{(2)}(\eta) \cos(2\zeta + 2\varphi_s) \\ & + 2\zeta \sin 2\varphi_s - \cos 2\varphi_s \Big], \end{aligned} \tag{6}$$

where $e_i = \widehat{e}_i / \beta_s$, $\nu_{n,s} = (k_n - k_s)/k_s$, $\mu_{n,s} = (k_n + k_s)/k_s$, $\iota_{n,s} = k_n / k_s$. $n,\ s,\ p \in \mathbb{N}_0$, φ_s is the synchronous particle phase, the functions of the dimensionless transverse coordinate being defined as

$$\begin{aligned} w_{n,s}^{(0)}(\eta) &= I_0^2(\iota_{n,s}\eta) + I_1^2(\iota_{n,s}\eta) - 1; \\ w_{n,s,p}^{(1)}(\eta) &= I_0(\iota_{n,s}\eta) I_0(\iota_{p,s}\eta) - I_1(\iota_{n,s}\eta) I_1(\iota_{p,s}\eta); \\ w_{n,s,p}^{(2)}(\eta) &= I_0(\iota_{n,s}\eta) I_0(\iota_{p,s}\eta) + I_1(\iota_{n,s}\eta) I_1(\iota_{p,s}\eta). \end{aligned} \tag{7}$$

POINCARÉ MAPPING

Most of accelerator design is based on the paraxial approximation and the resultant linearized equations. In order to investigate nonlinear beam particle interaction with the linac field we study the particle motion in the potential (6) by means of numerical simulations. For example, linac parameters presented in [1] were used to this purpose.

It is well known that chaotic particle motion can appear even in Hamiltonian systems with a few degrees of freedom, including potential (6), due to property of nonlinear systems to separate originally close particles trajectories in the restricted space exponentially fast. For sufficiently large perturbation strength, which appears far from linac axis or near it in the case of strong field (see (1)), the particle motion can be chaotic. It is unacceptable for accelerator operation because the particles go to sufficiently large amplitude that they are lost. Of some interest is the problem of finding the largest regular Kolmogorov-Arnold-Moser curve, as that defines the dynamic aperture inside which linac operation is at least potentially possible.

We plotted numerically a few Poincare section under linac parameters mentioned above. There are three Poincare sections, calculated for $\mathcal{H} = \mathcal{H}_{\rm sep}$ and $z = 0$, $z = L_{\rm gr}$, $z = L$ in Fig. 1, Fig. 2 & Fig. 3 respectively, where $L_{\rm gr}$ and L are the field amplitude increasing length and the linac total length. Based on the presented pictures one can define the dynamic aperture readily. As one can see there is no additional isolating motion integral (the so called third motion integral) in the given case. Furthermore, three qualitatively different types of motion are observed. The qualitative features are that for all time particle representation points either lie on regular closed smooth curves or lie on islands, jumping from one to other, or follow chaotic trajectories, jumping around erratically.

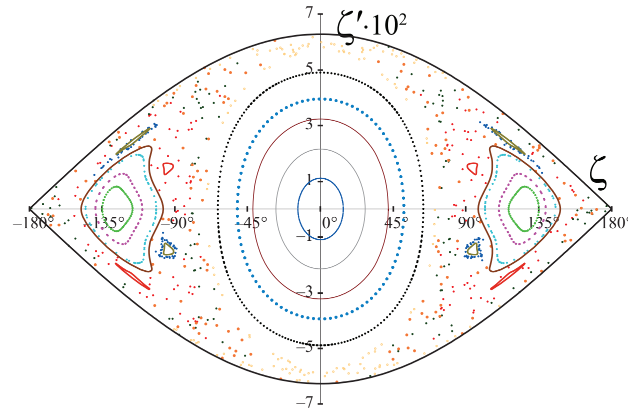

Figure 1: Poincaré sections at linac input.

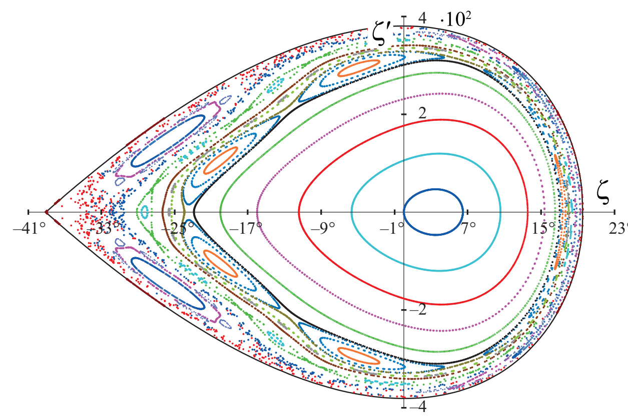

Figure 2: Poincaré sections at the end of bunching part.

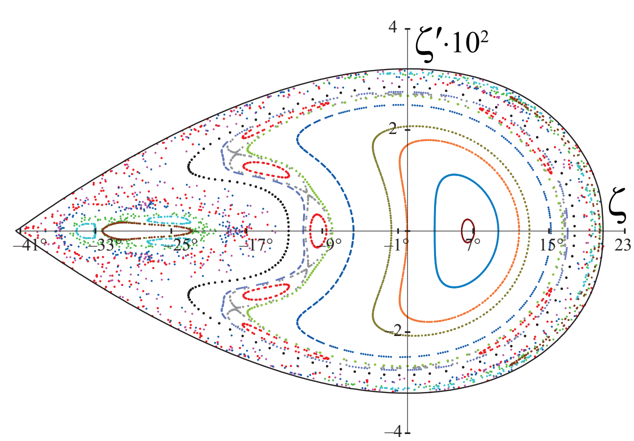

Figure 3: Poincaré sections at linac output.

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SUMMARY

Analytical beam dynamics investigation is carried out by means of the averaging method over the rapid oscillations period. Motion equation is presented in the form of the Hamilton's equations. Additional (third) motion integral is not found by means of Poincare mapping for linac parameters presented in [1]. Three qualitatively different types of motion are observed.

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