

SECOND-ORDER CORRECTION IN THE ISOCHRONOUS MODE OF THE COLLECTOR RING (CR) AT FAIR

S. Litvinov, C. Dimopoulou, A. Dolinskii, O. Gorda, F. Nolden, M. Steck, H. Weick
GSI, Darmstadt, Germany

Abstract

A challenge for nuclear physics is to measure masses of exotic nuclei up to the limits of nuclear existence which are characterized by low production cross-sections and short half-lives. The large acceptance Collector Ring (CR) [1] at FAIR [2] tuned in the isochronous ion-optical mode offers unique possibilities for such measurements. However, the mass-measurement resolution is inversely proportional to the transverse emittance. In order to reach a resolving power of 10^5 the transverse beam emittance would have to be limited up to 10 mm mrad in both planes, which drastically reduces the transmission of the exotic nuclei. We demonstrate here that the negative influence of the transverse emittance on the mass resolution can be significantly reduced by a proper second-order correction.

Isochronous Mode of the CR

The Collector Ring of the FAIR project is a symmetric, achromatic ring with two arcs, two straight sections and a total circumference of 221.5 meters. It is designed for operation at a maximum magnetic rigidity of 13 Tm. The CR will be operated in three ion-optical modes, two of them providing fast pre-cooling of either antiprotons or radioactive ion beams [1]. In the third mode (isochronous optics) the CR will be operated as a Time-Of-Flight (TOF) spectrometer for the mass measurement of exotic very short-lived nuclei ($T_{1/2} > 20\mu s$) produced and selected in flight with the Super-FRS fragment separator [3]. This technique for mass measurements has been developed at the ESR at GSI [4]. An advantage of this method is that a large number of nuclei can be measured in one experimental run.

The relative change of revolution time T due to different mass-to-charge ratio m/q and velocity v of the stored ions circulating in the ring is [5]:

$$\frac{\Delta T}{T} = \frac{1}{\gamma_t^2} \cdot \frac{\Delta(m/q)}{(m/q)} + \left(\frac{\gamma^2}{\gamma_t^2} - 1 \right) \frac{\Delta v}{v} - \frac{dT}{T}, \quad (1)$$

where γ is the relativistic Lorentz factor and γ_t is the transition energy of the ring. The isochronous condition is reached when $\gamma = \gamma_t$. It means, the second term in Eq. (1) vanishes and T defines the m/q . The resolution depends on the width of the time dT . Effects of nonlinear field errors, fringe fields of magnets, closed orbit distortion and transverse emittance negatively act on dT . Their influence has been investigated in [5].

Ions with different m/q are separated in time if their mean time separation ΔT is larger than the full time width of the beam.

$$\Delta T > dT. \quad (2)$$

Influence of Transverse Emittance

For good adjustment of γ_t the largest contribution to dT comes from the second-order geometric aberrations. In order to distinguish their influence, we consider a beam of one species in the ideal isochronous ring without higher-order field errors, fringe fields and closed orbit distortions. Only pure betatron motion exists. For such a ring the time spread is directly related to the transverse emittance ($\varepsilon_{x,y}$) [5]:

$$\left(\frac{dT}{T} \right)_{Emitt.} \approx \frac{1}{4} \left(\varepsilon_x \langle \gamma_x^{Twiss} \rangle + \varepsilon_y \langle \gamma_y^{Twiss} \rangle \right), \quad (3)$$

where $\langle \gamma_{x,y}^{Twiss} \rangle$ are the Twiss parameters averaged over the whole circumference of the ring.

Thus, from Eqs. (1, 3) one can derive the mass resolving power depending on the beam emittance [5]. For the CR, where acceptance is 100 mm mrad in both planes, the limit of the mass resolving power is about 10^4 , which is insufficient for precise mass measurements. Therefore, in order to reach the necessary resolving power of 10^5 the transverse emittance would have to be limited to 10 mm mrad in both planes. As a result, the transmission of the ions into the ring would be reduced drastically.

Revolution Time in Second-Order

However, the mass resolving power can be improved using second-order corrections and keeping the transverse emittance large. Let us assume a beam of one species circulating in the ring turn by turn. We observe it in the symmetry plane of the ring where the phase-space ellipse is upright ($\alpha^{Twiss} = 0$) and this condition is restored after each turn.

Statistical uncertainty gets reduced with increasing number of revolutions, and for accurate mass measurements it is essential to measure the revolution time of the particle over many turns. Therefore, the relative revolution time deviation between an arbitrary and the reference particle can be expressed in terms of the initial coordinates as a Taylor

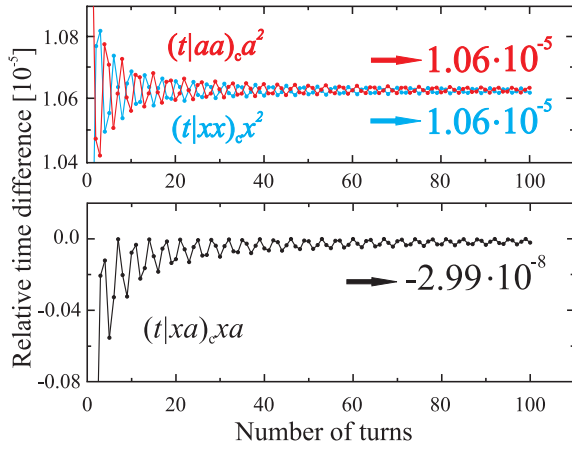


Figure 1: Evolution of the relative second-order geometric aberrations as a function of the number of turns in the CR. The aberrations $(t|xx)_c x^2$, $(t|xa)_c x a$, $(t|aa)_c a^2$ are marked in blue, black and red colors, respectively. The inserted numbers show the limit over many turns.

series in a second-order approximation [6, 7]:

$$\begin{aligned} \frac{dT}{T} = \frac{T - T_0}{T_0} = & (t|x)_c x + (t|a)_c a + (t|\delta)_c \delta + (t|xx)_c x^2 + \\ & + (t|xa)_c x a + (t|aa)_c a^2 + (t|yy)_c y^2 + (t|yb)_c y b + \\ & + (t|bb)_c b^2 + (t|x\delta)_c x \delta + (t|a\delta)_c a \delta + (t|\delta\delta)_c \delta^2, \quad (4) \end{aligned}$$

where (x, y) are the transverse coordinates and (a, b) their derivatives with respect to the longitudinal coordinate s . The index c stamps for the coefficients normalized by the total time-of-flight $t = nT_0$, where n is the number of turns. The fractional momentum deviation δ is given by $p = p_0(1 + \delta)$. T_0 and p_0 are the revolution time and the momentum of the reference particle, correspondingly.

In the first-order achromatic ring the first-order transverse matrix elements $(t|x)$ and $(t|a)$ simultaneously vanish [7, 8]. The necessary condition to be an isochronous ring in the first-order is $(t|\delta) = 0$ i.e. $\gamma = \gamma_t$. The second-order isochronous condition is fulfilled when $(t|\delta\delta) = 0$, which can be corrected with one family of sextupole magnets installed in a dispersive section of the ring. The mixed aberrations $(t|x\delta)_c x \delta$ and $(t|a\delta)_c a \delta$ usually do not contribute much to the revolution time. Their influence is negligible and we can skip them here.

Therefore, only the second-order geometric aberrations in time of Eq. (4) are significant. Without correction their contribution corresponds to the right part of Eq. (3) i.e. the transverse emittance. For simplicity we inspect only the horizontal plane, the arguments for the vertical plane are the same.

Geometric Aberrations and Chromaticity

The matrix elements $(t|xx)$, $(t|xa)$ and $(t|aa)$ can be expressed for the achromatic ring as [7, 9]:

$$(t|xx) = \lambda \cdot [(x|x)(a|x\delta) - (a|x)(x|x\delta)], \quad (5)$$

$$(t|xa) = \lambda \cdot [(x|x)(a|a\delta) - (a|x)(x|a\delta)], \quad (6)$$

$$(t|aa) = \lambda \cdot [(x|a)(a|a\delta) - (a|a)(x|a\delta)], \quad (7)$$

where λ is a constant with the dimension of inverse velocity. The coefficients in Eqs. (5-7) correspond to the terms of the transfer matrix M , which for the full circumference can be expressed by the betatron functions $\beta(s)$, $\alpha(s)$ and the betatron phase advance $\mu(s)$:

$$M = \begin{pmatrix} \cos \mu & \beta \sin \mu \\ -\frac{1}{\beta} \sin \mu & \cos \mu \end{pmatrix} \equiv \begin{pmatrix} (x|x) & (x|a) \\ (a|x) & (a|a) \end{pmatrix}, \quad (8)$$

Due to the oscillatory character of betatron motion the coefficients $(t|xx)_c$, $(t|xa)_c$, $(t|aa)_c$ fluctuate with the number of turns and after many revolutions average out (see Fig. 1). The aberration $(t|xa)_c x a$ becomes negligible. $(t|xx)_c x^2$ and $(t|aa)_c a^2$ aberrations become equal at a constant value, which can be explained by the mirror symmetry [10].

The terms $(x|a\delta)$ and $(a|x\delta)$ are connected via [9]:

$$(x|a\delta)(a|x) = (x|x)(a|a\delta) - (a|x\delta)(x|a) + (x|x\delta)(a|a). \quad (9)$$

Combining Eqs. (5, 7-9) we obtain:

$$(t|aa) + \beta^2(t|xx) = \frac{\lambda\beta}{\sin\mu} [(x|x\delta) + (a|a\delta)]. \quad (10)$$

On the other hand the relative natural chromaticity ξ_{1x} can be written as (see appendix):

$$\xi_{1x} = -\frac{1}{4\pi Q_{0x} \sin\mu} \cdot [(x|x\delta) + (a|a\delta)], \quad (11)$$

where Q_{0x} is the tune of the ring. Therefore, combining Eqs. (10, 11) one gets:

$$(t|aa) + \beta^2(t|xx) = -4\pi\lambda\beta Q_{0x} \xi_{1x}. \quad (12)$$

The β is constant at the symmetry point and due to $\alpha^{\text{Twiss}} = 0$ it can be expressed as $\beta = x/a$ and $\pi\varepsilon_x = \pi x a$. Then Eq. (12) can be written as:

$$(t|aa)a^2 + (t|xx)x^2 = -4\pi\lambda\varepsilon_x Q_{0x} \xi_{1x}. \quad (13)$$

In general, due to the mirror symmetry two chromatic matrix coefficients coincide in the achromatic ring: $(x|x\delta) = (a|a\delta)$ [11]. The natural chromaticity can be corrected using one family of sextupole magnets and therefore, we can reach a regime where the $(t|xx)$ and $(t|aa)$ contribution vanishes and in the limit of many turns the isochronous ring turns into a second-order achromatic system [12, 13].

Monte-Carlo Simulation

To check the analytic derivation described above a dedicated Monte-Carlo simulation has been performed with the program MOCADI [14]. In this program an ion-optical system is described by third-order transfer matrices which have been calculated with the GICOSY code [15].

In the simulations we have used a beam of 10^5 particles of one species with transverse emittance of 100 mm mrad

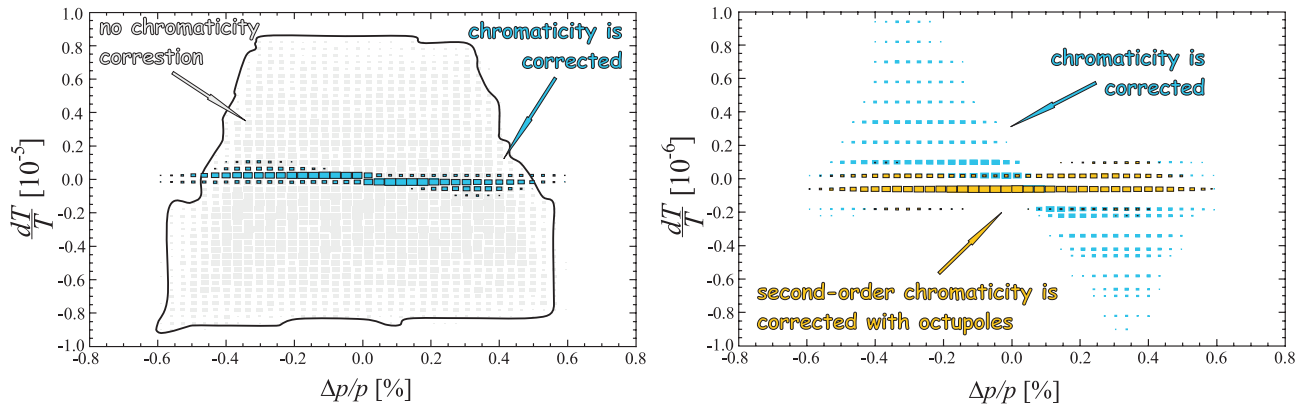


Figure 2: The revolution time spread as a function of the momentum deviation.

in both planes circulating 100 turns in the CR. The dependence of the revolution time spread on the momentum deviation in the ring has been calculated.

At first, we have corrected only the second-order isochronicity with one sextupole family (grey distribution in Fig. 2). Then, the natural chromaticity $\xi_{1x,y}$ with two additional sextupole families has been corrected (blue distribution in Fig. 2).

Moreover, applying the octupole magnets in the same manner as sextupoles, we have corrected third-order isochronicity ($(t|\delta\delta\delta) = 0$) and second-order chromaticity ($\xi_{2x,y}$) with three octupole families (yellow distribution in Fig. 2).

The broad grey time distribution in the left picture of Fig. 2 is governed by the large transverse emittance and corresponds to about $1.7 \cdot 10^{-5}$ in time resolution (or about $2 \cdot 10^{-4}$ of mass resolving power). By correcting in addition the natural chromaticity (blue distribution in the left picture of Fig. 2) the influence of the transverse motion is significantly reduced and a resolution of up to $dT/T \approx 2 \cdot 10^{-6}$ (or $m/\Delta m \approx 2 \cdot 10^5$) in the full momentum acceptance range can be achieved. Applying in addition the octupole correction (see the yellow distribution on the right picture in Fig. 2) one can reach a resolution of up to $dT/T \approx 3 \cdot 10^{-7}$, which corresponds to the mass resolving power of about 10^6 .

Appendix

The phase advance μ can be expanded with respect to δ as [16]:

$$\mu = \sum_{n=0}^{\infty} \mu_n \delta^n, \quad \text{or} \quad \cos \mu = \sum_{n=0}^{\infty} \chi_n \delta^n. \quad (14)$$

Since $\cos \mu$ is the trace of the transfer matrix M (see Eq. 8), which can also be expanded with respect to δ one can obtain $\chi_n = 0.5 \cdot \text{Tr}(M_n)$, where M_n is:

$$M_n = \begin{pmatrix} (x|x\delta^n) & (x|a\delta^n) \\ (a|x\delta^n) & (a|a\delta^n) \end{pmatrix}, \quad n = 0, 1, 2, 3, \dots \quad (15)$$

Thus, from Eqs. (14,15) one can extract the phase advance described in matrix coefficients and correspondingly obtain the chromaticity:

$$\xi_x = \frac{1}{\delta} \cdot \frac{\Delta Q}{Q_0} = \frac{\Delta \mu}{2\pi \delta Q_0} = \frac{1}{2\pi Q_0} \sum_{n=1}^{\infty} \mu_n \delta^{n-1}. \quad (16)$$

REFERENCES

- [1] A. Dolinskii et al., EPAC Proceedings, 572-574 (2002).
- [2] FAIR Baseline Technical Report, GSI, Darmstadt, 2006, <http://www.fair-center.de>
- [3] H. Geissel et. al., NIM B204, 71-85 (2003).
- [4] M. Hausmann et. al., NIM A 446, 569-580 (2000).
- [5] A. Dolinskii et al., NIM A 574, 207-212 (2007).
- [6] D. C. Carey, "The optics of charged particles beams", Harwood Academic Publisher, New York (1987).
- [7] M. Yavor, "Optics of Charged Particle Analyzers", Advances in Imaging and Electron Physics vol. 157, Elsevier Inc., 2009.
- [8] H. Wiedemann "Particle Accelerator Physics I (Basic Principles and Linear Beam Dynamics)", Springer-Verlag, Berlin, Heidelberg, New York (1998).
- [9] H. Wollnik, "Optics of charged particles", Academic Press, Florida (1987).
- [10] M. Yavor, Private Communications.
- [11] B. Erdelyi et al., Physical Review Special Topics - Accelerators and Beams 10, 064002 (2007).
- [12] K. L. Brown, PAC Proceedings, 3490-3492 (1979).
- [13] D. C. Carey, NIM 189, 365-367 (1981).
- [14] N. Iwasa et al., NIM B 126, 284-289 (1997), <http://www-linux.gsi.de/~weick/mocadi/>
- [15] H. Wollnik et al., AIP Proceedings, 177, 74-85 (1988), <http://www-linux.gsi.de/~weick/gicosy/>
- [16] M. Takao et al., Physical Review E 70, 016501 (2004).